

H-C and H*-C Semi compactness in bitopological-space

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Abstract: In this paper we introduce two new concepts, namely H-C-Semi compact and H*-C-Semi compact in bitopological space several propositions and examples about these concepts are introduced.

Key words: H-C , H*-C , Semi compactness , bitopological-space

Introduction
Let X be a non empty set. Let T1 and T2 be two topologies on X then the triple (X,T1,T2) is called a bitopological space, this concept was first introduced by Kelly [1]. In this work, we introduce new concepts namely H-C-Semi compact and H*-C semi compact in bitopological space.

Preliminaries
2-1 Remarks
i) If T1 is a topology on X and T2 is also a topology on X then T1 \cup T2 is not necessarily a topology on X.
ii) (T1 \cup T2) means the topology on X generated by T1 \cup T2.

Definition: [2]
Let (X, T1, T2) be a bitopological-space let A \subseteq X, we say that A is N-open if and only if is open in the space (X, T3) where T3 = T1 \cup T2.

Remarks and examples about these concepts are introduced.

3-H-C-Semi-compact-space
In this section, we introduce the concept of H-C-Semi-compact space several properties of this concept are proved.

First, we introduce the following definition:

Definition
Let (X, T1, T2) be a bitopological-space let A \subseteq X, we say that A is H-semi open in X iff it is semi-open in the space (X, T1 \cup T2).

Remarks and examples
i) The complement of H-open (S-open) is called N-closed (S-closed).
ii) Every N-open is H- semi-open but the converse is not necessarily true.
iii) X=(a, b, c)
T1={Ø, X, {a}}
T2={Ø, X, {b}}
(T1 \cup T2)={Ø, X, {a}, {b}, {a,b}}.
\{a,b\} is N-open but not S-open.
\{a\} is S-open, hence it's also N-open.
\{a, b\}c = \{c\} N-closed.
\{a\}c = \{b, c\} S-closed.
Definition [2], [3]
A bitopological-space (X, T1, T2) is called N-compact (S-compact) if every N-open cover (S-open cover) of X has a finite subcover.

Definition [2], [3]
Let (X, T) be a topological-space we say that X is C-compact if: for each closed set A \subseteq X and each open cover
F = \{W\alpha | \alpha \in \Omega \} of A, there exists \alpha_1, \alpha_2, ...., \alpha_n such that A \subseteq \bigcup \alpha_1 \cup \bigcup \alpha_2 \cup .... \bigcup \alpha_n (that is, there exist a finite sub family whose closures covers A).

3-H-C-Semi-compact-space
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First, we introduce the following definition:

Definition
Let (X, T1, T2) be a bitopological-space let A \subseteq X, we say that A is H- semi-open in X iff it is semi-open in the space (X, (T1 \cup T2)).
Let $A \subseteq X$

$F = \{ W^\alpha \mid \alpha \in \Omega \} \text{ is called H-semi – open cover of } A$ if

1. $W^\alpha$ is H- semi-open in $X$ for each $\alpha \in \Omega$
2. $A \subseteq \bigcup_{\alpha \in \Omega} W^\alpha$

Definition

i) A bitopological space $(X, T_1, T_2)$ is called H-semi compact iff every H-semi open cover of $X$ has a finite sub cover

ii) Let $A \subseteq X$. We say that $A$ is H- semi compact iff every H-semi open cover of $A$ has a finite sub cover

Definition

Let $(X, T)$ be a topological-space, $X$ is called C-semi compact if:

- Given a semi–closed subset $A \subseteq X$ and given a semi – open cover
- $F = \{ W^\alpha \mid \alpha \in \Omega \} \text{ of } A$
- Then there exist $\alpha_1, \alpha_2, \ldots, \alpha_n$ Such that $A \subseteq \bigcup_{\alpha_i \in \Omega} W^\alpha_i$

Definition

Let $(X, T)$ be abitopological space we say that $X$ is H-C-semi compact if given $H$ – semi compact set $A \subseteq X$ and given $F = \{ W^\alpha \mid \alpha \in \Omega \}$ where $F$ is an $H$ – semi open cover of $A$

- then there exist $\alpha_1, \alpha_2, \ldots, \alpha_n$ such that $A \subseteq \bigcup_{\alpha_i \in \Omega} (H - scl W^\alpha_i)$

Proposition

Every H- semi closed sub set of H-semi compact space is H-semi compact

Proof

Let $(X, T)$ be H-semi compact and let $A \subseteq X$ be H-semi closed subset of $X$

- $F = \{ W^\alpha \mid \alpha \in \Omega \} \text{ be an H-semi open cover of } A$

Now $A$ is H-semi closed, so $AC = X-A$ is H - semi open

Now $F^* = F \cup \{ A \}$ is an H-semi open cover of $X$, but $X$ is H-semi compact so $\exists \alpha_1, \alpha_2, \ldots, \alpha_n$

such that $X = W^\alpha_1 \bigcup W^\alpha_2 \bigcup \ldots \bigcup W^\alpha_n \bigcup A$

Hence $A \subseteq W^\alpha_1 \bigcup W^\alpha_2 \bigcup \ldots \bigcup W^\alpha_n \bigcup A$

i) Every compact space is C-compact

ii) Every semi compact space is C-semi compact

iii) Every H-semi compact space is H-C- semi compact

iv) the converse of (iii) is not necessarily true . as show by the following example.

Let $(N, T_1, T_2)$ be a bitopological space where 

$N = \{ N, \emptyset \}$

where $F = \{ W_n \mid W_n = \{ 1, 2, \ldots, n \}, n \in N \}$

Now $(N, T_1, T_2)$ is H- C- semi compact but not H-semi compact

3.9 Proposition

Let $(X, T_1, T_2)$ be H-C-semi compact then $(X, T_1)$ and $(X, T_2)$ are C- semi compact space.

Proof

Let $A \subseteq (X, T_1)$ be semi closed and let $F = \{ W^\alpha \mid \alpha \in \Omega \}$ be a $T_1$ semi open cover of $A$. 

Now $A$ is H-semi compact subset of $(X, T_1, T_2)$ and $F$ is an H-semi open cover of $A$

But $(X, T_1, T_2)$ is H – C- semi compact so $\exists \alpha_1, \alpha_2, \ldots, \alpha_n$

such that $A \subseteq (H - scl W^\alpha_1) \bigcup (H - scl W^\alpha_2) \bigcup \ldots \bigcup (H - scl W^\alpha_n)$.

Now $H - scl W^\alpha_1 \subseteq T_1 - scl W^\alpha_1$

$H - scl W^\alpha_1 \subseteq T_1 - scl W^\alpha_n$

So $A \subseteq (T_1 - scl W^\alpha_1) \bigcup \ldots$

$(T_1 - scl W^\alpha_n)$

So $(X, T_1)$ is C-semi compact

Similarly we prove that $(X, T_2)$ is C- semi compact

3.10 Remark

The converse of proposition (3-9) is not necessarily true as shown in the following

Example

Let $(N, T_1, T_2)$ be a bitopological space, let $T_1 = P(0+) \bigcup \{ N \}$ and $T_2 = P(0+) \bigcup \{ N \}$ where $P(O+) \text{ is the power set of } O+ \text{ is set of all odd natural numbers and } P(E+) \text{ is the power set of } E+ \text{ of all even natural numbers then } (N, T_1)$ and $(N, T_2)$ are C-semi compact but $(N, T_1, T_2)$ is not H-semi compact space

3.11 Definition

Let $f : (X, T_1, T_2) \rightarrow (Y, T_1, T_2)$ be a function, we say that $f$ is H- semi continuous if the inverse image of H- semi open set in $Y$ is H- semi open in $X$

3.12 Proposition:

The H- semi continuous image of an H-C-semi compact space is also H-C- semi compact
Proof:
Let \((X, T_1, T_2)\) be \(H\)-C semi compact we have to prove that \((Y, T_1, T_2)\) is also \(H\)-C semi compact

Let \(A \subseteq Y\) be \(H\)-semi closed Now \(B = f^1(A)\) is \(H\)-semi closed in \(X\)

Let \(F = \{ \{ W \alpha | \alpha \in \Omega \} \}\) be an \(H\)-semi open cover of \(Y\)

Hence \(F^* = \{ f^1 ( \{ W \alpha | \alpha \in \Omega \} ) \}\) is an \(H\)-semi open cover of \(B = f^1(A)\)

But \(X\) is \(H - C\) semi compact

So \(\exists \alpha_1, \alpha_2, \ldots, \alpha_n \in \Omega\)

\[ B \subseteq (H - scl f^1 W \alpha_1 ) \]

\[ \ldots \cup (H - scl f^1 W \alpha_n ) \]

Hence \(Y\) is \(H\)-C semi compact

4- \(H^*\)-C compact space

In this section we introduce the concept of \(H^*\)-C compact space

Definition

A sub set \(A\) of bitopological space \((X, T_1, T_2)\) is said to be \(H^*\)- semi open if it is \(T_1\)-semi open or \(T_2\) semi open

The complement of \(H^*\)- semi open set is called \(H^*\)- semi closed

Definition

Let \((X, T_1, T_2)\) be abitopological space , let \(A \subseteq X\)

A sub collection of the family \(T_1 \cup T_2\) is called \(H^*\)- semi open cover of \(A\) if the union of members of this sub collection contains \(A\).

Definition

A bitopological space \((X, T_1, T_2)\) is said to be \(H^*\)-semi compact if every \(H^*\)-semi open cover of \(X\) has finite sub cover.

4-4 Definition

A bitopological space \((X, T_1, T_2)\) is said to be \(H^*\)-C compact if give \(H^*\)-semi closed set \(A \subseteq X\)

Given \(F = \{ W \alpha | \alpha \in \Omega \}\)

Which is \(H^*\)-semi open cover of \(A\) then \(\exists \alpha_1, \alpha_2, \ldots, \alpha_n\) such that

\[ A \subseteq (H^*\text{-scl} W \alpha_1 \cup \ldots \cup (H^*\text{-scl} W \alpha_n) \]

The proof of the following propositions is similar to the previous one.

Proposition

Every \(H^*\)- semi closed subset of \(H^*\)-semi compact space is \(H^*\)-semi compact.

Proposition

Every \(H^*\)- semi compact space is \(H^*\)- C semi compact.

Proposition

Let \((X, T_1, T_2)\) be an \(H^*\)- C semi compact space, then \((X, T_1)\) and \((X, T_2)\) are both \(C\)- semi Compact

Example

Let \((X, T_1, T_2)\) be a bitopological space where \(X = [0,1]\) and

\[ T_1 = \{ X, \emptyset, \{0\} \cup \{0, n \} | n \in \mathbb{N} \} \]

\[ T_2 = \{ X, \emptyset, \{0\} \cup \{n, 1\} | n \in \mathbb{N} \} \]

Then \((X, T_1)\) and \((X, T_2)\) are \(C\)-semi compact but \((X, T_1, T_2)\) is not \(H^*\)- C semi compact

Remark

Let \((X, T_1, T_2)\) be a bitopological space and let \(A \subseteq X\) then

i) \(H\text{-scl} A \subseteq H^*\text{-scl} A \)

ii) \(H\text{- scl} A \subseteq T_1\text{- scl} A \)

iii) \(H\text{- scl} A \subseteq T_2\text{- scl} A \)

The proof of the following proposition is clear

4-10 proposition

Every \(H\)-C semi compact space is \(H^*\)-C semi compact

References


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