Strongly Semiopen Sets In Intuitionistic Special Fuzzy Topological Spaces
Hanna H. Alwan
Education Dyalaa, Dyalaa, Iraq
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Abstract
The aim of this paper is generalize the concepts of fuzzy strongly semi-open sets, and fuzzy strongly semicontinuous due to Bai shi Zhong [2 ] to intuitionistic fuzzy special topological space. And study the relation between strongly semi-open with semi-open and pre-open sets. Some of properties are studied using these concepts. We investigate several characterizing theorems.

1. Introduction
After the introduction of fuzzy set by Zadeh there have been number of generalization of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov is one among them. Using the notion of intuitionistic fuzzy sets Coker [3 ] introduced the notion of intuitionistic fuzzy topological spaces [4 ]. The concept is used to define intuitionistic fuzzy special set by Coker [7 ] and intuitionistic fuzzy special topological spaces are introduced.

Here in section 3 of this paper we introduce strongly semi-open and strongly semi-closed sets establish some of their properties we also discuss the relations between the open set, semi-open, pre-open sets and strongly semi-open sets. In this work, non-empty sets will be denoted by X,Y, etc. Intuitionistic fuzzy special sets of X denoted by A,B, C, etc. intA , clA , A , will denote respectively the interior, closure, and complement of the set A.

2. Preliminaries
First we shall present the fundamental definitions
Definition 2.1 [8]
Let X be a nonempty set. An intuitionistic fuzzy special set A is an object having the form A = \{x, A_1, A_2\}, where A_1 and A_2 are subsets of X satisfying A_1 \cap A_2 = \emptyset . The set A_1 is called the set of members of A , while A_2 is called the set of nonmembers of A.

Definition 2.2 [7]
Let X be a nonempty set and the intuitionistic fuzzy special set A and B be in the form A = \{x, A_1, A_2\}, B = \{x, B_1, B_2\} furthermore. Let \{A_i: i \in J\} be an arbitrary family of intuitionistic fuzzy special sets in X where \ A_i = \{x, A_i^{(1)}, A_i^{(2)}\}.

1. A \subseteq B \iff A_i \subseteq B_i \& B_2 \subseteq A_2.
2. A = B \iff A \subseteq B \& B \subseteq A.
3. The complement of A is denoted by \overline{A} and defined by \ A = \{x, A_2, A_1\}.
4. \bigcup A_i = \{x, \bigcup A_i^{(1)}, \bigcap A_i^{(2)}\}, \bigcap A_i = \{x, \bigcap A_i^{(1)}, \bigcup A_i^{(2)}\}.
5. \overline{F} = (x, \emptyset, X), \overline{A} = (x, A, \emptyset).

Definition 2.3 [7]
An intuitionistic fuzzy special topology on a nonempty set X is family T of intuitionistic fuzzy special sets in X containing \ \overline{F}, \overline{A} and closed under finite infima and arbitrary suprema in this case the pair (X,T) is called an intuitionistic fuzzy special topological space and any intuitionistic fuzzy special set in T known open set in X.

From now the word space means an intuitionistic fuzzy special topological space.

Definition 2.4
Let (X,T) be a space and let A be an intuitionistic fuzzy special set of X. Then A is called:
1. An intuitionistic fuzzy semi-open set (SOS, for short) iff A \subseteq cl(int(A)) [1].
2. An intuitionistic fuzzy pre-open set (POS, for short) iff A \subseteq int(cl(A)) [1].
3. An intuitionistic fuzzy semi-close set (SCS for short) iff cl(int(A)) \subseteq A. [6]
4. An intuitionistic fuzzy pre-closed set (PCS, for short) iff cl(cl(A)) \subseteq A. [5]

From now we denoted SO(X) to the family of intuitionistic fuzzy semi-open sets of a space (X,T).

Definition 2.5 [2]
Let A be an intuitionistic fuzzy special set of a space (X,T). Then
1. sintA = \bigcup \{B: B \subseteq A \& B \in SO(X)\} is called the semi-interior of A.
2. sclA = \bigcap \{B: A \subseteq B \& B \in SC(X)\} is called semi-closure of A.

Lemma 2.6 [3]
For any intuitionistic fuzzy special set A of a space (X,T),
1. int \overline{A} = \overline{\text{int } A}
2. cl \overline{A} = \overline{\text{cl } A}

Lemma 2.7 [2]
For any family \{A_x\} of intuitionistic fuzzy special sets of a space (X,T),
1. U cl A_x \subseteq cl(U A_x)
2. U int A_x \subseteq int(U A_x)
3. intuitionistic fuzzy special strongly semi-open and strongly semi-closed sets

Definition 3.1
Let A be an intuitionistic fuzzy special set of a space (X,T). Then A is called:
1. Intuitionistic fuzzy special strongly semi-open set of X iff there is open set B \in T such that B \subseteq A \subseteq cl(cl(B)).
2. Intuitionistic fuzzy special strongly semi-closed set of X iff there is closed set B in X such that cl(in(B)) \subseteq A \subseteq B.

Note
We denote SSO(X) to the family of strongly semi-open set of a space (X,T).
The following remarks conclusion from definition 3.1

**Remarks 3.2**
1. Every open set is strongly semi-open set.
2. Every strongly semi-open set is semi-open set.
3. Every strongly semi-open set is pre-open set.

Now the following examples show that the converse of remarks 3.2 is not true in general.

**Example 3.3**
Let $X=\{a,b,c\}$, $T=\{\emptyset,A\}$, where $A=\{a\}, \{b\}$
And let $B=\{a\}$

Let $B$ be pre-open set since $B \subseteq int(cl(B)) = X$. But $B$ is not strongly semi-open since $A \not\subseteq int(cl(B))$, but $B \not\subseteq A$.

**Example 3.4**
Let $X=\{a,b,c\}$, $T=\{\emptyset,A,B,C\}$, where $A=\{a\}, \{b\}, \{c\}$
And let $D=\{a\}$

Let $D=\{a\}, \{b\}, \{c\}$.

$D$ is semi-open set since $cl(int(D))=\{a\}$. But $D \not\subseteq A \not\subseteq int(cl(D))$, so $D$ is not strongly semi-open set.

The following theorem give a characterization of strongly semi-open set.

**Theorem 3.5**
Let $A$ be an intuitionistic fuzzy special set of a space $(X,T)$, then $A$ is strongly semi-open set if and only if $A \subseteq int(cl(int(A)))$.

**Proof**
Let $A$ be strongly semi-open set, then there is open set $B$ such that $B \subseteq A \subseteq int(cl(B))$.

Hence $B \subseteq int(A)$ and $intclB \subseteq intclintA$, since $A \subseteq intclB$.

We have $A \subseteq int(cl(int(A)))$.

Let $B \subseteq int(A)$, then $B$ is open set.

By $A \subseteq int(cl(int(A)))$.

$B=\{a\} \subseteq A \subseteq int(cl(int(A)))=int(cl(B))$.

Therefore $B \subseteq A \subseteq int(cl(B))$.

Thus $A$ is strongly semi-open set.

**Remark 3.6**

The complement of strongly semi-open set is strongly semi-closed set and defined as $cl(int(cl(A))) \subseteq A$.

**Theorem 3.7**
1. Any union of intuitionistic fuzzy special strongly semi-open sets is strongly semi-open set.
2. Any intersection of intuitionistic fuzzy special strongly semi-closed sets is strongly semi-closed set.

**Proof**
We prove (1)
Let $A_x \in SSO(X)$. Then for each $x \in B_x$ open set such that $B_x \subseteq A_x \subseteq intcl B_x$.

By using lemma 1.9 we get $\bigcup B_x \subseteq \bigcup A_x \subseteq \bigcup intcl B_x \subseteq \bigcup cl(intcl B_x) \subseteq \bigcup cl((UB_x))$.

Since $\cup B_x$ is open set, it follows that $\cup A_x \in SSO(X)$.

In the same way we can prove (2).

**Remark 3.8**
The intersection (union) of any two intuitionistic fuzzy special strongly semi-open (semi-closed) sets need not to be strongly semi-open (semi-closed) set. Even the intersection (union) of strongly semi-open (semi-closed) set with open (closed) set may fail to be a strongly semi-open (semi-closed) set. This shown by the following example.

**Example 3.9**
Let $X=\{a,b,c\}$, $T=\{\emptyset,\{a\}, A, B\}$, where $A=\{a\}, \{b\}, \{a\}$

$B=\{a\}$

Let $B$ be open set and since every open set is strongly semi-open (Re.3.2) therefore $B$ is strongly semi-open.

$A \cap B = B$, hence $A \cap B$=

$int cl (A \cap B) = \emptyset$ and $A \cap B \not\subseteq \emptyset$.

Now the following examples show that the converse of remarks 3.2 is not true in general.

**Example 3.3**
Let $X=\{a,b,c\}$, $T=\{\emptyset, A\}$, where $A=\{a\}, \{b\}$

And let $B=\{a\}$

We have $B=\{a\} \subseteq A \subseteq int(cl(int(A)))=int(cl(B))$.

Therefore $B \subseteq A \subseteq int(cl(B))$.

Thus $A$ is strongly semi-open set.
References

المجموعات شبه المفتوحة القوية في الفضاءات التبولوجية المضببة الحدسية الخاصة

هلا حسين علوان

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الملخص

أن الهدف من هذا البحث هو تعليم مفاهيم المجموعات شبه المفتولة (شبه المغلقة) القوية المضببة في المصدر (1) إلى الفضاءات التبولوجية المضببة الحدسية الخاصة.

وقد تم دراسة العلاقة بين المجموعات شبه المفتولة القوية والمجموعات شبه المفتولة القوية ودراسة بعض الخواص لهذه المفاهيم و لتحقيق معظم النظريات المكافئة.