WAVELET-BASED FILLING-IN IN A DECOMPOSITION SPACE OF
REGULARLY OR IRREGULARLY SAMPLED IMAGE*

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Abstract

Wavelet-Network (W N) model has been very recently proposed and applied to
image processing. In this paper a general model is proposed for the reconstruction of the
lost blocks of the irregularly sampled image. The proposed model is fast and it uses the
correlation between the lost block and its neighbors. An algorithm is introduced which
uses first thresholding to determine the presence or absence of edges in the lost block.
The interpolation scheme minimizes the square of the error between the border
coefficients of the lost block and those of its neighbors. The proposed method is able to
improve image quality in terms of both visual perception and image fidelity. The
architecture of the proposed neural network is fast forward multilayer perceptron and
in it’s learning it uses the resilient back-propagation training. The reconstruction error
is expressed in terms of (PSNR) When it is viewed on a monitor, there is virtually no
difference between the original and the image reconstructed using the proposed method.

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Introduction

Various channel/network errors will result in damage or loss of compressed video packages. Reliable reception of still images is imperative in providing robust multimedia transmission, such as ATM and other packet-based networks, satellite channels, and wireless networks. In addition, the industrial standard, developed by the joint photographic expert group (JPEG) is based on Discrete cosine Transform (DCT) applied to sub blocks in an image, this method is unacceptable because it introduces blocking artifacts due to independent errors across block boundaries [1]. Hence, the ringing effect is considered the major artifact around edges.

However, besides the ringing effect, edges have also been blurred by the low pass filtering effect introduced by the allocation of zero bit to high-frequency coefficient, several techniques have been proposed in the past ten years. Some techniques are rather intuitive, such as pixel Domain Interpolation, [2] Multi-directional Interpolation [3], etc.

Error concealment is an ill-posed problem, which has no unique recovery of the lost information. Many post-processing schemes have been developed, among which most techniques attempt to remove the blocking effect [4]-[8], and some methods focus on the suppression of the ringing effect [9] or both artifacts [10]-[13].

In this paper, our approach applies a general model to reconstruct the lost blocks of the irregularly sampled Image. Hence, fast schemes for wavelet-domain interpolation of irregularly lost blocks are presented.

We reconstruct the lost block in the wavelet domain using the correlation between the lost block and its neighbors. The proposed method is able to improve image quality in terms of both visual perception and image fidelity; a neural network (NN) classifier can handle problems of large dimension efficiency. Hence, the classifier is combined with the wavelet domain interpolation of irregularly lost blocks. The architecture of the proposed neural network is a fast forward multi-layer perceptron and in its learning it uses the resilient backpropagation training. It will be seen that computation results is very closer to that normalized desired input-output values.

Wavelet Decomposition
Wavelet transform can decompose images into various multiresolution subbands. It has been extensively used in various fields including texture analysis [14], VLSI architecture [15], image coding [16,17,18]. This is because it can decompose the image into several multiresolution subbands and perfectly reconstruct the original image from them. It has been used in subband image coding [16,17,18]. Mallat [19] developed the multiresolution architecture which is very suitable for image analysis.

Fig.1 shows, an image called (Pout: 296 x 240), while Fig.2 shows, a 10–subbands decomposed by two-dimensional wavelet transform with the filter proposed by Haar.
Fig. 1 An image called (Pout: 296 X 240)
Fig.2 An image called (Pout:296 X 240) is decomposed to 11-subbands with 2-dimensional 3-level wavelet transform.

Fig 3 and 4 show, the notation and the data structure for the subbands. The original image of 296 X 240 is decomposed into 4 sub-images of 148 X 120 in the subband decomposition of level -1. They are LL, LH, HL, and HH bands. The sub-image of the LL band is the coarse image of the original image.

Similarly, the LL band is decomposed into the LL, LH, HL, and HH bands of 74 X 60 in level-2. Finally, the LL band in level 2 is decomposed into the LL, LH, HL, and HH bands of 37 X 30 in level - 3. Therefore, the LL band in level - 3 is coarser than the LL band in level - 2 and so on. The edge in level - 3 is the coarsest edge and the edge in level - 1 is the finest edge.
Fig- 3 The notation for the corresponding sub-bands

Fig- 4 The data structure for the corresponding sub-bands

The Reconstruction Method
To recover the lost edge information, we test the missing blocks to see whether the block contains part of an edge or not. Fig -5, shows an example of a vertical edges in $V_1$ level.

A missing substream affects the 2X2 coefficients of the missing blocks, and tests whether the values are greater than certain threshold value. If any of the four tests meets the condition, then we decide the missing block is in part of vertical edge.

The coefficients left and right of the missing block are needed to recover horizontal detail, and diagonal neighboring coefficients are needed to recover the diagonal detail. For each high frequency subband, the matrix estimate of lost coefficients is given by:

$$X = W_1 A + W_2 B$$  \hspace{1cm} (1)
where, A and B are the adjacent coefficients, and \( W_1 \) and \( W_2 \) are scalar parameters. In the vertical edge case, A and B can be T (Top) and B (Bottom) respectively. Then, we need to find \( W_1 \) and \( W_2 \) where the error at the top and bottom borders is minimized.

**The Vertical, Horizontal and Diagonal Edgy Blocks**

For level \( V_0 \), X, T, B consist of only one coefficient, and hence X is the average of T and B. While, for level \( V_1 \), X, T, B are 2X2 matrices. Fig -6 will illustrate 2X2 lost blocks reconstruction.

\[
\begin{array}{c}
\text{t}_1 & \text{t}_2 \\
\begin{array}{c}
\text{t}_3 \rightarrow \\
\text{t}_4 \rightarrow \\
T_b \\
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\text{x}_1 & \text{x}_2 & \text{X}_t \\
\begin{array}{c}
\text{x}_3 \rightarrow \\
\text{x}_4 \\
X_b \\
\end{array}
\end{array}
\]

\[
\begin{array}{c}
\text{b}_1 & \text{b}_2 & \text{B}_t \\
\begin{array}{c}
b_3 \rightarrow \\
b_4 \\
\end{array}
\end{array}
\]

Lost – Block X
Fig -6 shows the V₁-level lost block reconstruction

where , x₁ and x₂ are solved, as :-

\[ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = w_1 \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} + w_2 \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{2} \]

and squared error at the top border is:

\[ \varepsilon_t = \| X - T_b \|^2 \tag{3} \]

where, \( T_b = \begin{bmatrix} t_3 \\ t_4 \end{bmatrix} \)

Similarly we can solve both x₃ and x₄ by:

\[ x = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = w_1 \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} + w_2 \begin{bmatrix} b_3 \\ b_4 \end{bmatrix} \tag{4} \]

and the squared error at the bottom border is :-

\[ \varepsilon_b = \| X - B_t \|^2 \tag{5} \]

where, \( X_b = \begin{bmatrix} X_3 \\ X_4 \end{bmatrix} \) and \( B_t = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \)

Non – edgy Blocks Reconstruction

In this paper the lost-blocks are reconstructed by all types of interpolation details of all neighbors, such as vertical, horizontal and diagonal, for 8X8 tiling, the finest level is V₂, which is a 4X4 matrix, for horizontal interpolation details where the reconstruction is done by splitting the columns of 4 coefficients into two groups of two coefficients, each, as shown in the Fig(7).
Fig.(7) shows the 4X4-Horizontal type details of lost block reconstruction.

Hence,

\[
\begin{bmatrix}
X_4 \\
X_{12}
\end{bmatrix} = W_1 \begin{bmatrix}
R_4 \\
R_{12}
\end{bmatrix} + W_2 \begin{bmatrix}
L_4 \\
L_{12}
\end{bmatrix}
\]  \hfill (6)

\[
\begin{bmatrix}
X_4 \\
X_{16}
\end{bmatrix} = W_1 \begin{bmatrix}
R_8 \\
R_{16}
\end{bmatrix} + W_2 \begin{bmatrix}
L_8 \\
L_{16}
\end{bmatrix}
\]  \hfill (7)

\[
\begin{bmatrix}
X_1 \\
X_9
\end{bmatrix} = W_1 \begin{bmatrix}
R_1 \\
R_9
\end{bmatrix} + W_2 \begin{bmatrix}
L_1 \\
L_9
\end{bmatrix}
\]  \hfill (8)

\[
\begin{bmatrix}
X_3 \\
X_{13}
\end{bmatrix} = W_1 \begin{bmatrix}
R_5 \\
R_{13}
\end{bmatrix} + W_2 \begin{bmatrix}
L_5 \\
L_{13}
\end{bmatrix}
\]  \hfill (9)

Similarly:

\[
\begin{bmatrix}
X_3 \\
X_{11}
\end{bmatrix} = W_1 \begin{bmatrix}
R_3 \\
R_{11}
\end{bmatrix} + W_2 \begin{bmatrix}
L_3 \\
L_{11}
\end{bmatrix}
\]  \hfill (10)

And so on ....

For horizontal edgy lost blocks, Fig -8 will illustrate the location pixels
Fig.(8) shows 2X2:Horizontal type reconstruction.

\[
\begin{bmatrix}
L_1 \\
X_1 \\
R_1 \\
L_2 \\
X_2 \\
R_2 \\
L_3 \\
X_3 \\
R_3 \\
L_4 \\
X_4 \\
R_4
\end{bmatrix}
\]

\[
\begin{bmatrix}
L_w \\
X_b \\
X_t \\
R_b
\end{bmatrix}
\]

\[
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} = W_1 \begin{bmatrix} R_1 \\
R_2
\end{bmatrix} + W_2 \begin{bmatrix} L_1 \\
L_2
\end{bmatrix}
\]

(11)

Hence,

\[
\begin{bmatrix}
X_3 \\
X_4
\end{bmatrix} = W_1 \begin{bmatrix} R_3 \\
R_4
\end{bmatrix} + W_2 \begin{bmatrix} L_3 \\
L_4
\end{bmatrix}
\]

(12)

**Wavelet Network Based Proposed Lost Block Reconstruction Procedure:**

The following procedure is followed in this proposed reconstruction method using the Wavelet Network. To the knowledge of the author, there is no method proposed using this technique. Thus this is the first time that a reconstruction procedure is followed using Wavelet Network.

**Step-1** Referring to the previous image processing procedure, the goal of the proposed tested methods is to minimize the squared error between the lost block and its neighbors. In this paper a novel selection type of the surrounding neighbor coefficients is achieved. Whether vertical, horizontal, or diagonal reconstruction process the same nearest values of these neighbors pixels are obtained.

**Step-2** Start with the filling-in processes a decomposition space of any selected image; the estimated matrix of any lost coefficients is given by means of equation 1. Next apply
equations 2 up-to 5 to recognize the vertical edgy and non-edgy (2X2) or (4X4) matrix missing of blocks reconstruction. Equations 6 up-to 14 will be used to recognize the horizontal and diagonal (2X2) or (4X4) missing matrix of blocks reconstruction.

**Step-3** To recognize the correlation process of step-2 between the lost block and its neighbors; a fast scheme forward multilayer perceptron with resilient back-propagation training is used. The classifier (NN) is combined with the wavelet domain interpolation.

(a) - Initialize the scalar parameters (i.e., weights) $W_1$ and $W_2$ of equation 1, then update these weights after presenting each training of tested image to reach nearest lost blocks values that are closer to the desired values.

(b) - Apply a set input coefficients in a form of matrix to that classifier (NN). These, coefficients represent the interpolation parts for both missing and surrounding blocks that are put under testing method, while the missing blocks are put as a desired matrix.

(c) - Because the decomposition extracted coefficients are under transient gray levels, hence, the matrix coefficients are sometimes either positive or negative. Therefore, the sigmoid function is used.

(d) - Because the extracted coefficients will be in a high range value, the input and the desired coefficients matrices must be normalized. In addition it is necessary to initialize the scalar parameter (i.e., $\lambda$) of the sigmoid function.

**Step-4** Apply the adaptation techniques based on the LMS algorithms. The implementation of the above steps (step 3-b up-to 3-d) is done with starting at minimum suggestion errors.

**Step-5** In case of the missing matrix coefficients in form of (4X4) matrix, the split coefficients process is used. So, steps (3.a-3.d) and step-4 are repeated four times in each quadrant sub-tested image.

**Step-6** Arrange the updating reconstruction blocks of each iteration in a special matrix. Then, make a comparison with that desired one.

**Step-7** Evaluate the peak-signal-to-noise-ratio (PSNR) of each iteration for both missing and the surrounding pixels. Hence, [11].

$$\text{PSNR} = 10 \times \log_{10} \left[ \frac{\sum_{x=1}^{W} \sum_{y=1}^{H} [(p(x, y) - p^*(x, y))^2]}{WXH} \right]$$

_(13)_

**Example**

In this example an image namely $P_{out}$ of 240X296 pixels is considered. In order to simplify the application of the proposed method, first of all a block of 2X2 is assumed to be
lost from the given image with vertical, horizontal, and diagonal test. Fig.(9) shows the proposed block diagram of the missing blocks.

![Block Diagram](image)

Fig. (9) The proposed block diagram of the missing blocks in the form of sub-image decomposition
Table (1) gives the location of the missing blocks with its neighbors. The total coefficients are 10X10 pixels, which represent the third depth decomposition at the assigned location.

| Table (2) represents the processing image $P_{\text{out}}$ of 240x296; (I) first sub-image, 3-depth decomposition coefficients (I1) with a desired sub-Matrix coefficients and (8-neighbors) for each V, H, and D |

<table>
<thead>
<tr>
<th>Location</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>94 97 95 94 95 98 99 102 103 105</td>
<td></td>
</tr>
<tr>
<td>93 95 97 95 97 99 99 105 106 106</td>
<td></td>
</tr>
<tr>
<td>93 94 94 95 98 98 101 105 106 107</td>
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</tr>
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</table>

<table>
<thead>
<tr>
<th>Location</th>
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<tbody>
<tr>
<td>78 83 84 84 80 78 78 82 83 84</td>
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<tr>
<td>78 83 84 84 82 77 78 82 82 82</td>
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<tr>
<td>93 94 93 93 93 97 97 98 101 101</td>
<td></td>
</tr>
<tr>
<td>94 97 95 94 95 98 99 102 103 105</td>
<td></td>
</tr>
<tr>
<td>93 95 97 95 97 99 99 105 106 106</td>
<td></td>
</tr>
<tr>
<td>93 94 94 95 98 98 101 105 106 107</td>
<td></td>
</tr>
</tbody>
</table>

Table (3) represents the resultant output of (NN) with V1, H1, and D1 analysis procedure as a filling-in coefficients with a corresponding (PSNR).
<table>
<thead>
<tr>
<th></th>
<th>Vertical-Test</th>
<th>Horizontal-Test</th>
<th>Diagonal-Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Or1</td>
<td>87.04</td>
<td>96.98</td>
<td>86.78</td>
</tr>
<tr>
<td></td>
<td>87.02</td>
<td>87.06</td>
<td>86.89</td>
</tr>
<tr>
<td>PSNR1</td>
<td>95.9282</td>
<td>95.9573</td>
<td>95.9180</td>
</tr>
</tbody>
</table>
**Fig. (10)** shows an image called (clown: 200x320) with a simulated loss

![Level-1 Decomposition](image1)

![Level-2 Decomposition](image2)

![Level-3 Decomposition](image3)

**Fig. (11)** shows an image called (clown: 200x320) in the form of the second-quadrant sub-image, 3-depth decomposition with a simulation loss.
Fig. (12) shows an image called (clown: 200x320) Filling-in is in a decomposition space

Conclusions

In the classical method, the wavelet transform is used as a very efficient multi-resolutions modeling to translate the image into four small sub-images in each depth. These trees of images contain the split orthogonal features, which are distributed in this multi-resolutions multi-scale model.

1- The reconstruction of sub band coefficients iteratively generated lost coefficients to minimize the mean squared error between the correctly received coefficients and the coefficients resulting from the analysis of the synthesized reconstructed image. In general, the reconstructed LH and HL bands provide accurate information for edge placement.

2- Low frequency reconstruction performs well on horizontal, vertical, and strong diagonal edges. Hence, the accurate edge placement is achieved by adapting the interpolation grid in both the H, V, and the D directions as determined by the edges present.

3- The sub band decomposition provides natural framework through which relationships between low and high frequency sub bands can be used to characterize the low frequency signal for accurate edge reconstruction. In the case of reconstruction, coefficients so that edges are accurately synthesized are crucial to providing visually acceptable images. Where, Some low frequency coefficients are missing, so processing of the low frequency band cannot be used to accurately identify edge locations.

4- In the proposed method, any setting lost coefficients to zero produces an image with dark “holes” spread out to an extent determined by the number of decomposition levels.
and the filter length. High frequency loss without reconstruction may or may not be visible, depending on where the loss occurs. Better and the more accurate reconstruction is achieved when all coefficients used in interpolation and derivative estimation are known, and if vectors are staggered down rows or columns.

5- The performance of our proposed method was evaluated by reconstructing random loss of three coefficient groupings across all sub bands at all decomposition levels: single coefficients, vectors of length 4, and blocks of size 2X2. Hence, the image reconstruction consists of two parts: reconstruction of lost coefficients in the visually most important lowest frequency band and reconstruction of coefficients in all other bands.

6- The lost blocks are generated as a linear combination of available adjacent blocks, when the data matches the assumptions, the reconstructed image quality can be quite good. However, when the data does not match the assumptions, in the best case the reconstructed blocks may be visually acceptable but may differ greatly from the original blocks, where the randomly lost coefficients are most likely to have the highest number of known coefficients required in interpolation present. Conversely, blocks require the most coefficients estimation for use in interpolation process.

7- The proposed method has been tested in the context of intermediate view reconstruction where the reconstruction error will be expressed in terms of image fidelity as it is shown in the proposed tables. It is seen that the proposed method clearly outperforms the nearest-neighbors and the polynomial reconstructions. When viewed on a monitor, there is virtually no difference between the original and the reconstructed image.

References


