ON GENERALIZED ALMOST CONTRA CONTINUOUS FUNCTIONS AND SOME RELATIONS WITH ANOTHER KINDS OF CONTINUITY ON INTUITIONISTIC TOPOLOGICAL SPACES

Yunis J. Yaseen    Ali M. Jasim
College of Education - Tikrit University

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Abstract: We study in this paper the concept of almost contra continuous functions and generalized them in intuitionistic topological spaces and we studied the relations of each kind of these function by properties, examples and a diagram to summarize these functions. Also we study some relation between almost contra continuous function and some continuous functions.

Key words: ALMOST CONTRA CONTINUOUS FUNCTIONS, CONTINUITY ON INTUITIONISTIC TOPOLOGICAL SPACES

Introduction

Almost contra continuous functions were introduced by Joseph and Kwack [4], almost contra pre continuous function was introduced by Ekici [3]. So we are going generalized them on ITS's.

In this paper we investigate definitions of almost contra continuous, almost contra semi continuous, almost contra pre continuous, almost contra α continuous, almost contra β continuous, almost contra γ continuous, almost contra δ continuous, almost contra ζ continuous, almost contra η continuous, almost contra ξ continuous, almost contra η continuous, almost contra ξ continuous functions and we show the relations of each kind of these functions by topological space (ITS, for short), any IS in T is known as an intuitionistic open set (IOS, for short) in X. The complement of IOS is called intuitionistic closed set (ICS, for short), so the interior and closure of A are denoted by

\[ \text{int}(A) \] and \[ \text{cl}(A) \]

respectively and defined by

\[ \text{int}(A) = \bigcup \{ G \mid G \subseteq T \text{ and } G \subseteq A \} \] and \[ \text{cl}(A) = \bigcap \{ F \mid F \text{ is ICS in } X \text{ and } A \subseteq F \} \]

So \[ \text{int}(A) \] is the largest IOS contained in A, and \[ \text{cl}(A) \] is the smallest ICS contain A, a set A is called intuitionistic regular-closed set (IRCS, for short) if

\[ A = \text{cl}(A) \]

intuitionistic α-closed set (Iα-CS, for short) if

\[ \text{cl}_\alpha(A) \subseteq A \]

intuitionistic semi-closed set (ISCS, for short) if

\[ \text{int}_\alpha(A) \subseteq A \]

pre-closed set (IPCS, for short) if

\[ \text{cl}(A) \subseteq A \]

intuitionistic β-closed set (IBCS, for short) if

\[ \text{cl}(A) \subseteq A \]

intuitionistic ζ-closed set (Iζ-CS, for short) if

\[ \text{cl}(A) \subseteq A \]

intuitionistic η-closed set (Iη-CS, for short) if

\[ \text{cl}(A) \subseteq A \]
for short) if $\text{intclint} A \subseteq A$. The complement of IRCS (resp. $I_\alpha$CS, ISCS, IPCS and $I_\beta$CS) is called intuitionistic regular-open set (resp. intuitionistic $\alpha$-open set, intuitionistic semi-open set, intuitionistic pre-open set and intuitionistic $\beta$-open set) in $X$. (IROS, $I_\alpha$OS, ISOS, IPOS and $I_\beta$OS, for short), $A$ is said to be intuitionistic semi-regular set (ISRS, for short) [6] if $A$ is ISOS and ISCS in $X$, so $A$ is called intuitionistic B-set (IBS, for short) [6] if $A$ is the intersection of an IOS and ISCS and $A$ is said to be an intuitionistic $\theta$-closed set ($I_\alpha$CS, for short) if $A = \text{cl}_\theta A$, where $A = \{x \in X : \text{cl}(U) \cap A = \emptyset, U \in T \text{ and } x \in U\}$.

A is called intuitionistic $\theta$ generalized-closed set ($I_\alpha$g-closed for short) if $\text{cl}_\theta A \subseteq U$, whenever $A \subseteq U$ and $U$ is IOS.

3. Generalized almost contra continuous functions on ITS's.

The definitions of almost contra continuous functions which appears in general topology by [2],[5] and [6], so we generalized them on ITS's.

Definition 3.1. Let $(X,T)$ and $(Y,\mathcal{U})$ be two ITS's and let $f: X \to Y$ be a function then $f$ is said to be:

An intuitionistic almost contra continuous (I almost contra cont., for short) function if the inverse image of each IROS in $Y$ is ICS in $X$.

An intuitionistic almost contra semi-continuous (I almost contra semi-cont., for short) function if the inverse image of each IROS in $Y$ is ISCS in $X$.

An intuitionistic almost contra pre-continuous (I almost contra pre-cont., for short) function if the inverse image of each IROS in $Y$ is IPCS in $X$.

An intuitionistic almost contra $\theta$-continuous (I almost contra $\theta$-cont., for short) function if the inverse image of IROS in $Y$ is $\theta$CS in $X$.

An intuitionistic almost contra $\alpha$-continuous (I almost contra $\alpha$-cont., for short) function if the inverse image of each IROS in $Y$ is $I_\alpha$CS in $X$.

An intuitionistic almost contra $\beta$-continuous (I almost contra $\beta$-cont., for short) function if the inverse image of each IROS in $Y$ is $I_\beta$CS in $X$.

An intuitionistic almost contra $\gamma$-continuous (I almost contra $\gamma$-cont., for short) function if the inverse image of each IROS in $Y$ is $I_\gamma$CS in $X$. (For short) if $f$ is said to be an intuitionistic almost contra $\sigma$ cont. (resp. almost contra $\theta$-cont., almost contra $I_\alpha$CS-cont., almost contra $I_\beta$CS-cont., almost contra $I_\gamma$CS-cont., almost contra $I_\delta$CS-cont., almost contra $I_\varepsilon$CS-cont., and almost contra $I_\zeta$CS-cont. functions if the inverse image of each IROS in $Y$ is $I_\sigma$-closed (resp. $I_\theta$-closed, $I_\alpha$-closed, $I_\beta$-closed, $I_\gamma$-closed, $I_\delta$-closed, $I_\varepsilon$-closed, $I_\zeta$-closed and $I_\zeta$-closed) set in $X$.

Proposition 3.3. Let $(X,T)$ and $(Y,\mathcal{U})$ be two ITS's and let $f: X \to Y$ be a function then:

1- If $f$ is I almost contra cont. function then $f$ is I almost contra $g$ cont. function.

2- If $f$ is I almost contra $\alpha$ cont. function then $f$ is I almost contra cont. function.

3- If $f$ is I almost contra $\sigma$ cont. function then $f$ is I almost contra semi-cont. function.

4- If $f$ is I almost contra semi-cont. function then $f$ is I almost contra pre-cont. function.

5- If $f$ is I almost contra pre-cont. function then $f$ is I almost contra $g$ cont. function.

6- If $f$ is I almost contra $\alpha$ cont. function then $f$ is I almost contra $g$ cont. function.

7- If $f$ is I almost contra semi-cont. function then $f$ is I almost contra $\beta$ cont. function.

8- If $f$ is I almost contra $\beta$ cont. function then $f$ is I almost contra semi-cont. function.

9- If $f$ is I almost contra semi-cont. function then $f$ is I almost contra $\sigma$ cont. function.

10- If $f$ is I almost contra $\sigma$ cont. function then $f$ is I almost contra $g$ cont. function.

11- If $f$ is I almost contra $\sigma$ cont. function then $f$ is I almost contra $\alpha$ cont. function.

12- If $f$ is I almost contra $\alpha$ cont. function then $f$ is I almost contra $g$ cont. function.

13- If $f$ is I almost contra $g$ cont. function then $f$ is I almost contra $\alpha$ cont. function.

14- If $f$ is I almost contra $g$ cont. function then $f$ is I almost contra $\sigma$ cont. function.
15- If \( f \) is I almost contra \( g \)-cont. function then \( f \) is I almost contra \( g \)-cont. function.

16- If \( f \) is I almost contra sg-cont. function then \( f \) is I almost contra gs-cont. function.

17- If \( f \) is I almost contra pg-cont. function then \( f \) is I almost contra gp-cont. function.

18- If \( f \) is I almost contra pre-cont. function then \( f \) is I almost contra g-cont. function.

19- If \( f \) is I almost contra g-cont. function then \( f \) is I almost contra pre-cont. function.

20- If \( f \) is I almost contra \( g \)-cont. function then \( f \) is I almost contra \( g \)-cont. function.

21- If \( f \) is I almost contra \( g \)-cont. function then \( f \) is I almost contra \( g \)-cont. function.

22- If \( f \) is I almost contra g-cont. function then \( f \) is I almost contra g-cont. function.

23- If \( f \) is I almost contra g-cont. function then \( f \) is I almost contra g-cont. function.

24- If \( f \) is I almost contra g-cont. function then \( f \) is I almost contra g-cont. function.

Proof:

We are give the proof of (21) as example and others can be proved in a similar way.

Let \( V \) be IROS in \( Y \) then \( f^{-1}(V) \) is I closed set in \( X \) (since \( f \) is I almost contra \( g \)-cont. function). So for each IOS \( A \) in \( X \) and \( f^{-1}(V) \subseteq A \) then \( \text{osl}(f^{-1}(V)) \subseteq A \). Now since every ICS is ISCS then \( \text{osl}(f^{-1}(V)) \subseteq f^{-1}(V) \subseteq A \). So we have that for each IOS \( A \) in \( X \) and \( f^{-1}(V) \subseteq A \) then \( f^{-1}(V) \subseteq A \). Therefore, \( f^{-1}(V) \) is ICS set in \( X \) and hence \( f \) is I almost contra \( g \)-cont. function.

We start with example to show that I almost contra \( g \)-cont. is not imply I almost contra \( g \)-cont.

**Example 3.4.** Let \( X = \{a, b, c, d\} \) and \( T = \{\emptyset, \{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}, \{b, c\}, \{b\}, \{c\}, \{d\}\} \) and let \( Y = \{1, 2, 3\} \) and \( \sigma = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\} \) and let \( D = \{\emptyset, \{1\}\} \), \( E = \{\{1\}, \{2\}\} \) and \( F = \{\{1\}\} \) and \( H = \{\emptyset, \{1\}\} \).

Define a function \( f: X \rightarrow Y \) by \( f(a) = f(b) = 1 \) and \( f(c) = 2 \) and \( f(d) = 3 \).

**Example 3.5.** Let \( Y = \{a, b, c\} \) and let \( T = \{\emptyset, \emptyset, A, B, \{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}\} \) where \( A = \{x: \{a, b\}, \{c\}\} \), \( B = \{x, \{b, d\}, \{a\}\} \), \( C = \{x, \{b\}, \{c\}\} \) and \( D = \{x, \{a, b, c\}, \emptyset\} \) and let \( Y = \{1, 2, 3\} \) and \( \sigma = \{\emptyset, \{1\}, \{2\}\} \) where \( E = \{y, \{1\}, \{2, 3\}\} \) and \( F = \{y, \{1\}, \{2\}\} \). Define a function \( f: X \rightarrow Y \) by \( f(a) = f(b) = 1 \) and \( f(c) = 2 \) and \( f(d) = 3 \).

**Example 3.6.** Let \( T = \{\emptyset, \emptyset, A, B, \{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}\} \) and let \( Y = \{1, 2, 3\} \) and \( \sigma = \{\emptyset, \{1\}, \{2\}\} \) where \( E = \{y, \{1\}, \{2\}\} \) and \( F = \{y, \{1\}, \{2\}\} \). Define a function \( f: X \rightarrow Y \) by \( f(a) = f(b) = 1 \) and \( f(c) = 2 \) and \( f(d) = 3 \).

Now let \( ROY = \{\emptyset, \emptyset, D\} \) and \( \sigma = \{\emptyset, \emptyset, E, F\} \) where \( G = f^{-1}(D) = \{x, \{a, c\}, \emptyset\} \) then \( G \) is I closed set in \( X \) since the only IOS containing \( G \) is \( X \) and \( clG = X \subseteq X \) but \( G \) is not ICS in \( X \) since \( G = \emptyset \neq X \). So \( f \) is I almost contra \( g \)-cont. function but not I almost contra cont. function.

In this example we are going to show I almost contra \( g \)-cont. function is not imply I almost contra cont. function.

**Example 3.5.** Let \( X = \{a, b, c, d\} \) and let \( T = \{\emptyset, \emptyset, A, B, \{a\}, \{b\}, \{c\}, \{d\}\} \) where \( A = \{x: \{a, b\}, \{c\}\} \), \( B = \{x, \{b, d\}, \{a\}\} \), \( C = \{x, \{b\}, \{c\}\} \) and \( D = \{x, \{a, b, c\}, \emptyset\} \) and let \( Y = \{1, 2, 3\} \) and \( \sigma = \{\emptyset, \emptyset, E, F\} \) where \( E = \{y, \{1\}, \{2\}\} \) and \( F = \{y, \{1\}, \{2\}\} \). Define a function \( f: X \rightarrow Y \) by \( f(a) = 1 \), \( f(b) = 2 \) and \( f(c) = 3 \).
almost contra $θ^g$-cont. function. But $f$ is not $I$ almost contra $θ$-cont. function.

The next example shows that:
1. $I$ almost contra semi-cont. is not imply $I$ almost contra $θ$-cont.
2. $I$ almost contra semi-cont. is not imply $I$ almost contra pre-cont.
3. $I$ almost contra semi-cont. is not imply $I$ almost contra cont.

Example 3.7. Let $X = \{a, b, c, d\}$ and $T = \{\emptyset, X, A, B\}$ where $A = \langle x, \{a\}, \{b\} \rangle$ and $B = \langle x, \emptyset, \{a, b\} \rangle$ and let $Y = \{1, 2, 3\}$ and $\sigma = \{\emptyset, Y, C, D\}$ where $C = \langle y, \emptyset, \{1\} \rangle$ and $D = \langle y, \emptyset, \{1, 2\} \rangle$. Define a function $f : X \to Y$ by $f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 0$. $\text{RO} = \{\emptyset, Y, C\}$.

Now let $G = \{x, \{a\}, \{b\}\}$ is ISCS in $X$ since $\text{ÎncI}(G) = \{x, \emptyset, \{a\}\}$ is not $I$-CS (resp. IPCS and ICS) in $X$. Therefore, $f$ is I almost contra semi-cont. function but $f$ is not I almost contra $θ$-cont. function (resp. $I$ almost contra pre-cont. and $I$ almost contra cont.) function.

We are going to show that:
1. $I$ almost contra cont. is not imply $I$ almost contra $θ$-cont.
2. $I$ almost contra cont. is not imply $I$ almost contra $g$-cont.
3. $I$ almost contra $g$-cont. is not imply $I$ almost contra $θ^g$-cont.
4. $I$ almost contra $g$-cont. is not imply $I$ almost contra $θ$-cont.

Example 3.8. Let $X = \{a, b, c\}$ and $T = \{\emptyset, X, A, B\}$ where $A = \langle x, \{a\}, \{b, c\} \rangle$ and $B = \langle x, \{a\}, \{b, c\} \rangle$ and let $Y = \{1, 2, 3\}$ and $\sigma = \{\emptyset, Y, C, D\}$ where $C = \langle y, \emptyset, \{1\} \rangle$ and $D = \langle y, \emptyset, \{1, 2\} \rangle$. Define a function $f : X \to Y$ by $f(a) = 1, f(b) = 2, f(c) = 3$. $\text{RO} = \{\emptyset, Y, C\}$.

Now let $G = \{x, \{a\}, \{b, c\}\}$, then $G$ is IPCS in $X$ since $\text{ÎncI}(G) = \emptyset$ but $G$ is not $I$-CS (resp. ISCS and ICS) in $X$ since $\text{ÎncI}(G) = \emptyset$. So the inverse image of each IROS in $Y$ is ISCS in $X$. Therefore, $f$ is I almost contra pre-cont. function but $f$ is not I almost contra $θ$-cont. (resp. $I$ almost contra semi-cont. and $I$ almost contra con.) function.

The following example shows that:
1. $I$ almost contra $g$-cont. is not imply $I$ almost contra cont.
2. $I$ almost contra $g$-cont. is not imply $I$ almost contra pre-cont.
3. $I$ almost contra $g$-cont. is not imply $I$ almost contra semi-cont.
4. $I$ almost contra $g$-cont. is not imply $I$ almost contra $θ$-cont.

Example 3.9. Let $X = \{a, b, c\}$ and $T = \{\emptyset, X, A, B\}$ where $A = \langle x, \{a\}, \emptyset \rangle$, $B = \langle x, \{a\}, \emptyset \rangle$, and let $Y = \{1, 2, 3\}$ and $\sigma = \{\emptyset, Y, C, D\}$ where $C = \langle y, \emptyset, \{1\} \rangle$ and $D = \langle y, \emptyset, \{1, 2\} \rangle$. Define a function $f : X \to Y$ by $f(a) = 1, f(b) = 2, f(c) = 3$. $\text{RO} = \{\emptyset, Y, C\}$.

Now let $G = \{x, \{a\}, \{b, c\}\}$, then $G$ is IPCS in $X$ since $\text{ÎncI}(G) = \emptyset$ but $G$ is not $I$-CS (resp. ISCS and ICS) in $X$ since $\text{ÎncI}(G) = \emptyset$. Therefore, $f$ is I almost contra pre-cont. function but $f$ is not I almost contra $θ$-cont. (resp. $I$ almost contra semi-cont. and $I$ almost contra con.) function.

The following example shows that:
1. $I$ almost contra $g$-cont. is not imply $I$ almost contra cont.
2. $I$ almost contra $g$-cont. is not imply $I$ almost contra pre-cont.
3. $I$ almost contra $g$-cont. is not imply $I$ almost contra semi-cont.
4. $I$ almost contra $g$-cont. is not imply $I$ almost contra $θ$-cont.
Define a function \( f : X \rightarrow Y \) by
\[ f(a) = f(d) = 2, f(b) = 1 \] and
\[ f(c) = 3. \]
Then a set \( G = \{x, (a, d), (b)\} \) is IC in \( X \) since \( G \in I \) but \( G \) is not IC (resp. IGCS, IPCS and ISCS) in \( X \) since
\[ G \notin I \] as \( I \) of \( X \). Then \( f \) is I almost contra \( \beta \)-cont. function but \( f \) is not I almost contra \( \beta \)-cont. (resp. I almost contra semi-\( \beta \)-cont., I almost contra \( \beta \)-cont. and I almost contra pre-\( \beta \)-cont.) function.

We are going in the following example to show that:

1. I almost contra \( g \)-cont. is not imply I almost contra \( g \)-cont.
2. I almost contra \( g \)-cont. is not imply I almost contra \( g \)-cont.
3. I almost contra \( g \)-cont. is not imply I almost contra \( g \)-cont.

Example 3.11. Let \( X = \{a, b, c\} \) and
\[ T = \{\emptyset, X, A, B, C\} \] where
\( A = \{x, \{a, b\}\} \), \( B = \{X, \{b\}\} \) and \( C = \{x, \{a, b\}, \emptyset\} \) where \( D = \{y, \{2, 3\}\} \) and
\( H = \{y, \emptyset, \{2, 3\}\}. \) Define a function
\[ f: X \rightarrow Y \] by
\[ f(a) = 1, f(b) = 2 \text{ and } f(c) = 3. \]
\[ ROY = \{\emptyset, \emptyset, X, D, E\} \] and
\[ SOX = \{\emptyset, X, A, B, C, E, F\} \] where
\( E = \{x, \{c\}, \{b\}\} \) and \( F = \{x, \{a, b\}, \{c\}\}. \) So \( \alpha OX = T. \) We have \( E = f^{-1}(D) \) is \( g \)-closed and \( g \)-closed in \( X \) since the only IOS containing \( G \) is \( X \) and
\[ aclG = aclG = B \subseteq X \] and \( aclG = N \subseteq X \) but \( G \) is not \( g \)-closed since \( G \subseteq F \) where \( F \) is \( I \) of \( X \) but
\[ aclG = aclG = B \subseteq X. \] Then the inverse image of each IROS in \( Y \) is \( g \)-closed (resp. \( g \)-closed and \( g \)-closed) set in \( X \). So \( f \) is I almost contra \( g \)-cont. (resp. I almost contra \( g \)-cont.) but not I almost contra \( g \)-cont. function.

We are going to show I almost contra \( g \)-cont. is not imply I almost contra \( \alpha \)-cont.

Example 3.12. Let \( X = \{a, b, c\} \) and
\[ T = \{\emptyset, X, A, B, C\} \] where
\( A = \{x, \{a, b, c\}, B = \{X, \{b\}\} \) and \( C = \{x, \{a, b\}, \{c\}\} \) and let \( Y = \{1, 2, 3\} \) and \( \sigma = \{\emptyset, Y, D, E\} \) where \( D = \{y, \{3\}, \{1\}\} \) and
\( E = \{y, \emptyset, \{1, 3\}\}. \) Define a function
\[ f: X \rightarrow Y \] by
\[ f(a) = 1, f(b) = 2 \text{ and } f(c) = 3. \]
\[ ROY = \{\emptyset, Y, D\} \] and
\[ SOX = \{\emptyset, X, A, B, C, G, K, I, N, F\} \] where
\( G = \{x, \{a, b\}\}, K = \{x, \{a, b\}, \emptyset\}, I = \{x, \{a, c\}, \{b\}\}, N = \{x, \{b\}, \{a\}\} \) and \( F = \{x, \{b, c\}, \{a\}\}. \) So \( \alpha OX = T. \) We have \( G = f^{-1}(D) \) is \( g \)-closed (resp. \( g \)-closed, \( g \)-closed) set in \( X \) since the only IOS containing \( G \) is \( X \) and
\[ aclG = aclG = B \subseteq X \] and \( aclG = N \subseteq X \) but \( G \) is not \( g \)-closed since \( G \subseteq F \) where \( F \) is \( I \) of \( X \) but
\[ aclG = aclG = B \subseteq X. \] Then the inverse image of each IROS in \( Y \) is \( g \)-closed (resp. \( g \)-closed and \( g \)-closed) set in \( X \). So \( f \) is I almost contra \( g \)-cont. (resp. I almost contra \( g \)-cont.) but not I almost contra \( g \)-cont. function.

We are going to show I almost contra \( g \)-cont. is not imply I almost contra \( \alpha \)-cont.

Example 3.13. Let \( X = \{a, b, c, d\} \) and
\[ T = \{\emptyset, X, A, B, C\} \] where
\( A = \{x, \{b\}, \{a, c\}\} \) and \( B = \{x, \{a\}, \{b, c\}\} \) and \( C = \{x, \{a, b\}, \{c\}\} \) and let \( Y = \{1, 2, 3\} \) and
\[ \sigma = \{\emptyset, Y, D, E, F, H\} \]
where
\[ D = \{y, \{2\}, \{1,3\}\}, E = \{y, \{1,2\}, 0\}, F = \{y, \{1\}, 0\}\] and \( H = \{y, \{0\}, \{1,3\}\}\).

Define a function \( f : X \rightarrow Y \) by \( f(a) = 2, f(b) = f(c) = 1 \) and \( f(d) = 3 \).

\( ROY = \{\emptyset, Y, H\} \) and \( \sigma_{DX} = \{\emptyset, X, A, B, C, E, D, E, H\} \) where \( K = \{x, \{a, b\}, 0\} \). So a set \( G = f^{-1}(F) = \{x, \{b, c\}, 0\} \) is \( I_{\sigma_{DX}} \)-closed set in \( X \) since the only IOS containing \( G \) is \( X \) and but \( G \) is not \( I_{\sigma_{DX}} \)-closed set in \( X \) since \( L \) is ISOS in \( X \) and \( \sigma_{DX} \).

Then the inverse image of each IOS in \( Y \) is \( I_{\sigma_{DX}} \)-closed set in \( X \) so \( f \) is \( I_{\sigma_{DX}} \)-contra-seq. function but not \( I \) almost contra \( g_{\sigma_{DX}} \)-cont. function.

The following example shows that \( I \) almost contra \( g_{\sigma_{DX}} \)-cont. is not imply \( I \) almost contra \( g_{\sigma_{DX}} \)-cont.

Example 3.15. Let \( X = \{a, b, c\} \) and let \( T = \{\emptyset, X, A, B, C\} \) where \( A = (x, \{a\}, \{b, c\}), B = (x, \{a\}, \{b, c\}) \) and \( C = (x, \{a, b\}, \{c\}) \) and let \( Y = \{1,2,3\} \) and \( v = (\emptyset, Y, D, E) \) where \( D = \{y, \{1\}, \{2\}\} \) and \( E = \{y, 0, \{1,2\}\} \).

Define a function \( f : X \rightarrow Y \) by \( f(a) = 1, f(b) = f(c) = 2 \).

\( ROY = \{\emptyset, Y, D\} \) and \( \sigma_{DX} = \{\emptyset, X, A, B, C, F, H, K, L, M, O, N, G, V, J\} \) where \( F = (x, \{b\}, \{a\}), H = (x, \{b\}, \{c\}), K = (x, \{b\}, 0), L = (x, \{a\}, \{b\}), I = (x, \{a\}, \{c\}), M = (x, \{a\}, 0), O = (x, \{b\}, \{a\}), N = (x, \{a\}, \{b\}), G = (x, \{b\}, \{a\}), V = (x, \{a\}, \{b\}) \) and \( J = (x, \{a\}, 0) \).

\( \sigma_{DX} = \{\emptyset, X, A, B, C, H, K, L, M, O, N, G, V, J\} \).

Now a set \( A = f^{-1}(D) \) is \( I_{\sigma_{DX}} \)-closed set in \( X \) since \( A \) is IOS and \( \beta_{\sigma_{DX}} \sigma_{DX} \sigma_{DX} = A \). But \( A \) is not \( I_{\sigma_{DX}} \)-closed set since \( \beta_{\sigma_{DX}} \sigma_{DX} \sigma_{DX} = A \). Then \( f \) is \( I_{\sigma_{DX}} \)-contra- seq. function since the inverse image of each IOS in \( Y \) is \( I_{\sigma_{DX}} \)-closed set in \( X \), so \( f \) is not \( I_{\sigma_{DX}} \)-contra- seq. function.

We are going to show that:

1. \( I \) almost contra \( pre_{\sigma_{DX}} \)-cont. is not imply \( I \) almost contra \( g_{\sigma_{DX}} \)-cont.
2. \( I \) almost contra \( \beta_{\sigma_{DX}} \)-cont. is not imply \( I \) almost contra \( g_{\sigma_{DX}} \)-cont.
3. \( I \) almost contra \( \beta_{\sigma_{DX}} \)-cont. is not imply \( I \) almost contra \( g_{\sigma_{DX}} \)-cont.
4. \( I \) almost contra \( g_{\sigma_{DX}} \)-cont. is not imply \( I \) almost contra \( g_{\sigma_{DX}} \)-cont.
5. \( I \) almost contra \( g_{\sigma_{DX}} \)-cont. is not imply \( I \) almost contra \( g_{\sigma_{DX}} \)-cont.
6. \( I \) almost contra \( g_{\sigma_{DX}} \)-cont. is not imply \( I \) almost contra \( g_{\sigma_{DX}} \)-cont.
7. I almost contra \(g\)-cont. is not imply I almost contra \(g\)-cont.

**Example 3.16.** Let \(X = \{a, b, c\}\) and let \(T = \langle \emptyset, X, A, B \rangle\) where \(A = \{x, \{c\}, \{b\}\}\) and \(B = \{x, \{b, c\}, \emptyset\}\) and let \(Y = \{1, 2, 3\}\) and \(\sigma = \langle \emptyset, Y, C, D \rangle\) where \(C = \{y, \{1\}\}\) and \(D = \{y, \{1, 2\}\}\). Define a function \(f: X \rightarrow Y\) by
\[
f(a) = 2, \quad f(b) = 3 \quad \text{and} \quad f(c) = 1.
\]

\[\text{RO} = \langle \emptyset, Y, C, D \rangle \quad \text{and} \quad \text{SO} = \emptyset\]
so
\[\text{RO} = \text{SO}\]
and
\[\text{RO} = \text{SO} = T \cup \{K_1, K_2, K_3\} \]
where
\[K_1 = \{x, \{a\}, \{b\}\}, \quad K_2 = \{x, \{c\}, \emptyset\}, \quad K_3 = \{x, \{b, c\}, \emptyset\}\]
and
\[K_{12} = \{x, \{a\}, \{b\}\}, \quad K_{13} = \{x, \{a\}, \emptyset\}, \quad K_{23} = \{x, \{b\}, \emptyset\}\]
\[K_{12} = \{x, \{a\}, \{b\}\}, \quad K_{13} = \{x, \{b\}, \emptyset\}, \quad K_{23} = \{x, \{c\}, \emptyset\}\]
and
\[K_{13} = \{x, \{c\}, \emptyset\}, \quad K_{23} = \{x, \{c\}, \emptyset\}\]
so
\[\text{cloK}_2 = X \cup \{B\} \quad \text{and} \quad \text{cloK}_3 = X \cup \{B\}\]
There for, \(f\) is I almost contra \(\text{pre-cont.}\) (resp. I almost contra \(g\)-cont., I almost contra \(g\)-cont. and I almost contra \(g\)-cont.). but \(f\) is not I almost contra \(g\)-cont. (resp. I almost contra \(g\)-cont., I almost contra \(g\)-cont. and I almost contra \(g\)-cont.) function.

The following example shows that:

1. I almost contra \(g\)-cont. is not imply I almost contra \(g\)-cont.
2. I almost contra \(g\)-cont. is not imply I almost contra \(g\)-cont.
3. I almost contra \(g\)-cont. is not imply I almost contra \(g\)-cont.
4. I almost contra \(g\)-cont. is not imply I almost contra \(g\)-cont.
5. I almost contra \(g\)-cont. is not imply I almost contra \(g\)-cont.
6. I almost contra \(g\)-cont. is not imply I almost contra \(g\)-cont.
7. I almost contra \(g\)-cont. is not imply I almost contra \(g\)-cont.
8. I almost contra \(g\)-cont. is not imply I almost contra \(g\)-cont.

**Example 3.17.** Let \(X = \{a, b, c\}\) and \(T = \langle \emptyset, X, A, B \rangle\) where \(A = \{x, \{b, c\}, \emptyset\}\) and \(B = \{x, \{a\}, \emptyset\}\) and let \(Y = \{1, 2, 3\}\) and \(\sigma = \langle \emptyset, Y, C, D \rangle\) where
\[C = \{y, \{1\}\}, \quad D = \{y, \{1, 2\}\}, \quad E = \{y, \{2\}, \{1, 3\}\}\]
and
\[F = \{y, \emptyset, \{1, 3\}\}\]
Define a function \(f: X \rightarrow Y\) by
\[f(a) = f(b) = 1 \quad \text{and} \quad f(c) = 2.
\]
\[\text{RO} = \langle \emptyset, Y, C, D \rangle \quad \text{and} \quad \text{SO} = \emptyset\]
so
\[\text{SO} = \emptyset\]
and
\[\text{SO} = \emptyset\]
and
\[\text{SO} = \emptyset\]
so
\[\text{cloK}_2 = X \cup \{B\} \quad \text{and} \quad \text{cloK}_3 = X \cup \{B\}\]
There for, \(f\) is almost contra \(\text{pre-cont.}\) (resp. almost contra \(g\)-cont., almost contra \(g\)-cont. and almost contra \(g\)-cont.). but \(f\) is not almost contra \(g\)-cont. (resp. almost contra \(g\)-cont., almost contra \(g\)-cont. and almost contra \(g\)-cont.) function.

Now a set \(K = f^{-1}(C)\) is \(1g\)-closed (resp. \(1g\)-closed and \(1g\)-closed) set in \(X\) since the only IOS containing \(K\) is \(X\) and
but it's not IPCS (resp. I-CS, Isg-closed set, Ipg-closed) set in X since clutKg = IntclutKg = X ⊆ X. Then f is I almost contra g-cont. (resp. I almost contra g-cont. and I almost contra g-cont.) function but it's not I almost contra pre-cont. (resp. I almost contra p-cont, I almost contra p-cont. and I almost contra p-cont.) function.

In the next example we show that I almost contra s-cont. is not imply I almost contra semi-cont.

Example 3.18. Let X = {a, b, c} and 
\[ T = \{ \emptyset, \{a\}, \{b, c\}, \{a, b, c\} \} \]
where
\[ A = \{x_a, \{a, b, c\}\}, B = \{x_b, \{a\}\}, C = \{x_c, \{b, c\}\} \]
and let 
\[ Y = \{1, 2, 3\} \]
and 
\[ \sigma = \{\emptyset, \{a, c\}, \{b, c\}, \{a, b, c\}\} \]
Define a function \( f: X \to Y \) by 
\[ f(a) = 1, f(b) = 2, f(c) = 3. \]
ROX = \{x_a, x_b, x_c, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}.
SOK = \{\emptyset, \{x_a\}, \{x_b\}, \{x_c\}, \{x_a, x_b\}, \{x_a, x_c\}, \{x_b, x_c\}, \{x_a, x_b, x_c\}\}.
Now let 
\[ G = f^{-1}(\{1\}) = \{x_a, \{a, b, c\}\}. \]
G is Isg-cont. set in X since the only ISOS containing G in X is A and N and G is not ISCS in X since IntclG = \{x_a, x_b, x_c\} but G is not Iscs in X since clutG = \{x_a, x_b, x_c\} and the inverse image of each IOS in Y is Isg-cont. set in X and we have f is I almost contra s-cont. function but not I almost contra semi-cont. function.

In the last example we show that:

1. I almost contra g-cont. is not imply I almost contra g-cont.

2. I almost contra g-cont. is not imply I almost contra g-cont.

Example 3.19. Let X = \{a, b, c\} and 
\[ T = \{\emptyset, \{a\}, \{b\}, \{a, b\}\} \]
where
\[ A = \{x_a, \{a, b\}\}, B = \{x_b, \{a, b\}\}, C = \{x_c, \{a, b\}\} \]
and let 
\[ Y = \{1, 2, 3\} \]
and 
\[ \sigma = \{\emptyset, \{a, c\}, \{b, c\}, \{a, b, c\}\} \]
Define a function \( f: X \to Y \) by 
\[ f(a) = 1, f(b) = 2, f(c) = 3. \]
\[ \alpha \times \sigma = \{\emptyset, \{x_a, x_b\}, \{x_a, x_c\}, \{x_b, x_c\}, \{x_a, x_b, x_c\}\} \]
and 
\[ \beta = \{\emptyset, \{x_a, x_b\}, \{x_a, x_c\}, \{x_b, x_c\}, \{x_a, x_b, x_c\}\} \]
Define a function \( f: X \to Y \) by 
\[ f(a) = 1, f(b) = 2, f(c) = 3. \]
}\[ ROY = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\} \]
and 
\[ N = \{x_a, \{a\}, \emptyset, \{a, b, c\}\}. \]
Now let 
\[ G = f^{-1}(\{1\}) = \{x_a, \{a, b, c\}\} \]
Then the inverse image of each IOS in Y is Isg-cont. closed set in X and hence f is I almost contra g-cont. function and I almost contra g-cont. function but it's not I almost contra g-cont. function.

We summarized the above result by the following diagram.

Diagram 3.20. The following implications are true and not reversed:
4-Some relations among almost contra continuous functions and another kinds of continuity.

We introduce the following definitions.

**Definition 4.1.** Let \((X, T)\) and \((Y, U)\) be two ITS's and let \(f : X \rightarrow Y\) be a function then \(f\) is said to be I almost SR-\(\text{cont.}\) (resp. I almost \(\text{cont.}\), I almost RC-\(\text{cont.}\) and I regular irresolute) if the inverse image of each IROS in \(Y\) is ISRS (resp. IRCS and IROS) in \(X\).

**Remark 4.2.** The notions I almost contra cont. function and I almost cont. function are independent.

The following examples shows this cases.

**Example 4.3.** Let \(X = \{a, b, c\}\) and let \(T = \{\emptyset, X, A, B\}\) where \(A = \{x, \{b\}, \{c\}\}\) and \(B = \{x, \emptyset, [a, c]\}\) let \(Y = \{1, 2, 3\}\) and \(\mathcal{g} = \{\emptyset, Y, C, D\}\) where \(C = \{y, \{3\}, [1]\}\) and \(D = \{y, \emptyset, [1, 3]\}\). Define a function \(h : X \rightarrow Y\) by \(h(a) = 2, h(b) = 1\) and \(h(c) = 3\). \(\text{ROY} = \{\emptyset, Y, C\}\). It's easy to verify \(f\) is I almost contra cont. function but not I almost cont. function.

**Example 4.4.** Let \(X = \{a, b, c\}\) and \(T = \{\emptyset, X, A, B\}\) where \(A = \{x, \{a\}, \{b\}\}\) and \(B = \{x, \emptyset, [a, b]\}\) let \(Y = \{1, 2, 3\}\) and \(\mathcal{g} = \{\emptyset, Y, C, D\}\) where \(C = \{y, \emptyset, [1, 2]\}\) and \(D = \{y, \emptyset, [1, 2]\}\). Define a function \(h : X \rightarrow Y\) by \(h(a) = 1, h(b) = 2\) and \(h(c) = 3\).

**Proposition 4.5.** Let \((X, T)\) and \((Y, U)\) be two ITS's and let \(f : X \rightarrow Y\) be a function then the following statements are equivalent:

1. \(f\) is I almost RC-\(\text{cont.}\) function.
2. \(f\) is I almost RC-\(\text{cont.}\) function and I almost contra cont. function.

**Proof** 1 \(\Rightarrow\) 2 Let \(V\) be IROS in \(Y\) then \(f^{-1}(V)\) is IRCS in \(X\) (since \(f\) is I almost RC-cont. function) then \(\text{clutcf}^{-1}(V) = f^{-1}(V)\) hence \(f^{-1}(V)\) is ICS in \(X\) so \(f^{-1}(V) \subseteq \text{clutcf}^{-1}(V)\) and \(f^{-1}(V) \subseteq \text{clutcf}^{-1}(V)\) imply \(f^{-1}(V) \subseteq \text{clutcf}^{-1}(V)\). Therefore, \(f^{-1}(V)\) is ICS and hence \(f\) is I almost contra cont. function and I almost RC-cont. function.

2 \(\Rightarrow\) 1 Let \(U\) be IROS in \(Y\) then \(f^{-1}(U)\) is IRCS in \(X\) (by hypothesis) then \(f^{-1}(U) \subseteq \text{clutcf}^{-1}(U)\) and \(\text{clutcf}^{-1}(U) = f^{-1}(U)\) imply \(f^{-1}(U) \subseteq \text{clutcf}^{-1}(U)\) and \(\text{clutcf}^{-1}(U) = f^{-1}(U)\) imply \(f^{-1}(U) = \text{clutcf}^{-1}(U)\). Therefore, \(f^{-1}(U)\) is IRCS in \(X\). Hence \(f\) is I almost RC-cont. function.
Proposition 4.6. Let \((X,T)\) and \((Y,\mathcal{A})\) be two \(\mathcal{M}\)-topological spaces and let \(f: X \to Y\) be a function then the following statements are equivalent:

1. \(f\) is \(\mathcal{M}\)-almost continuous function.
2. \(f\) is \(\mathcal{M}\)-almost \(\mathcal{A}\)-continuous function and \(\mathcal{M}\)-almost contra semi-continuous function.

Proof 1 \(\Rightarrow\) 2 Suppose that \(V\) be any IROS in \(Y\) then \(f^{-1}(V)\) is isometric in \(X\) (by hypothesis) then \(f^{-1}(V)\) is IPOS and ISCS so \(\text{Int} f^{-1}(V) \subseteq \text{cl\textit{Int}} f^{-1}(V)\) and \(\text{Int} f^{-1}(V) \subseteq f^{-1}(V)\). Now since \(f^{-1}(V) \subseteq \text{cl\textit{Int}} f^{-1}(V)\) imply \(f^{-1}(V) \subseteq \text{cl\textit{Int}} f^{-1}(V)\). There fore, \(f^{-1}(V)\) is IPOS and ISCS in \(X\). Hence \(f\) is \(\mathcal{M}\) almost contra semi-continuous function.

Proposition 4.7. Every \(\mathcal{M}\) almost contra \(\mathcal{A}\)-continuous function and \(\mathcal{M}\) almost \(\mathcal{A}\)-continuous function is \(\mathcal{M}\) almost contra semi-continuous function.

Proof 1 \(\Rightarrow\) 2 Suppose that \(U\) be IROS in \(Y\) then \(f^{-1}(U)\) is IPOS and ISCS in \(X\) (by hypothesis) then \(f^{-1}(U) \subseteq \text{cl\textit{Int}} f^{-1}(U)\) and \(\text{Int} f^{-1}(U) \subseteq f^{-1}(U)\). Now we have \(\text{Int} f^{-1}(U) \subseteq f^{-1}(U) \subseteq \text{cl\textit{Int}} f^{-1}(U)\), then \(f^{-1}(U)\) is ISCS in \(X\) also IPOS is ISCS in \(X\). There fore, \(f^{-1}(U)\) is ISRS in \(X\) and hence \(f\) is \(\mathcal{M}\) almost contra semi-continuous function.

Corollary 4.7. Every \(\mathcal{M}\) almost contra continuous function and \(\mathcal{M}\) almost \(\mathcal{A}\)-continuous function is \(\mathcal{M}\) almost semi-continuous function.

Proof: Let \((X,T)\) and \((Y,\mathcal{A})\) be two \(\mathcal{M}\)-topological spaces and let \(f: X \to Y\) an \(\mathcal{M}\) almost contra continuous function and \(\mathcal{M}\) \(\mathcal{A}\)-continuous function, so for any IOS \(V\) in \(Y\) then \(f^{-1}(V)\) is ISCS in \(X\) imply \(f^{-1}(V) = \text{cl\textit{Int}} f^{-1}(V)\) and \(f^{-1}(V) \subseteq \text{cl\textit{Int}} f^{-1}(V)\). Now we have \(\text{Int} f^{-1}(V) \subseteq f^{-1}(V) \subseteq \text{cl\textit{Int}} f^{-1}(V)\), then \(f^{-1}(V)\) is ISCS in \(X\). Hence \(f\) is \(\mathcal{M}\) almost semi-continuous function.

Proposition 4.8. Let \((X,T)\) and \((Y,\mathcal{A})\) be two \(\mathcal{M}\)-topological spaces and let \(f: X \to Y\) be a function then the following statements are equivalent:

1. \(f\) is \(\mathcal{M}\)-irresolute function.
2. \(f\) is \(\mathcal{M}\)-almost pre-continuous function and \(\mathcal{M}\) almost contra semi-continuous function.

Proof 1 \(\Rightarrow\) 2 Suppose that \(V\) be any IROS in \(Y\) then \(f^{-1}(V)\) is ISCS in \(X\) (by hypothesis) then \(f^{-1}(V) \subseteq \text{cl\textit{Int}} f^{-1}(V)\) and \(\text{Int} f^{-1}(V) \subseteq f^{-1}(V)\). Now since \(f^{-1}(V) \subseteq \text{cl\textit{Int}} f^{-1}(V)\) imply \(f^{-1}(V) \subseteq \text{cl\textit{Int}} f^{-1}(V)\). There fore, \(f^{-1}(V)\) is ISCS in \(X\). Hence \(f\) is \(\mathcal{M}\)-almost pre-continuous and \(\mathcal{M}\)-almost contra semi-continuous function.

Proposition 4.9. Let \((X,T)\) and \((Y,\mathcal{A})\) be two \(\mathcal{M}\)-topological spaces and let \(f: X \to Y\) be a function then the following statements are equivalent:

1. \(f\) is \(\mathcal{M}\) almost contra semi-continuous function.
2. \(f\) is \(\mathcal{M}\)-almost semi-coneontinuous function and \(\mathcal{M}\) almost contra semi-continuous function.

Proof 1 \(\Rightarrow\) 2 Suppose that \(V\) be any IOS in \(Y\) then \(f^{-1}(V)\) is ISCS in \(X\) (by hypothesis). Now let \(A\) be IOS in \(X\) and \(f^{-1}(V) \subseteq A\) then \(f^{-1}(V) = A \cap f^{-1}(V)\) imply \(f^{-1}(V)\) is IBS, so \(f^{-1}(V) = \text{cl\textit{Int}} f^{-1}(V)\) since \(\text{cl\textit{Int}} f^{-1}(V) = f^{-1}(V) \cup \text{Int} f^{-1}(V)\) and \(\text{Int} f^{-1}(V) \subseteq f^{-1}(V)\). Hence for each IOS \(A\) in \(X\) and \(f^{-1}(V) \subseteq A\) then \(\text{cl\textit{Int}} f^{-1}(V) \subseteq A\). There fore, \(f^{-1}(V)\) is ICS closed set and IBS in X, so \(f\) is \(\mathcal{M}\) almost contra semi-continuous function and I almost B-continuous function.

2 \(\Rightarrow\) 1 Suppose that \(U\) be any IOS in \(Y\) then \(f^{-1}(U)\) is IBS and ISCS in \(X\) (by hypothesis). Then \(f^{-1}(U) = A \cup G\) where \(A\) is IOS containing \(f^{-1}(U)\) in \(X\) and \(G\) is ISCS in \(X\). So \(\text{cl\textit{Int}} f^{-1}(U) \subseteq A\) since \(f^{-1}(U)\) is ICS closed set. Now \(\text{Int} f^{-1}(U) \subseteq \text{Int} f^{-1}(U) = \text{Int}(A \cap \text{cl\textit{Int}} f^{-1}(U)) \subseteq \text{Int} f^{-1}(U) \cap \text{Int} f^{-1}(U) = \text{Int} f^{-1}(U) \cap \text{Int} G \subseteq \text{Int} f^{-1}(U) \cap G\) since \(G\) is ISCS. So \(\text{Int} f^{-1}(U) \subseteq A\).
and \( A \subseteq \text{Int} \mathcal{A} \). We have \( \text{Int}f^{-1}(U) \subseteq A \cap G = f^{-1}(U) \). Therefore, \( f^{-1}(U) \) is ISCS in X and hence \( f \) is I almost contra semi-cont. function.

**Corollary 4.10.** Let \((X,T)\) and \((Y,\sigma)\) be two ITS's and let \( f : X \rightarrow Y \) be a function then the following statements are equivalent:

1. \( f \) is I almost SR-cont. function.
2. \( f \) is I almost \( \mathcal{B} \)-cont. function, I almost \( \mathcal{B} \)-cont. function and I almost contra \( \mathcal{B} \)-cont. function.

**Proof** 1 \( \Rightarrow \) 2 Let \( V \) be IOS in \( Y \) then \( f^{-1}(V) \) is ISRS in \( X \) (since \( f \) is I almost contra SR-cont. function). Then \( f^{-1}(V) \) is ISCS and ISOS in \( X \), that is \( \text{Int}f^{-1}(V) \subseteq f^{-1}(V) \) and \( f^{-1}(V) \subseteq \text{cl} \text{Int}f^{-1}(V) \) imply \( f^{-1}(V) \subseteq \text{cl} \text{Int}f^{-1}(V) \) then \( f^{-1}(V) \) is \( \mathcal{B} \)-OS. Now let \( A \) be IOS in \( X \) and \( f^{-1}(V) \subseteq A \) imply \( f^{-1}(V) = A \cap f^{-1}(V) \) then \( f^{-1}(V) \) is I almost contra \( \mathcal{B} \)-cont. function. Hence for each IOS \( A \) in \( X \) and \( f^{-1}(V) \subseteq A \) then \( f^{-1}(V) \subseteq \mathcal{A} \). There fore, \( f^{-1}(V) \) is Igs-closed set, IBS and IBOS in \( X \) and hence \( f \) is I almost \( \mathcal{B} \)-cont. function, I almost \( \mathcal{B} \)-cont. function and I almost contra \( \mathcal{B} \)-cont. function.

2 \( \Rightarrow \) 1 Let \( U \) be IOS in \( Y \) then \( f^{-1}(U) \) is \( \mathcal{B} \)-OS, IBS and Igs-closed set in \( X \) (by hypothesis) then \( f^{-1}(U) \subseteq \text{cl} \text{Int}f^{-1}(U) \) and \( f^{-1}(U) = A \cap G \) where \( A \) is IOS containing \( f^{-1}(U) \) in \( X \) and \( G \) is ISCS in \( X \) so \( f^{-1}(U) \subseteq A \) since \( f^{-1}(U) \) is Igs-closed set. Now \( \text{Int}f^{-1}(U) = \text{Int}(A \cap G) \subseteq \text{Int}(G) = \text{Int}A \cap \text{Int}G \) since \( G \) is ISCS. So \( \text{Int}f^{-1}(U) \cap A = \text{Int}A \cap A \) since \( A \) is IOS containing \( f^{-1}(U) \) in \( X \) and \( G \) is ISCS in \( X \) so \( \text{Int}f^{-1}(U) \cap A \subseteq A \) since \( A \) is IOS containing \( f^{-1}(U) \) in \( X \). Hence \( \text{Int}f^{-1}(U) \subseteq A \cap G = f^{-1}(U) \).

**Corollary 4.11.** Let \((X,T)\) and \((Y,\sigma)\) be two ITS's and let \( f : X \rightarrow Y \) be a function then the following statements are equivalent:

1. \( f \) is I R-irresolute function.
2. \( f \) is I almost pre-cont. function, I almost \( \mathcal{B} \)-cont. function and I almost contra \( \mathcal{B} \)-cont. function.

**Proof** 1 \( \Rightarrow \) 2 Suppose that \( V \) is IOS in \( Y \) then \( f^{-1}(V) \) is IROS in \( X \) (by hypothesis). That is \( f^{-1}(V) = \text{Int}f^{-1}(V) \) and \( f^{-1}(V) \subseteq \text{cl} \text{Int}f^{-1}(V) \) imply \( f^{-1}(V) \subseteq \text{cl} \text{Int}f^{-1}(V) \) then \( f^{-1}(V) \) is IPOS. Now let \( A \) be IOS in \( X \) and \( f^{-1}(V) \subseteq A \) imply \( f^{-1}(V) = A \cap f^{-1}(V) \) then \( f^{-1}(V) \) is I almost contra \( \mathcal{B} \)-cont. function. Hence for each IOS \( A \) in \( X \) and \( f^{-1}(V) \subseteq A \) then \( f^{-1}(V) \subseteq \mathcal{A} \). There fore, \( f^{-1}(V) \) is Igs-closed set, IBS and IBOS in \( X \) and hence \( f \) is I almost \( \mathcal{B} \)-cont. function, I almost \( \mathcal{B} \)-cont. function and I almost contra \( \mathcal{B} \)-cont. function.

2 \( \Rightarrow \) 1 Suppose that \( U \) is IOS in \( Y \) then \( f^{-1}(U) \) is IPOS, IBS and Igs-closed set in \( X \) (by hypothesis) then \( f^{-1}(U) \subseteq \text{Int}f^{-1}(U) \) and \( f^{-1}(U) = A \cap G \) where \( A \) is IOS containing \( f^{-1}(U) \) in \( X \) and \( G \) is ISCS in \( X \) so \( f^{-1}(U) \subseteq A \) since \( f^{-1}(U) \) is Igs-closed set. Now \( \text{Int}f^{-1}(U) = \text{Int}(A \cap G) \subseteq \text{Int}(G) = \text{Int}A \cap \text{Int}G \) since \( G \) is ISCS. So \( \text{Int}f^{-1}(U) \cap A = \text{Int}A \cap A \) since \( A \) is IOS containing \( f^{-1}(U) \) in \( X \) and \( G \) is ISCS in \( X \) so \( \text{Int}f^{-1}(U) \cap A \subseteq A \) since \( A \) is IOS containing \( f^{-1}(U) \) in \( X \). Hence
\( f^{-1}(U) \) is ISCS in X, then we have
\[ \text{Int}(f^{-1}(U)) \subseteq f^{-1}(\text{Int}(U)) \]
and
\[ f^{-1}(U) \subseteq \text{Int}(f^{-1}(U)) \]
imply
\[ f^{-1}(U) = \text{Int}(f^{-1}(U)). \]
Therefore, \( f^{-1}(U) \) is IROS in X and hence f is I R-irresolute function. \( \Box \)

The following definition is given in [1] by general topology, we generalized it on ITS's.

**Definition 4.12.** Let \((X,T)\) and \((Y,G)\) be two ITS's then a function \( f : X \rightarrow Y \) is said to be intuitionistic contra continuous (resp. intuitionistic contra semi-continuous if the inverse image of each IOS in Y is ICS (resp. ISCS) in X. 

The following definition is given in [5] by general topology, we generalized it on ITS's.

**Definition 4.13.** Let \((X,T)\) and \((Y,G)\) be two ITS's then a function \( f : X \rightarrow Y \) is said to be intuitionistic regular set connected if the inverse image of each IOS in Y is clopen in X.

**Proposition 4.14.** Let \((X,T)\) and \((Y,G)\) be two ITS's then and let \( f : X \rightarrow Y \) be a function then:

1. If \( f \) is I perfectly cont. function then \( f \) is ISR-cont. function.

2. If \( f \) is ISR-cont. function then \( f \) is I contra semi-cont. function.

3. If \( f \) is I contra cont. function then \( f \) is I contra semi-cont. function.

4. If \( f \) is I perfectly cont. function then \( f \) is I regular set connected function.

5. If \( f \) is I regular set connected function then \( f \) is I almost contra cont. function.

6. If \( f \) is I contra cont. function then \( f \) is I almost contra semi-cont. function.

7. If \( f \) is I contra semi-cont. function then \( f \) is I almost contra semi-cont. function.

8. If \( f \) is I almost contra semi-cont. function then \( f \) is I contra semi-cont. function.

**Proof:**

1. Let \( V \) be IOS in Y then \( f^{-1}(V) \) is clopen set in X (since \( f \) is I perfectly cont. function) so
\[ f^{-1}(V) = \text{cl}(f^{-1}(V)) \]
and
\[ f^{-1}(V) = \text{Int}(f^{-1}(V)) \]
imply
\[ f^{-1}(V) = \text{Int}(f^{-1}(V)). \]
Therefore, \( f^{-1}(V) \) is ISCS and ISOS. There fore, \( f^{-1}(V) \) is ISR-cont. function. \( \Box \)

2. Suppose that \( V \) be IOS in Y then \( f^{-1}(V) \) is ISRS in X (since \( f \) is ISR-cont. function) so
\[ f^{-1}(V) = \text{Int}(f^{-1}(V)) \]
and
\[ f^{-1}(V) \subseteq \text{cl}(\text{Int}(f^{-1}(V))) \]
implies
\[ f^{-1}(V) = \text{Int}(f^{-1}(V)). \]
Therefore, \( f^{-1}(V) \) is IROS in X and hence f is I R-irresolute function. \( \Box \)

3. For any IOS V in Y then \( f^{-1}(V) \) is ICS in X (since \( f \) is I contra cont. function) so
\[ \text{cl}(f^{-1}(V)) = f^{-1}(V) \]
and
\[ \text{Int}(f^{-1}(V)) \subseteq f^{-1}(V) \]
implies
\[ f^{-1}(V) = \text{Int}(f^{-1}(V)). \]
Therefore, \( f^{-1}(V) \) is ISCS in X and hence f is I contra semi-cont. function. \( \Box \)

4. Let \( V \) be IOS in Y then \( f^{-1}(V) \) is clopen set in X (since \( f \) is I perfectly cont. function). Now since every IROS is IOS that is the inverse image of each IROS in Y is clopen set in X, so \( f \) is I regular set connected function. \( \Box \)

5. Suppose that \( V \) be IROS in Y then \( f^{-1}(V) \) is clopen set in X (by hypothesis). That is \( f^{-1}(V) \) is IOS and ICS in X and hence \( f \) is I almost contra cont. function. \( \Box \)

6. Let \( V \) be IOS in Y then \( f^{-1}(V) \) is ICS in X (since \( f \) is I contra cont. function). Now since every IROS is IOS that is the inverse image of each IROS in Y is ICS in X. Hence \( f \) is I contra semi-cont. function. \( \Box \)

7. For any IROS V in Y then \( f^{-1}(V) \) is ICS in X (by hypothesis) then
\[ f^{-1}(V) = \text{cl}(f^{-1}(V)) \]
imply
\[ f^{-1}(V) = \text{Int}(f^{-1}(V)), \]
so \( f^{-1}(V) \) is ISCS in X and hence \( f \) is I almost contra semi-cont. function. \( \Box \)

8. Let \( V \) be IOS in Y then \( f^{-1}(V) \) is ISCS in X (since \( f \) is I contra semi-cont. function). Now since every IROS is IOS then the inverse image of each IROS in Y is ISCS in X. Hence \( f \) is I contra semi-cont. function. \( \Box \)

We start with example to show that ISR-cont. function is not imply I perfectly cont. function.

**Example 4.15.** Let \( X = \{a, b, c\} \) and \( T = \{\emptyset, X, A, B\} \) where \( A = \{a, \{a\}, \{b\}\} \) and \( B = \{\emptyset, \{a\}, \{b, c\}\} \) and let \( Y = \{1, 2, 3\} \) and \( g = \{\emptyset, Y, C\} \) where \( C = \{c, \{2\}, \{3\}\} \).

Define a function \( f : X \rightarrow Y \) by
\[ f(a) = 1, f(b) = 2 \text{ and } f(c) = 3. \]
Now let \( U = f^{-1}(V) = \{a, \{a\}, \{b\}\} \) then
\[ \text{Int}(U) = G \subseteq G \text{ and } G \subseteq \text{cl}(G) = B, \]
that is G is ISCS and ISOS imply G is ISRS in X but G is not clopen set in X since \( \text{Int}(G) = B \neq G \) so \( \text{cl}(G) = B \neq G \). Then F is ISR-cont. function but f is I perfectly cont. function.

The next example shows that:
Example 4.16. Let $X = \{a, b, c\}$ and $T = \{\emptyset, X, A, B, C, D\}$ where $A = \{x, \{a\}, \{b\}\}$, $B = \{x, \{a, b\}, \{c\}\}$, $C = \{x, \{a, b\}, \emptyset\}$ and $D = \{x, \{a\}, \{b, c\}\}$ and let $Y = \{a, b, c\}$ and $\sigma = \{\emptyset, Y, E\}$ where $E = \{y, \{1\}, \{2\}\}$. Define a function $f : X \to Y$ by $f(a) = 1, f(b) = 2$, and $f(c) = 2$. Now let $G = f^{-1}(E) = \{x, \{a\}, \{b\}\}$ then $G$ is ISCS in X since $\text{Int} \cap G$, but $G$ is not ISOS since $G \cap \text{Int} G = \emptyset$ so $G$ is not ISRS in X as well as $G$ is not ICS in X since $\text{cI} G = D = G$, then the inverse image of each IOS in Y is ISCS in X.

We are going to show that I regular set connected function is not imply I perfectly connected function.

Example 4.17. Let $X = \{a, b, c\}$ and $T = \{\emptyset, X, A, B, C, D\}$ where $A = \{x, \{a\}, \{b\}\}$, $B = \{x, \emptyset, \{a, b\}\}$, $C = \{x, \{a, b\}, \emptyset\}$ and $D = \{x, \{a\}, \{b\}\}$ and let $Y = \{1, 2, 3\}$ and $\sigma = \{\emptyset, Y, E, F\}$ where $E = \{y, \{1\}, \{2\}\}$ and $F = \{y, \emptyset, \{1, 2\}\}$. Define a function $h : X \to Y$ by $h(a) = 1, h(b) = 2$, and $h(c) = 3$. Now a set $G = h^{-1}(E) = \{x, \{a\}, \{b\}\}$ then $G$ is ISCS in X but not IOS so it’s not clopen in X. Therefore, f is I almost contra cont. function but not I regular set connected.

The next example shows I almost contra cont. is not imply I contra cont.

Example 4.19. Let $X = \{a, b, c\}$ and $T = \{\emptyset, X, A, B\}$ where $A = \{x, \{a\}, \{b\}\}$ and $B = \{x, \{a, c\}, \emptyset\}$ and let $Y = \{1, 2, 3\}$ and $\sigma = \{\emptyset, Y, D\}$ where $C = \{y, \{3\}, \{1\}\}$ and $D = \{y, \emptyset, \{1, 2\}\}$. Define a function $h : X \to Y$ by $h(a) = 1, h(b) = 3$, and $h(c) = 2$. Now let $H = h^{-1}(C) = \{x, \{a\}, \{b\}\}$ then $H$ is not ICS in X since $\text{cl} H \neq X \neq H$, then $f$ is not I contra cont. function but $f$ is I almost contra cont. function.

The following example shows that I almost contra semi-cont. is not imply I almost contra cont.

Example 4.20. Let $X = \{a, b, c\}$ and $T = \{\emptyset, X, A, B\}$ where $A = \{x, \{a\}, \{b\}\}$ and $B = \{x, \emptyset, \{a, b\}\}$ and let $Y = \{1, 2, 3\}$ and $\sigma = \{\emptyset, Y, C\}$ where $C = \{y, \emptyset, \{1\}\}$. Define a function $h : X \to Y$ by $h(a) = 1, h(b) = 2$, and $h(c) = 3$. Now a set $G = h^{-1}(C) = \{x, \emptyset, \{a\}\}$ then $G$ is ISCS in X since $\text{Int} G \subseteq G$, but $G$ is not closed since $G \neq \emptyset$ so $G$ is not ICS in X and $\text{cI} G = A \neq G$, hence $f$ is I almost contra semi-cont. function but not I almost contra cont. function.

In the last example we show I almost contra semi-cont. is not imply I contra semi-cont.

Example 4.21. Let $X = \{a, b, c\}$ and $T = \{\emptyset, X, A, B\}$ where $A = \{x, \{a\}, \emptyset\}$ and $B = \{x, \{c\}, \{b\}\}$ and let $Y = \{1, 2, 3\}$ and $\sigma = \{\emptyset, Y, C\}$ where $C = \{y, \{3\}, \{1\}\}$. Define a function $h : X \to Y$ by $h(a) = 1, h(b) = 2$, and $h(c) = 3$. Now let $G = h^{-1}(B) = \{x, \emptyset, \{a\}\}$ then $G$ is ISCS in X but $G$ not IOS so it’s not clopen in X. Therefor, f is I almost contra cont. function but not I regular set connected.

We summarized the above result by the following diagram.

Diagram 4.22. The following implications are true and not reversed:
Proposition 4.23. Let \((X, T)\) and \((Y, \sigma)\) be two ITS's and let \(f: X \rightarrow Y\) be a function then \(I\) contra cont. and \(I\) almost contra cont. are equivalent if:

1. \((Y, \sigma)\) is discrete.
2. \((Y, \sigma)\) is indiscrete.
3. \((Y, \sigma)\) is disconnected.

REFERENCES


E.mail: scianb@yahoo.com