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المجموعات – α - شبه المنتظمة المغلقة

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الخلاصة

لقد قمنا في هذا البحث بتقديم ودراسة نوع جديد من المجموعات المغلقة في الفضاءات التوسيعية يدعى بالمجموعات – α - شبه المنتظمة المغلقة، إذ أن هذا النوع من المجموعات المغلقة تشوي مجموعات شبه مغلقة - α - وتكون محتوية في المجموعات قبل شبه المغلقة. وكما قمنا ودرسنا نوعا جديدا من الدوال المستمرة والمترددة تعني دالة من النمط - α - شبه المنتظمة المستمرة وبلا من النمط - α - شبه المنتظمة المترددة. كما وجدنا أن الاستمرارية من النمط - α - شبه المنتظمة تكون واقعة تماماً بين الاستمرارية من النمط - α - والاستمرارية من النمط قبل الشبه.

الكلمات المفتاحية: المجموعة من النمط - α - شبه المنتظمة المغلقة، الدالة من النمط - α - شبه المنتظمة المستمرة، الدالة من النمط - α - شبه المنتظمة المترددة.
α - Semi-Regular Closed Sets

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Abstract

In this paper, a new class of sets, namely α- semi-regular closed sets is introduced and studied for topological spaces. This class properly contains the class of semi-α-closed sets and is property contained in the class of pre-semi-closed sets. Also, we introduce and study αsr-continuity and αsr-irresolute. We showed that αsr-continuity falls strictly in between semi-α- continuity and pre-semi-continuity.

Key words: α- semi-regular closed set, α- semi-regular continuous, α- semi-regular irresolute.

Introduction

Najasted [1] and Levine [2] introduced α-open sets and generalized closed sets, Kummar introduced α-generalized regular closed set and pre-semi closed set, see [3] and [4]. Alot of work was done in the field of generalized closed sets. In this paper we employ a new technique to obtain a new class of sets, called α-semi-regular closed sets. This class is obtained by semi-α-closed set and regular open set. It is shown that the class of α-semi-regular closed sets properly contains the class of semi-α-closed sets and is properly contained in the class of pre-semi-closed sets. We also introduce and study two classes of maps, namely, α-semi-regular continuity and α-semi-regular irresoluteness, α-semi-regular continuity falls strictly in between semi-α-continuity and pre-semi-continuity.

1- Preliminaries

Throughout this paper (X,τ) and (Y,τ') represent non-empty topological spaces. For a subset A of a space (X,τ), cl(A) and int(A) represent the closure of A and the interior of A respectively.

1.1 Definition:

(1) an α-open set [1], [5] if A ⊆ int(cl(int(A))) and α-closed if cl(int(cl(A))) ⊆ A.
(2) a semi-α-open set [6], [7] if A ⊆ cl(int(cl(int(A)))) and semi-α-closed if int(cl(int(cl(A)))) ⊆ A.
(3) a semi-preopen set [8], [9] if A ⊆ cl(int(cl(A))) and semi-preclosed if int(cl(int(cl(A)))) ⊆ A.
(4) a regular open set [10], [11] if A = int(cl(A)) and regular closed if A = cl(int(A)).
(5) a generalized closed set (briefly g-closed) [2], [12] if cl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ). The complement of a g-closed set is called a g-open set.
(6) an α-generalized closed set (briefly αg-closed) [13] if α cl(A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
(7) a generalized α-closed set (briefly ga-closed) [14] if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in (X,τ).
(8) a generalized α*-closed set (briefly ga*-closed) [14] if αcl(A) ⊆ int(U) whenever A ⊆ U and U is α-open in (X,τ).
an $\alpha\ast\ast$-generalized closed set (briefly $\alpha\ast\ast$-g-closed) [14] if $\alpha\ cl(A) \subseteq \text{int}(cl(U))$ whenever $A \subseteq U$ and $U$ is open in $(X,\tau)$.

(10) a generalized $\alpha\ast\ast$-closed set (briefly $g\alpha\ast\ast$-closed) [14] if $\alpha\ cl(A) \subseteq \text{int}(cl(U))$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X,\tau)$.

(11) an $\alpha$-generalized regular closed set (briefly $\alpha gr$-closed) [3] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $(X,\tau)$.

(12) a regular generalized closed set (briefly rg-closed) [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $(X,\tau)$.

(13) a generalized semi-preclosed set (briefly gsp-preclosed) [16] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X,\tau)$.

(14) a pre-semi-closed set [4] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is g-open in $(X,\tau)$.

The semi-$\alpha$-closure (resp. $\alpha$-closure, semi-pre-closure) of $A$ in $(X,\tau)$ is the intersection of all semi-$\alpha$-closed (resp. $\alpha$-closed, semi-pre-closure) sets of $(X,\tau)$ that contain $A$ and is denoted by $S_\alpha cl(A)$ (resp. $\alpha cl(A), spcl(A)$).

1.2 Proposition:

(1) Every $\alpha$-closed set is semi-$\alpha$-closed set, not conversely, [6].

(2) Every closed set is $\alpha$-closed set, so it is semi-$\alpha$-closed set, not conversely, [6].

(3) Every closed (resp. $\alpha$-closed, g-closed, $g\alpha$-closed) set is an $\alpha gr$-closed set, [3].

(4) Every $g\alpha^\ast$-closed (resp. $\alpha\ast\ast$-g-closed, $g\alpha\ast\ast$-closed) set is an $\alpha gr$-closed set, [3].

(5) Every pre-semi-closed set is a gsp-closed set [4].

(6) Every semi-$\alpha$-closed set is semi-pre-closed set.

The proof follows directly from the definitions.

1.3 Remark: [6]

Let $X$ be a topological space, $A$ and $B$ be two subsets of $X$, then

(1) $A$ is semi-$\alpha$-closed set if and only if $A = S_\alpha cl(A)$.

(2) $A \subseteq S_\alpha cl(A) \subseteq \alpha cl(A) \subseteq cl(A)$.

(3) $S_\alpha cl(A) \subseteq S_\alpha cl(B)$, whenever $A \subseteq B$.

1.4 Definition:

A function $f(X,\tau) \longrightarrow (Y,\tau')$ is said to be:

(1) semi-$\alpha$-continuous [6], [7] if $f^{-1}(V)$ is a semi-$\alpha$-closed set in $(X,\tau)$ for every closed set $V$ of $(Y,\tau')$.

(2) g-continuous [17] if $f^{-1}(V)$ is a g-closed set in $(X,\tau)$ for every closed set $V$ of $(Y,\tau')$.

(3) $\alpha$g-continuous [18] if $f^{-1}(V)$ is an $\alpha$g-closed set in $(X,\tau)$ for every closed set $V$ of $(Y,\tau')$.

(4) ga -continuous [14] if $f^{-1}(V)$ is a ga-closed set in $(X,\tau)$ for every closed set $V$ of $(Y,\tau')$.

(5) $\alpha gr$-continuous [3] if $f^{-1}(V)$ is an $\alpha gr$-closed set in $(X,\tau)$ for every closed set $V$ of $(Y,\tau')$.

(6) pre-semi-continuous [4] if $f^{-1}(V)$ is a pre-semi-closed set in $(X,\tau)$ for every closed set $V$ of $(Y,\tau')$.

(7) gsp-continuous [16] if $f^{-1}(V)$ is a gsp-closed set in $(X,\tau)$ for every closed set $V$ of $(Y,\tau')$.

(8) semi-$\alpha$-irresolute [6] if $f^{-1}(V)$ is a semi-$\alpha$-closed set in $(X,\tau)$ for every semi-$\alpha$-closed set $V$ of $(Y,\tau')$.

(9) $\alpha gr$-irresolute [3] if $f^{-1}(V)$ is an $\alpha gr$-closed set in $(X,\tau)$ for every $\alpha gr$-closed set $V$ of $(Y,\tau')$.

(10) regular irresolute [19] if $f^{-1}(V)$ is a regular open set in $(X,\tau)$ for every regular open set $V$ of $(Y,\tau')$.

(11) semi-$\alpha^\ast$-closed [6] if $f(U)$ is a semi-$\alpha$-closed set in $(Y,\tau')$ for every semi-$\alpha$-closed set $U$ in $(X,\tau)$.

1.5 Proposition:

(1) Every $\alpha$g-continuous map is $\alpha gr$-continuous map [3].

(2) Every g-continuous (resp. ga-continuous) map is an $\alpha gr$-continuous map [3].

(3) Every pre-semi-continuous map is gsp-continuous map [4].
(4) Every $\alpha gr$-irresolut map is $\alpha gr$-continuous map [3].

(5) Every continuous and open map is semi-$\alpha$-irresolute map [20].

2- $\alpha$-Semi-Regular Closed Sets

In this section we introduce the class of $\alpha$-semi-regular closed sets and study some of its basic properties.

2.1 Definition:

A subset $A$ of $(X, \tau)$ is called $\alpha$-semi-regular closed set (briefly $\alpha sr$-closed) if $S_\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $(X, \tau)$.

$\alpha SRC(X)$ denotes the collection of all $\alpha sr$-closed subset of $(X, \tau)$.

2.2 Proposition:

Every semi-$\alpha$-closed set is an $\alpha sr$-closed set.

Proof: Let $A$ be a semi-$\alpha$-closed set, let $U$ be a regular open set of $(X, \tau)$ such that $A \subseteq U$. Since $S_\alpha cl(A) = A$ for any semi-$\alpha$-closed set (by part 1 of remark 1.3), then $S_\alpha cl(A) \subseteq U$. Therefore $A$ is also an $\alpha sr$-closed set.

The following example shows that the converse of the above proposition is not true in general.

2.3 Example:

Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, c\}\}$. Let $A = \{a, c\}$, $X$ is the only regular open set containing $A$. It is clear $A$ is an $\alpha sr$-closed set. But $A$ is not semi-$\alpha$-closed set since $S_\alpha cl(\{a, c\}) = X \neq \{a, c\}$.

Thus the class of $\alpha sr$-closed set properly contains the class of semi-$\alpha$-closed sets.

2.4 Proposition:

Every $\alpha gr$-closed set is an $\alpha sr$-closed set.

Proof: Let $A$ be an $\alpha gr$-closed set, let $U$ be a regular open set of $(X, \tau)$ such that $A \subseteq U$. Since $A$ is $\alpha gr$-closed set and $S_\alpha cl(A) \subseteq cl(A)$ (by part (2) of remark 1.3), then $S_\alpha cl(A) \subseteq U$. Therefore $A$ is also an $\alpha sr$-closed set.

The following example shows that the $\alpha sr$-closed set need not to be an $\alpha gr$-closed set.

2.5 Example:

Let $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a, b\}\}$. Let $A = \{b\}$, let $\{b\}$ is the regular open set containing $A$. Trivially $A$ is an $\alpha sr$-closed set since $S_\alpha cl(\{a, c\}) = X \neq \{a, c\}$. But $A$ is not $\alpha gr$-closed set since $\alpha cl(A) = \{b, c\} \not\subseteq \{b\}$.

2.6 Corollary:

Every closed (resp. $\alpha$-closed, g-closed, $\alpha g$-closed) set is an $\alpha sr$-closed set.

Proof: Since every $\alpha gr$-closed set is an $\alpha sr$-closed set, part (4) of proposition 1.2 is applicable.

The following example shows that the reverse implications in the above corollary are not true in general.

2.7 Example:

Let $X, \tau$ and $A$ be as in example 2.3. $A$ is neither closed (since $cl(A) = X \neq A$) nor $\alpha$-closed (since $\alpha cl(A) = X \neq A$) and also it is neither g-closed (since $A = \{a, c\} \subseteq \{a, c\}$ whenever $\{a, c\} \in \tau$, but $cl(A) = X \not\subseteq \{a, c\}$) nor $\alpha g$-closed (since $\alpha cl(A) = X \not\subseteq \{a, c\}$) whenever $\{a, c\} \in \alpha O(X)$, but $\alpha cl(A) = X \not\subseteq \{a, c\}$.

2.8 Corollary:

Every $\alpha g**$-closed (resp. $\alpha**$ g-closed, $\alpha** g$-closed) set is an $\alpha sr$-closed set.

Proof: Since every $\alpha gr$-closed set is an $\alpha sr$-closed set, part (4) of proposition 1.2 is applicable.

The following example shows that an $\alpha sr$-closed set needs not to be a $\alpha g**$-closed set.
2.9 Example:
Let \( X = \square \) and \( \tau = \tau_U \), let \( A = (a,b) \) is \( \alpha \)-\( sr \)-closed set but not a \( \alpha \)-\( * \)-closed set, since \( (a,b) \subseteq (a,b) \) and \( (a,b) \) is \( \alpha \)-open set in \( (\square, \tau_U) \), but \( \alpha \text{cl}(A) = [a,b] \not\subseteq (a,b) \).

2.10 Proposition:
Every \( r \)-\( g \)-closed is an \( \alpha \)-\( sr \)-closed set.

Proof: Let \( A \) be a regular generalized closed set of \( (X, \tau) \). Let \( U \) be a regular open set of \( (X, \tau) \) such that \( A \subseteq U \). Then \( \text{cl}(A) \subseteq U \) since \( A \) is \( r \)-\( g \)-closed set. Since every closed set is semi-\( \alpha \)-closed set, then \( S_\alpha \text{cl}(A) \subseteq \text{cl}(A) \) (part 2 of remark 1.3). Thus \( S_\alpha \text{cl}(A) \subseteq U \), therefore \( A \) is an \( \alpha \)-\( sr \)-closed set.

The converse of above proposition is not always true as the following example shows.

2.11 Example:
Let \( X \) and \( \tau \) be as in example 2.3, let \( A = \{c\} \) and \( U = \{c\} \) is regular open set containing \( A \).
It is clear \( A \) is an \( \alpha \)-\( sr \)-closed set since \( S_\alpha \text{cl}(A) = \{c\} \subseteq \{c\} \). But is not \( r \)-\( g \) closed set since \( \text{cl}(A) = \{b,c\} \not\subseteq \{c\} \).

2.12 Proposition:
Every \( \alpha \)-\( g \)-closed set is an \( \alpha \)-\( sr \)-closed set

Proof: Let \( A \) be an \( \alpha \)-\( g \)-closed set, let \( U \) be a regular open set of \( (X, \tau) \) such that \( A \subseteq U \). Since \( A \) is \( \alpha \)-\( g \)-closed and every regular open set is an open set, then \( \alpha \text{cl}(A) \subseteq U \). But \( S_\alpha \text{cl}(A) \subseteq \alpha \text{cl}(A) \) since every \( \alpha \)-closed set is semi-\( \alpha \)-closed set. Therefore \( A \) is also an \( \alpha \)-\( sr \)-closed set.

The converse in the above proposition is not true as it can be seen from the following example.

2.13 Example:
In example 2.3 \( \alpha \text{cl}(A) = X \not\subseteq \{a,c\} \). Thus \( A \) is not \( \alpha \)-\( g \)-closed set, but it is \( \alpha \)-\( sr \)-closed set.

2.14 Proposition:
Let \( A \) be an \( \alpha \)-\( sr \)-closed set of \( (X, \tau) \). Then \( S_\alpha \text{cl}(A)-A \) does contain any non-empty regular closed set.

Proof: Let \( F \) be any regular closed set of \( (X, \tau) \) such that \( F \subseteq S_\alpha \text{cl}(A)-A \). Then \( F \subseteq X-A \) implies that \( A \subseteq X - F \). Since \( A \) is \( \alpha \)-\( sr \)-closed and \( X - F \) is a regular open set of \( (X, \tau) \), then \( S_\alpha \text{cl}(A) \subseteq X - F \), so \( F \subseteq X - S_\alpha \text{cl}(A) \). Therefore \( F \subseteq S_\alpha \text{cl}(A) \cap (X - S_\alpha \text{cl}(A)) = \emptyset \). Hence \( S_\alpha \text{cl}(A) - A \) does not contain any non-empty regular closed set.

2.15 Proposition:
Every \( \alpha \)-\( sr \)-closed set is a \( \alpha \)-pre-semi-closed set.

Proof: Let \( A \) be a \( \alpha \)-\( sr \)-closed set of \( (X, \tau) \), let \( U \) be a regular open set of \( (X, \tau) \) such that \( A \subseteq U \). Then \( S_\alpha \text{cl}(A)-A \) does contain any non-empty regular closed set.

2.16 Corollary:
Every \( \alpha \)-\( sr \)-closed set is a \( \alpha \)-\( gr \)-closed set.

Proof: Follows from the above proposition and part (5) of proposition 1.2.

2.17 Corollary:
Every \( \alpha \)-\( gr \)-closed set is \( \alpha \)-\( sr \)-closed set.

Proof: Follows from the fact every \( \alpha \)-\( gr \)-closed set is \( \alpha \)-\( sr \)-closed and proposition 2.15.

2.18 Proposition:
If \( A \) is regular open and \( \alpha \)-\( sr \)-closed set then \( A \) is \( \alpha \)-\( gr \)-closed set.

Proof: It is clear.

2.19 Proposition:
Let \( A \) be an \( \alpha \)-\( sr \)-closed subset of \( (X, \tau) \). If \( B \subseteq X \) such that \( A \subseteq B \subseteq S_\alpha \text{cl}(A) \), then \( B \) is \( \alpha \)-\( sr \)-closed set.
Proof: Let $U$ be a regular open set of $(X, \tau)$ such that $B \subseteq U$. Then $A \subseteq U$, since $A$ is $\alpha_{sr}$-closed set, $S_\alpha c(A) \subseteq U$. Now, $S_\alpha c(B) \subseteq S_\alpha c(S_\alpha c(A)) = S_\alpha c(A) \subseteq U$. Therefore $B$ is also an $\alpha_{sr}$-closed set.

Fig. (1) shows the relations among the different types of weakly closed sets that were studied in this section.

3- $\alpha$-Semi Regular Continuous Maps and $\alpha$-Semi-Regular-Irresolute Maps

3.1 Definition:
A function $f: (X, \tau) \rightarrow (Y, \tau')$ is called an $\alpha$-semi-regular continuous map (briefly $\alpha_{sr}$-continuous if $f^{-1}(V)$ is an $\alpha_{sr}$-closed set of $(X, \tau)$ for every closed set $V$ of $(Y, \tau')$.

3.2 Proposition:
Every semi-$\alpha$-continuous map is $\alpha_{sr}$-continuous.

Proof: Follows from proposition 2.2.

We show that the class of $\alpha_{sr}$-continuous maps properly contains the class of $\alpha_{gr}$-continous maps.

3.3 Proposition:
Let $f: (X, \tau) \rightarrow (Y, \tau')$ be an $\alpha_{gr}$-continuous map. Then $f$ is an $\alpha_{sr}$-continuous map.

Proof: Let $V$ be a closed set of $(Y, \tau')$. Since $f$ is an $\alpha_{gr}$-continuous map, then $f^{-1}(V)$ is an $\alpha_{gr}$-closed set of $(X, \tau)$. By proposition 2.4 $f^{-1}(V)$ is an $\alpha_{sr}$-closed set of $(X, \tau)$. Thus $f$ is an $\alpha_{sr}$-continuous map.

The implications in proposition 3.3 is not reversible. Follows from the following example.

3.4 Example:
Let $X = \{a,b,c\} = Y$, $\tau = \{X,\emptyset,\{a\},\{b\},\{a,b\}\}$ and $\tau' = \{Y,\emptyset,\{a,c\}\}$. Define $f: (X, \tau) \rightarrow (Y, \tau')$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$, $\{b\}$ is a closed set of $(Y, \tau')$ but $f^{-1}(\{b\}) = \{a\}$ is not $\alpha_{gr}$-closed set of $(X, \tau)$. So $f$ is not $\alpha_{gr}$-continuous map. However $f$ is an $\alpha_{sr}$-continuous map.

3.5 Corollary:
Every $\alpha_{g}$-continuous map is $\alpha_{sr}$-continuous.

Proof: Follow from part (1) of proposition 1.5 and proposition 3.3.

The converse of the above corollary is not true in general as we see in the following example.

3.6 Example:
Let $X$, $Y$, $\tau$ and the definition of $f$ as in example 3.4, let $\tau' = \{Y,\emptyset,\{a\},\{b,c\}\}$. $f$ is not $\alpha_{g}$-continuous map since $\{b,c\}$ is a closed set of $(Y, \tau')$ but $f^{-1}(\{b,c\}) = \{a\}$ is not $\alpha_{g}$-closed set of $(X, \tau)$. However $f$ is an $\alpha_{sr}$-continuous map.

3.7 Corollary:
Every $\alpha_{gr}$-irresolute map is an $\alpha_{sr}$-continuous.

Proof: Necessity follows from part (4) of proposition 1.5 and proposition 3.3.

The converse of the above corollary is not true in general as we see in the following example.

3.8 Example:
See example 3.4 $f$ is $\alpha_{sr}$-continuous map but not $g$-continuous map.

3.9 Corollary:
Every $\alpha_{gr}$-irresolute map is an $\alpha_{sr}$-continuous.

Proof: Necessity follows from part (4) of proposition 1.5 and proposition 3.3.

The converse of the above corollary is not true in general as we see in the following example.
3.10 Example:
Let \( X = \{a, b, c\} = Y \), \( \tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\} \) and \( \tau' = I \). Define \( f : X \rightarrow Y \) by \( f(a) = c \), \( f(b) = b \) and \( f(c) = a \). \( \{b\} \) is an \( \alpha \text{-sr-closed} \) set of \((Y, \tau')\) but \( f^{-1}(\{b\}) = \{b\} \) is not \( \alpha \text{gr-closed} \) set of \((X, \tau)\). So \( f \) is not \( \alpha \text{gr-irresolute} \) map. However \( f \) is an \( \alpha \text{sr-continuous} \) map.

3.11 Theorem:
Let \( f : (X, \tau) \rightarrow (Y, \tau') \) be an \( \alpha \text{sr-continuous} \) map. Then \( f \) is a pre-semi-continuous map.

Proof : Let \( V \) be a closed set of \((Y, \tau')\). Since \( f \) is \( \alpha \text{sr-continuous} \) map, then \( f^{-1}(V) \) is an \( \alpha \text{sr-closed} \) set of \((X, \tau)\). By proposition (2.15) \( f^{-1}(V) \) is a pre-semi-closed set of \((X, \tau)\). Thus \( f \) is a pre-semi-continuous map.

3.12 Corollary:
Every \( \alpha \text{sr-continuous} \) map is gsp-continuous.

Proof: Follows from the above proposition and part (3) of proposition 1.5.

3.13 Definition:
A function \( f : (X, \tau) \rightarrow (Y, \tau') \) is called an \( \alpha \text{-semi-regular} \) irresolute (briefly \( \alpha \text{sr-irresolute} \) ) if \( f^{-1}(V) \) is an \( \alpha \text{sr-closed} \) set of \((X, \tau)\) for every \( \alpha \text{sr-closed} \) set of \((Y, \tau')\).

3.14 Proposition:
Let \( f : (X, \tau) \rightarrow (Y, \tau') \) be an \( \alpha \text{sr-irresolute} \) map. Then \( f \) is an \( \alpha \text{sr-continuous} \) map.

Proof: Let \( V \) be a closed set of \((Y, \tau')\). By corollary 2.6 \( V \) is an \( \alpha \text{sr-closed} \) set of \((Y, \tau')\). Since \( f \) is an \( \alpha \text{sr-irresolute} \) map, \( f^{-1}(V) \) is an \( \alpha \text{sr-closed} \) set of \((X, \tau)\). Therefore \( f \) is an \( \alpha \text{sr-continuous} \) map.

Thus the class of \( \alpha \text{sr-continuous} \) maps property continuous the class of \( \alpha \text{sr-irresolute} \) map.

3.15 Corollary:
Every \( \alpha \text{sr-irresolute} \) map is a pre-semi-continuous.

Proof: Follows from the above proposition and corollary 3.12.

3.16 Corollary:
Every \( \alpha \text{sr-irresolute} \) is a gsp- continuous.

Proof: Follows from proposition 3.14 and corollary 3.12.

3.17 Theorem:
Let \( f : (X, \tau) \rightarrow (Y, \tau') \) be a regular irresolute and semi-\( \alpha \)-irresolute map. Then \( f \) is \( \alpha \text{sr-irresolute} \) map.

Proof: Let \( A \) be an \( \alpha \text{sr-closed} \) set of \((Y, \tau')\), then there exists a regular open set \( U \) of \( Y \) such that \( S_{\alpha \text{cl}}(A) \subseteq U \) whenever \( A \subseteq U \). By taking the inverse image we get \( f^{-1}(S_{\alpha \text{cl}}(A)) \subseteq f^{-1}(U) \). Since \( f \) is regular irresolute map, then \( f^{-1}(U) \) is regular open subset of \( X \). Since \( f \) is semi-\( \alpha \)-irresolute map, then \( f^{-1}(S_{\alpha \text{cl}}(A)) \) is semi-\( \alpha \)-closed subset of \( X \). This implies \( S_{\alpha \text{cl}}(f^{-1}(S_{\alpha \text{cl}}(A))) = f^{-1}(S_{\alpha \text{cl}}(A)) \) (by part (1) of remark 1.3), then \( S_{\alpha \text{cl}}(f^{-1}(S_{\alpha \text{cl}}(A))) \subseteq S_{\alpha \text{cl}}(f^{-1}(S_{\alpha \text{cl}}(A))). \) Thus \( S_{\alpha \text{cl}}(f^{-1}(A)) \subseteq f^{-1}(U) \). Therefore \( f^{-1}(A) \) is \( \alpha \text{sr-closed} \) set in \( X \). Therefore \( f \) is \( \alpha \text{sr-irresolute} \) map.

3.18 Corollary:
Every continuous, open and regular irresolute map is \( \alpha \text{sr-irresolute} \).

Proof: It is clear by part (5) of proposition 1.5 and the above theorem.

3.19 Definition:
Let \( f : (X, \tau) \rightarrow (Y, \tau') \) be a function, then \( f \) is said to be:

(1) \( \alpha \)-semi-regular closed (briefly \( \alpha \text{sr-closed} \)) if \( f(A) \) is an \( \alpha \text{sr-closed} \) set of \((Y, \tau')\) for every closed set \( A \) of \((X, \tau)\).

(2) \( \alpha \text{*}-semi-regular closed (briefly \( \alpha \text{*sr-closed} \)) if \( f(A) \) is an \( \alpha \text{sr-closed} \) set of \((Y, \tau')\) for every \( \alpha \text{sr-closed} \) set \( A \) of \((X, \tau)\).

3.20 Remark:
It is clear that every closed function is \( \alpha \)-semi-closed function, but the converse is not true in general as the following example shows:
3.21 Example:
Let \( X = \{a, b, c, d\} \), \( \tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \). Define \( f:(X, \tau) \longrightarrow (X, \tau) \) by \( f(a) = a, f(b) = b, f(c) = f(d) = d \) we observe \( f \) is \( \alpha \)-semi-regular closed function which is not closed function since \( \{a, c, d\} \) is closed set in \( X \), but \( f(\{a, c, d\}) = \{a\} \) is not closed set in \( X \). Hence \( f \) is \( \alpha \)-semi-regular closed function, which is not closed function.

Finally, we prove the following theorem.

3.22 Theorem:
Let \( f:(X, \tau) \longrightarrow (Y, \tau') \) be a regular irresolute and semi-\( \alpha \)-*-closed map. Then \( f \) is \( \alpha \)-*-semi-regular closed map.

Proof: Let \( A \) be an \( \alpha \)-sr-closed set of \( (X, \tau) \), let \( U \) be a regular open set of \( (Y, \tau') \) such that \( f(A) \subseteq U \). Since \( f \) is regular irresolute, then \( f^{-1}(U) \) is a regular open set of \( (X, \tau) \). Since \( A \subseteq f^{-1}(U) \) and \( A \) is an \( \alpha \)-sr-closed, then \( S_\alpha \text{cl}(A) \subseteq f^{-1}(U) \). This implies \( f(S_\alpha \text{cl}(A)) \subseteq U \). Since \( f \) is semi-\( \alpha \)-*-closed map, then \( f(S_\alpha \text{cl}(A)) = S_\alpha \text{cl}(f(S_\alpha \text{cl}(A))) \). So \( S_\alpha \text{cl}(f(A)) \subseteq S_\alpha \text{cl}(f(S_\alpha \text{cl}(A))) = f(S_\alpha \text{cl}(A))) \subseteq U \). Therefore \( f(A) \) is an \( \alpha \)-sr-closed set of \( (Y, \tau') \).

3.23 Corollary:
Let \( f:(X, \tau) \longrightarrow (Y, \tau') \) be a regular irresolute and semi-\( \alpha \)-*-closed map. Then \( f(A) \) is a pre-semi-closed set of \( (Y, \tau') \) for every \( \alpha \)-sr-closed set of \( (X, \tau) \).

Proof: It is clear.

Fig. (2) explains the relationships among the different types of weakly continuous function.

References

Fig. (1) the relations among the different types of weakly closed sets

Fig. (2) the relationships among the different types of weakly continuous function.