ON WEAKLY $\lambda$-CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

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Abstract
As a generalization of $\lambda$-continuous functions, we introduce and study several properties of weakly $\lambda$-continuous functions in Bitopological spaces and we obtain its several characterizations.

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1. Introduction
The notion of $\lambda$-open sets due to al-talkany[1], semi-preopen sets due to Andrijević [2] plays a significant role in general topology. In [3] the concept of $\lambda$-continuous functions is introduced and further Popa and Noiri[5] studied the concept of weakly $\lambda$-continuous functions. In this paper, we introduce and study the notion of weakly $\lambda$-continuous functions in bitopological spaces further and investigate the properties of these functions.

Throughout the present paper, $(X,T,T^{a})$ denotes a bitopological space. Let $(X,\tau)$ be a topological space and $A$ be a subset of $X$. The closure and interior of $A$ are denoted by $Cl(A)$ and $Int(A)$ respectively.

Let $(X,T,T^{a})$ be a bitopological space and let $A$ be a subset of $X$. The closure and interior of $A$ with respect to $T$ or $T^{a}$ are denoted by $Cl_{T}(A)$, $int_{T}(A)$ or $Cl_{T}^{a}(A)$ and $Int_{T}^{a}(A)$ respectively.

2. basic definition

In this section we give all basic definition and some theorems and lemma we needs in this paper.

Definition 2.1 [1]. A subset $A$ of a bitopological space $(X,T,T^{a})$ is said to be
(i)regular open if $A=Int_{T}^{a}(Cl_{T}(A))$.
(ii)regular closed if $A=Cl_{T}(Int_{T}^{a}(A))$.
(iii)preopen if $A\subseteq Int_{T}^{a}(Cl_{T}(A))$.

Remark 2.1:
1. $\lambda$-interior mean that the interior w.r.t. $\lambda$-open set.
2. $\lambda$-$cl$ mean clouser w.r.t. $\lambda$-open set.

Definition 2.2. [1] A subset $A$ of a bitopological space $(X,T,T^{a})$ is said to be $\lambda$-open if there exist $T^{a}$-open set $U$ such that $A\subseteq U$, $A\subseteq Cl_{T}(U)$.

Lemma 2.1. [1] Let $(X,\tau_{1},\tau_{2})$ be a bitopological space and $A$ be a subset of $X$. Then
(i) $A$ is $\lambda$-open if and only if $A=\lambda Int(A)$.
(ii) $A$ is $\lambda$-closed if and only if $A=\lambda Cl(A)$.
Lemma 2.2. For any subset A of a bitopological space \((X,T,T^a)\), \(x \in \lambda Cl(A)\) if and only if 
\(U \cap A \neq \emptyset\) for every \(\lambda\)-open set \(U\) containing \(x\).

Definition 2.3. [4] A function \(f:(X,T,T^a) \rightarrow (Y,K,K^a)\) is said to be \(\lambda\)-continuous if \(f^{-1}(V)\) is
\(\lambda\)-open in \(X\) for each \(K\)-open set \(V\) of \(Y\).

3. Weakly \(\lambda\)-continuous

In this section we define weakly \(\lambda\)-continuous with some theorems

Definition 3.1. (i). A function \(f:(X,T,T^a) \rightarrow (Y,K,K^a)\) is said to be weakly precontinuous if for each \(x \in X\) and each \(K\)-open set \(V\) of \(Y\) containing \(f(x)\), there exists a \(\lambda\)-open set \(U\) containing \(x\) such that \(f(U) \subseteq Cl^a(V)\).

(ii). A function \(f:(X,T,T^a) \rightarrow (Y,K,K^a)\) is said to be weakly \(\lambda\)-continuous if for each \(x \in X\) and each \(K\)-open set \(V\) of \(Y\) containing \(f(x)\), there exists a \(\lambda\)-open set \(U\) containing \(x\) such that \(f(U) \subseteq Cl^a(V)\).

Example 3.1. Let \(X=\{a,b,c,d\}, T=\{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}\), \(T=T^a\)
\(\lambda\)-open \((X) = \emptyset\)
\(Y=\{1,2,3\}, K=\{Y, \phi, \{1\}\}, K^a=\{X, \phi, \{1\}, \{1,2\}, \{1,3\}\}\)
let \(f:(X,T,T^a) \rightarrow (Y,K,K^a)\) defined by \(f(a)=1, f(b)=f(c)=2\) then \(f\) is weakly \(\lambda\)-continuous.

Remark 3.1. The composition of two weakly \(\lambda\)-continuous is not necessary weakly \(\lambda\)-continuous.

Theorem 3.2. For a function \(f:(X,T,T^a) \rightarrow (Y,K,K^a)\), the following properties are equivalent:

(i). \(f\) is weakly \(\lambda\)-continuous.
(ii). \(\lambda Cl(f^{-1}(Int^a(Cl^a(B)))) \subseteq f^{-1}(Cl^a(B))\) for every subset \(B\) of \(Y\).
(iii). \(\lambda Cl(f^{-1}(Int^a(F))) \subseteq f^{-1}(F)\) for every regular closed set \(F\) of \(Y\).
(iv). \(\lambda Cl(f^{-1}(Cl(V))) \subseteq f^{-1}(Cl^a(V))\) for every \(K\)-open set \(V\) of \(Y\).
(v). \(f^{-1}(V) \subseteq \lambda Int(f^{-1}(Cl^a(V)))\) for every \(K\)-open set \(V\) of \(Y\).

Proof. (i) \(\rightarrow\) (ii). Let \(B\) be any subset of \(Y\). Assume that \(x \in X\) \(\sim f^{-1}(Cl^a(B))\). Then \(f(x) \in Y \sim Cl^a(B)\) and so there exists a \(K\)-open set \(V\) of \(Y\) containing \(f(x)\) such that \(V \cap B = \emptyset\), so \(V \cap \lambda Int^a(Cl^a(B)) = \emptyset\) and hence \(\lambda Cl^a(V) \cap \lambda Int^a(Cl^a(B)) = \emptyset\). Therefore, there exists a \(\lambda\)-open set \(U\) containing \(x\) such that \(f(U) \subseteq Cl^a(V)\).

Hence we have \(U \cap f^{-1}(Int^a(Cl^a(B))) = \emptyset\) and \(x \in X \sim \lambda Cl(f^{-1}(((Int^a(Cl^a(B))))))\) by Lemma 2.3. Thus we obtain \(\lambda Cl(f^{-1}(((Int^a(Cl^a(B)))))) \subseteq f^{-1}(Cl^a(B))\).

(ii) \(\rightarrow\) (iii). Let \(F\) be any regular closed set of \(Y\). Then \(F = Cl^a(Int^a(F))\) and we have \(\lambda Cl(f^{-1}((Int^a(Cl^a(Int^a(F))))) \subseteq f^{-1}(Cl^a(Int^a(F)))\).

(iii) \(\rightarrow\) (iv). For any \(K\)-open set \(V\) of \(X\). \(Cl^a(V)\) is regular closed. Then \(\lambda Cl(f^{-1}(V)) \subseteq \lambda Cl(f^{-1}(Int^a(Cl^a(V)))) \subseteq f^{-1}(Cl^a(V))\).

(iv) \(\rightarrow\) (v) Let \(V\) be any \(K\)-open set of \(Y\). The \(Y/Cl^a(V)\) is \(K\)-open set in \(Y\) and we have \(\lambda Cl(f^{-1}(Y/Cl^a(V))) \subseteq f^{-1}(Cl^a(Y/Cl^a(V)))\) and hence \(X/\lambda Int(f^{-1}(Cl^a(V))) \subseteq X/\lambda Int(f^{-1}(Cl^a(V)))\).

4. Weakly*-quasi continuous

Now we define the regular in the topological space \((X,T,T^a)\) with some theorems

Definition 4.1. A bitopological space \((X,T,T^a)\) is said to be regular if for each \(x \in X\) and each \(T\)-open set \(U\) containing \(x\), there exists a \(T\)-open set \(V\) such that \(x \in V \subseteq Cl^a(V)\) \(\subseteq U\).

Definition 4.2. A function \(f:(X,T,T^a) \rightarrow (Y,K,K^a)\) is said to be weakly*-quasi continuous (briefly \(w^*\)-continuous) if for every \(K\)-open set \(V\) of \(Y\), \(f^{-1}(Cl^a(V) \sim V)\) is biclosed in \(X\).
Theorem 4.3. If a function \( f:(X,T,T^a) \rightarrow (Y,K,K^a) \) is weakly-\( \lambda \)-continuous and w*-q.c, then \( f \) is \( \lambda \)-continuous.

Proof. Let \( x \in X \) and \( V \) be any \( K \)-open set of \( Y \) containing \( f(x) \). Since \( f \) is weakly-\( \lambda \)-continuous, there exists an \( \lambda \)-open set \( U \) of \( X \) containing \( x \) such that \( f(U) \subseteq Cl_{T^a}(V) \). Hence \( x \in f^{-1}(Cl_{T^a}(V)) \). Therefore, \( x \in U \cap f^{-1}(Cl_{T^a}(V)) \). Since \( U \) is \( \lambda \)-open and \( X \) is \( \lambda \)-open, then \( U \cap f^{-1}(Cl_{T^a}(V)) \) is \( \lambda \)-open. Then \( x \in f^{-1}(Cl_{T^a}(V)) \) and hence \( f(y) \in V \). Therefore, \( f \) is \( \lambda \)-continuous.

5. Almost \( \lambda \)-continuous

In this section we define almost \( \lambda \)-continuous with some theorems

Definition 5.1. A function \( f: (X,T,T^a) \rightarrow (Y,K,K^a) \) is said to have a \( \lambda \) interiority condition if \( \lambda \text{ Int}(f^{-1}(Cl_{T^a}(V))) \subseteq f^{-1}(V) \) for every \( K \)-open set \( V \) of \( Y \).

Definition 5.2. A function \( f: (X,T,T^a) \rightarrow (Y,K,K^a) \) is said to be almost \( \lambda \)-continuous if for each \( x \in X \) and each \( K \)-open set \( V \) containing \( f(x) \), there exists an \( \lambda \)-open set \( U \) of \( X \) containing \( x \) such that \( f(U) \subseteq Cl_{T^a}(V) \).

Lemma 5.1. A function \( f: (X,T,T^a) \rightarrow (Y,K,K^a) \) is almost \( \lambda \)-continuous if and only if \( f^{-1}(V) \) is \( \lambda \)-open for each regular open set \( V \) of \( Y \).

Definition 5.3. A bitopological space \( (X,T,T^a) \) is said to be almost regular if for each \( x \in X \) and each regular open set \( U \) containing \( x \), there exists a regular open set \( V \) of \( X \) such that \( x \in V \subseteq Cl_{T^a}(V) \subseteq U \).

Theorem 5.4. Let a bitopological space \( (Y,K,K^a) \) be almost regular. Then a function \( f: (X,T,T^a) \rightarrow (Y,K,K^a) \) is almost \( \lambda \)-continuous if and only if it is weakly-\( \lambda \)-continuous.

Proof. Necessity this is obvious

Sufficiency. Suppose that \( f \) is weakly-\( \lambda \)-continuous. Let \( V \) be any regular open set of \( Y \) and \( x \in f^{-1}(V) \). Then we have \( f(x) \in V \). By the almost-regularity of \( Y \), there exists a regular open set \( V_0 \) of \( Y \) such that \( f(x) \in V_0 \subseteq Cl_{T^a}(V_0) \subseteq V \). Since \( f \) is weakly-\( \lambda \)-continuous, there exists an \( \lambda \)-open set \( U \) of \( X \) containing \( x \) such that \( f(U) \subseteq Cl_{T^a}(V_0) \subseteq V \). This implies that \( x \in U \subseteq f^{-1}(V) \). Therefore we have \( f^{-1}(V) \subseteq \lambda \text{ Int}(f^{-1}(V)) \) and hence \( f^{-1}(V) = \lambda \text{ Int}(f^{-1}(V)) \). By Lemma 2.2, \( f^{-1}(V) \) is \( \lambda \)-open and by Lemma 5.1, \( f \) is almost \( \lambda \)-continuous.

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