On Jeffery Prior Distribution in Modified Double Stage Shrinkage-Bayesian Estimator for Exponential Mean  

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Abstract  
This paper is concerned with Modified Double Stage Shrinkage Bayesian (DSSB) Estimator for lowering the mean squared error of classical estimator $\hat{\theta}$ for the scale parameter ($\theta$) of an Exponential Distribution in suitable region $(R)$ around available prior knowledge ($\theta_0$) about the actual value ($\theta$) as initial estimate as well as to reduce the cost of observation. In situation where the observations are time consuming or very costly, a “Double Stage procedure “can be used to reduce the Expected Sample Size needed to obtain the estimator. This estimator has been showing a smaller Mean Squared Error for certain choice of the shrinkage weight factor $\psi(\cdot)$ and for acceptance region $R$. Expressions for Bias, Mean Square Error (MSE), Expected sample size $[E(n/\theta,R)]$, Expected sample size proportion $[E(n/\theta,R)/n]$, probability for avoiding the second sample $p(\hat{\theta}_1 \in R)$ and percentage of overall sample saved $\frac{100}{n}[p(\hat{\theta}_1 \in R)]$ for the proposed estimator are derived. Numerical results and discussions are established when the consider estimator (DSSB) are testimator of level of significance $\alpha$. Comparisons with the classical estimator as well as with some existing studies were made to shown the usefulness of the proposed estimator.

1. Introduction

1.1 The Model:
Exponential distribution is one of the most useful and widely exploited model. Epstein [1] remarks that the exponential distribution plays as important a role in life experiments as the part played by the normal distribution in agricultural experiments. It is applied in a very wide variety of statistical procedures. Among the most prominent applications are those in the field of life testing and reliability theory. The scale parameter ($\theta$) is known as mean life time. The maximum likelihood estimator (MLE; $\hat{\theta}$) is the sample mean which is the minimum variance unbiased estimator. The one parameter exponential distribution has the following probability density function (p.d.f.)

\[ f(t; \theta) = \begin{cases} \frac{1}{\theta} \exp\left(\frac{-t}{\theta}\right) & t \geq 0, \theta > 0 \\ 0 & \text{o.w.} \end{cases} \]  

(1)

where $\theta$ is the average or the mean life or mean time to failure (MTTF) and it is also acts as scale parameter, see [1].

Furthermore, the Reliability function $R(t)$ is defined as:

\[ R(t) = \exp(- t/\theta), t > 0, \theta > 0. \]

Note that the maximum likelihood estimator $\hat{\theta}$ of the scale parameter $\theta$ of the mentioned distribution is $\hat{\theta} = \frac{\sum t_i}{n}$.

1.2 Jeffery Prior distribution (Bayesian Estimator):
Consider the one parameter Exponential Distribution which is define in (1).

\[ \frac{1}{\theta} \exp\left(\frac{-t}{\theta}\right) \]

...
Based on the rule proposed by Jeffery, one can get the prior distribution of $\theta \ [g(\theta)]$ as below, see[2]

\[ g(\theta) \propto \sqrt{I(\theta)}, \text{ where } I(\theta) \text{ is fisher information such that} \]

\[ I(\theta) = -nE\left(\frac{\partial^2 \ln f(t, \theta)}{\partial \theta^2}\right) = \frac{n}{\theta^2}, \quad \text{ ... (2)} \]

\[ \therefore g(\theta) \propto \sqrt{n} \Rightarrow g(\theta) = k \sqrt{n} \]

\[ L(t_1, t_2, \ldots, t_n) = \prod_{i=1}^{n} f(t_i | \theta) = \frac{1}{\theta^n} \exp \left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) \]

The joint probability density function $H(t_1, t_2, \ldots, t_n, \theta)$ is given by

\[ H(t_1, t_2, \ldots, t_n, \theta) = \prod_{i=1}^{n} f(t_i | \theta) \ g(\theta) = L(t_1, t_2, \ldots, t_n, \theta) \ g(\theta) = \frac{1}{\theta^n} \exp \left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) \]

The marginal probability density function of $(t_1, t_2, \ldots, t_n)$ is given by

\[ p(t_1, t_2, \ldots, t_n) = \int_\theta H(t_1, t_2, \ldots, t_n, \theta) \ d\theta = \frac{n}{\theta^n} \exp \left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) \]

Let $\frac{\partial R(\hat{\theta}, \theta)}{\partial \theta} = 0$, then

\[ \hat{\theta}_B = \frac{\sum_{i=1}^{n} t_i}{(n-1)!} \int_0^\theta \theta^n \exp \left(-\frac{\sum_{i=1}^{n} t_i}{\theta}\right) \ d\theta \]

And the condition probability density function of $\theta$ given the data $(t_1, t_2, \ldots, t_n)$ is given by

\[ \Pi(\theta | t_1, t_2, \ldots, t_n) = \frac{H(t_1, t_2, \ldots, t_n, \theta)}{p(t_1, t_2, \ldots, t_n)} \]
1. Double Stage Shrinkage Procedure:

A Double Stage Shrinkage Procedure is defined as follows; see [3], [4], [5], [6]. Let \( x_{1i}; i = 1, 2, \ldots, n_1 \) be a random sample of \( n_1 \) from exponential distribution and \( \hat{\theta}_i \) be a classical estimator (MLE) of \( \theta \) based on \( n_1 \) observation. Construct a preliminary test region \( (R) \) in the parameter space based on prior estimate \( \theta_0 \) and an appropriate criterion. If \( \hat{\theta}_i \in R \) shrink \( \hat{\theta}_i \) towards \( \theta_0 \) by shrinkage weight factor \( \psi(\hat{\theta}_i); 0 \leq \psi(\hat{\theta}_i) \leq 1 \) and use the shrinkage estimator \( \hat{\theta}_i + (1 - \psi(\hat{\theta}_i))\theta_0 \), for estimate \( \theta \).

If \( \hat{\theta}_i \notin R \), obtain \( x_{2i}; i = 1, 2, \ldots, n_2 \), an additional sample of size \( n_2 \), and use a pooled estimator \( \hat{\alpha}_p \) of \( \alpha \) based on combined sample of size \( n = n_1 + n_2 \), i.e.; \( \hat{\alpha}_p = \frac{n\hat{\theta}_i + n_i\hat{\theta}_0}{n} \).

Thus, the Double Stage Shrinkage Estimator (DSSE) will be

\[
\hat{\theta}_{DS} = \begin{cases} 
\psi(\hat{\theta}_i)\hat{\theta}_i + (1 - \psi(\hat{\theta}_i))\theta_0 & \text{if } \hat{\theta}_i \in R \\
\hat{\theta}_p & \text{if } \hat{\theta}_i \notin R
\end{cases}
\] (8)

To motivation of this study was provided by the work of [3], [4], [5], [6], [7], [8], [9], [10] and others.

The aim of this paper is to employ Bayesian estimator which is defined in (5) in the form of double stage shrinkage estimator (DSSE) which is defined in (8) for estimate the scale parameter \( (\theta) \) of Exponential Distribution.

2. Modified Double Stage Shrinkage-Bayesian Estimator

This section is concern with pooling approach between shrinkage estimation that uses a prior information about unknown parameter as initial value and Bayesian estimation that uses a prior information about unknown parameter as a prior distribution for the scale parameter \( (\theta) \) of exponential distribution using special shrinkage weight factors as well suitable region \( (R) \) when a prior information about \( (\theta) \) is available as initial value \( (\theta_0) \).

In the present work we establish modified Double Stage Shrinkage-Bayesian Estimator (DSSBE) which has the following form:-

\[
\hat{\theta}_{DS} = \begin{cases} 
\psi(\hat{\theta}_i)\hat{\theta}_{IB} + (1 - \psi(\hat{\theta}_i))\theta_0 & \text{if } \hat{\theta}_i \in R \\
\frac{n_1\hat{\theta}_{IB} + n_2\hat{\theta}_{2B}}{n} & \text{if } \hat{\theta}_i \notin R
\end{cases}
\] (9)

where \( \hat{\theta}_{IB} (i = 1, 2) \) represent to Bayes estimator for \( \theta \) on \( n_{i1} = 1, 2 \) observations, \( R \) is suitable region (say pretest region) and \( \psi(\hat{\theta}_i); 0 \leq \psi(\hat{\theta}_i) \leq 1 \) is shrinkage weight factor which may be a function of \( \hat{\theta}_i \) or constant, see for example : [3], [4], [7] and [10].

The Expressions for Bias, Mean Square Error (MSE), Relative Efficiency [R.Eff(-)], Expected sample size, Expected sample size proportion, probability for avoiding the second sample and percentage of overall sample saved are derived and obtained for the proposed estimator.

Numerical results and discussions due mentioned expressions including some constants are performed and displayed in annexed tables.

Comparisons between the proposed estimator with the classical estimator \( (\hat{\theta}) \) and with some of the last studies are demonstrated.

2.1 Modified DSSBE(\( \hat{\theta}_{DS} \)) Using Constant Shrinkage Weight Factor:

Using the form (9) above, the proposed DSSBE \( \hat{\theta}_{DS} \) has the following forms:

\[
\hat{\theta}_{DS} = \begin{cases} 
\theta_0 & \text{if } \hat{\theta}_i \in R \\
\hat{\theta}_{pB} & \text{if } \hat{\theta}_i \notin R
\end{cases}
\] (10)

i.e.; we place \( \psi(\hat{\theta}_i) = 0 \) (constant).
Where \( R \) is pretest region for acceptance of size \( \alpha \) for testing the hypothesis \( H_0: \theta = \theta_0 \) Vs. the hypothesis \( H_A: \theta \neq \theta_0 \) using test statistic
\[
T(\hat{\theta}_1|\theta) = \frac{2n_1\hat{\theta}_1}{\theta_0}
\]
In that,
\[
R = \left[ \frac{\theta_0}{2n_1}X^2_{\alpha/2,n_1}, \frac{\theta_0}{2n_1}X^2_{\alpha/2,n_1} \right]
\]
Assume that, \( R = [a, b] \), \( a < b \).
i.e. \( a = \frac{\theta_0}{2n_1}X^2_{\alpha/2,n_1} \) and \( b = \frac{\theta_0}{2n_1}X^2_{\alpha/2,n_1} \)
where \( X^2_{\alpha/2,n_1} \) and \( X^2_{\alpha/2,n_1} \) are respectively lower and upper \( 100(\alpha/2) \) percentile point of Chi-square distribution with degree of freedom \( (2n_1) \).
The Expression for Bias is given below
\[
\text{Bias}(\hat{\theta}_{DS} | \theta, R) = E(\hat{\theta}_{DS}) - \theta
\]
\[
= \int_{\hat{\theta}_1 = a}^{\hat{\theta}_1 = b} \int_{\hat{\theta}_2 = a}^{\hat{\theta}_2 = b} (\hat{\theta}_1 - \theta) + \prod_{i=1}^2 f(\hat{\theta}_i; \theta)d\hat{\theta}_1d\hat{\theta}_2
\]
where \( \hat{\theta}_i \) is the complement region of \( R \) in real space and \( f(\hat{\theta}_i; \theta) \) for \( i=1,2 \) is a p.d.f. of \( \hat{\theta}_i \) which has the following form:
\[
f(\hat{\theta}_i; \theta) = \begin{cases} 
\frac{[\hat{\theta}_i]^{n-1}\exp[-n_1\hat{\theta}_i/\theta_{\lambda}]}{\Gamma(n_1)[\theta/\theta_{\lambda}]^{n_1}}, & \text{for } 0 < \hat{\theta}_i < \infty \\
0, & \text{o.w}
\end{cases}
\]
We conclude:
\[
\text{Bias}(\hat{\theta}_{DS} | \theta, R) = \theta \left\{ \lambda - 1 \right\} J_0(a^*, b^*) +
\]
\[
\left\{ \frac{1}{(1+u)(n_1 - 1)} + \frac{u}{(1+u)(n_1 - 1)} \right\} -
\]
\[
\left\{ \frac{1}{(1+u)} \left\{ \frac{n_1}{n_1 - 1} J_1(a^*, b^*) - J_0(a^*, b^*) \right\} \right\} -
\]
\[
\left\{ \frac{u}{(1+u)(n_1 - 1)} J_0(a^*, b^*) \right\}
\]
\[
\ldots (13)
\]
where,
\[
\lambda = \theta_0 / \theta, \quad y = n_1 \hat{\theta}_1, \quad u = n_2/y, \quad n = n_1 + n_2,
\]
\[
J_1(a^*, b^*) = \frac{1}{n_1[\Gamma(n_1)]_a^b} y^{-n_1-1} e^{y} dy \quad \ldots (14)
\]
And \( a^* = \lambda X^2_{\alpha/2,n_1} \), \( b^* = \lambda X^2_{\alpha/2,n_1} \) \ldots (15)
The Bias ratio \( B(\cdot) \) of \( \hat{\theta}_{DS} \) is defined as below
\[
B(\hat{\theta}_{DS} | \theta, R) = \frac{\text{Bias(\hat{\theta}_{DS} | \theta, R)}}{\theta} \quad \ldots (16)
\]
See [6], [7] and [9].
The expression of mean square error [MSE] of \( \hat{\theta}_{DS} \) is as follows
\[
\text{MSE}(\hat{\theta}_{DS} | \theta, R) = E(\hat{\theta}_{DS} - \theta)^2
\]
\[
= \theta^2 \left\{ \lambda - 1 \right\} J_0(a^*, b^*) +
\]
\[
\frac{1}{(1+u)^2} \left\{ \frac{n_1 + 1}{(n_1 - 1)(n_1 - u)} \right\} -
\]
\[
\frac{1}{(1+u)} \left\{ \frac{n_1^2}{(n_1 - 1)^2} J_1(a^*, b^*) -
\]
\[
\frac{2}{(1+u)} \left\{ \frac{n_1}{n_1 - 1} J_0(a^*, b^*) + J_0(a^*, b^*) \right\} -
\]
\[
\frac{u}{(1+u)} \left\{ \frac{n_1}{n_1 - 1} J_1(a^*, b^*) +
\]
\[
J_0(a^*, b^*) \right\} \ldots (17)
\]
The Relative Efficiency of Estimator \( \hat{\theta}_{DS} \) relative to classical estimator \( (\hat{\theta}_1) \) is defined as below:-
\[
\text{R.Eff}(\hat{\theta}_{DS} | \theta, R) = \frac{\text{MSE}(\hat{\theta}_1)}{\text{MSE}(\hat{\theta}_{DS} | \theta, R) E(n | \theta, R) / n} \quad \ldots (18)
\]
where \( E(n | \alpha, R) \) is the Expected sample size, which is defined as:
\[
E(n | \theta, R) = n \left\{ 1 - \frac{u}{1+u} J_0(a^*, b^*) \right\} .
\]
See for example [3], [6], [7], [9] and [10].
As well as, the Expected sample size proportion \( E(n|\alpha R)/n \) equal to
\[
1 - \frac{u}{1 + u} J_0(a^*, b^*) \quad \text{...}(19)
\]
See [6],[7]and [9]. Also, it is necessary to define the percentage of the overall sample saved (P.O.S.S) of \( \hat{\theta}_{DS} \) as:
\[
P.O.S.S = \frac{n_2 J_0(a^*, b^*) \times 100}{n} \quad \text{...}(20)
\]
See [6],[7]and [9].

And, finally, \( p(\hat{\theta}_1 \in R) \) represent the probability of a voiding the second sample.

3. Numerical Results and Discussion:

The computations of Relative Efficiency \([\text{R.Eff}(\cdot)]\) and Bias Ratio \([B(\cdot)]\), Expected sample size \([E(n|\theta R)]\), Expected sample size proportion \([E(n|\theta R)/n]\), Percentage of the overall sample saved (P.O.S.S.) and probability of avoiding the second sample \( p(\hat{\theta}_1 \in R) \) were used for the estimator \( \hat{\theta}_{DS} \). These computations (using Mat. LAB programs) were performed for \( n_1 = 4, 6, 8, 10, 12, 16, u = (n_2/n_1) = 0.5, 1, 2, 3, 9, 12, \lambda = (\theta_0/\theta) = 0.25(0.25)2, \alpha = 0.01, 0.05, 0.1 \).

Some of these computations are given in tables (1) - (12).

The observation mentioned in the tables leads to the following results:

i. The Relative Efficiency \([\text{R.Eff}(\cdot)]\) of \( \hat{\theta}_{DS} \) are adversely proportional with small value of \( \alpha \) especially when \( \lambda = 1 \), i.e. \( \alpha = 0.01 \) yield highest efficiency. see table (1) ,(2).

ii. The Relative Efficiency \([\text{R.Eff}(\cdot)]\) of \( \hat{\theta}_{DS} \) has maximum value when \( \theta = \hat{\theta}_0(\lambda=1) \), for each \( n_1 \) and \( \alpha \), and decreasing otherwise \( (\lambda \neq 1) \). This feature shown the important usefulness of prior knowledge which given higher Effects of proposed estimator as well as the important role of shrinkage technique and its philosophy. see table (1) ,(2).

iii. Bias Ratio \([B(\cdot)]\) of \( \hat{\theta}_{DS} \) are reasonably small when \( \theta = \hat{\theta}_0 \) for each \( n_1 \), \( \alpha \), and increases otherwise. This property shown that the proposed estimator \( \hat{\theta}_{DS} \) is very closely to unbiased ness property especially when \( \theta = \theta_0 \). See table (1) ,(2).

iv. The Effective interval of \( \hat{\theta}_{DS} \) [the value of \( \lambda \) which makes \( \text{R.Eff}(\cdot) \) of \( \hat{\theta}_{DS} \) greater than one] is approximately \([0.75, 1.25]\). See table (1) ,(2).

v. Bias Ratio \([B(\cdot)]\) of \( \hat{\theta}_{DS} \) are reasonably small with small value of \( u \). see table (1) ,(2).

vi. \( \text{R.Eff}(\hat{\theta}_{DS}) \) is decreasing function with increasing of the first sample size \( n_1 \), for each \( \alpha \) and \( \lambda \). See table (1) ,(2).

vii. The Expected value of sample size of \( \hat{\theta}_{DS} \) is close to \( n_1 \), especially when \( 0.5 \leq \lambda < 1 \) and start faraway otherwise. see table (3) ,(4),(5),(8), (9) ,(10).

viii. Percentage of the overall sample saved \( \frac{n_2 J_0(a^*, b^*) \times 100}{n} \) is increasing value with increasing value of \( u \) \((u = n_2/n_1)\) and decreasing value with increasing value of \( \lambda \geq 0.5 \). see table (6) ,(11).

ix. \( \text{R.Eff}(\hat{\theta}_{DS}) \) is an increasing function with respect to \( u \). This property shown the effective of proposed estimator using small \( n_1 \) relative to \( n_2 \) (or large \( n_2 \)) which given higher efficiency and reduce the observation cost. See table (1) ,(2).

x. the Probability of avoiding second sample is very suitable especially when \( \theta = \theta_0 \) see table (7) ,(12).

xi. The considered estimator \( \hat{\theta}_{DS} \) is better than the classical estimator \( \hat{\theta} \) especially when \( \theta = \theta_0 \), this will given the Effective of \( \hat{\theta}_{DS} \) Relative to \( \hat{\theta} \) and also given an important weight of prior knowledge, and the augmentation of efficiency may be reach to tens times. see table (1) ,(2).

xii. The considered estimator \( \hat{\theta}_{DS} \) is more efficient than the estimators introduced by [6] and [7] in the sense of higher efficiency.
References:


Table 1

Shown Bias Ratio \([B(\cdot)]\) and Relative Efficiency \([\text{R.Eff.}(\cdot)]\) of \(\tilde{\theta}_{DS}\) w.r.t. \(u, n_1, \lambda\) and \(\lambda\) when \(\alpha=0.01\)

<table>
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<tr>
<th>(u)</th>
<th>(n_1)</th>
<th>(\lambda)</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
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<td>-0.0078</td>
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Table (2)

Shown Bias Ratio [B(·)] and Relative Efficiency [R.Eff. (·)] of $\hat{\theta}_{DS}$ w.r.t. u, n₁ and λ when α=0.05

Abbasi Najim Salman  Assel Hussein Ali  Maha A. Mohammed
Table (3)
Shown Expected Sample Size \([E]\) and Expected Sample Size Proportion \([Ep]\) of \(\tilde{\theta}_{DS}\) w.r.t. \(u\) and \(\lambda\) when \(n_1 = 4, \alpha = 0.01\)

<table>
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Table (4)
Shown Expected Sample Size \([E]\) and Expected Sample Size Proportion \([Ep]\) of \(\tilde{\theta}_{DS}\) w.r.t. \(u\) and \(\lambda\) when \(n_1 = 6, \alpha = 0.01\)

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Table (5)
Shown Expected Sample Size \([E]\) and Expected Sample Size Proportion \([Ep]\) of \(\tilde{\theta}_{DS}\) w.r.t. \(u\) and \(\lambda\) when \(n_1 = 8, \alpha = 0.01\)

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Table (6)
Shown the Percentage of overall Sample Saved (P.O.S.S.) of $\theta_{DS}$ w.r.t. $u$, $n_1$ and $\lambda$ when $\alpha=0.01$

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Table (7)
Shown the Probability of avoiding Second Sample [Av] w.r.t. $u$, $n_1$ and $\lambda$ when $\alpha=0.01$

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Table (8)
Shown Expected Sample Size \([E]\) and Expected Sample Size Proportion \([Ep]\) of \(\tilde{\theta}_{DS}\) w.r.t. \(u\) and \(\lambda\)
when \(n_1 = 4, \alpha = 0.05\)

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Table (9)
Shown Expected Sample Size \([E]\) and Expected Sample Size Proportion \([Ep]\) of \(\tilde{\theta}_{DS}\) w.r.t. \(u\) and \(\lambda\)
when \(n_1 = 6, \alpha = 0.05\)

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Table (10)
Shown Expected Sample Size \([E]\) and Expected Sample Size Proportion \([Ep]\) of \(\tilde{\theta}_{DS}\) w.r.t. \(u\) and \(\lambda\)
when \(n_1 = 8, \alpha = 0.05\)

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Table (11)

Shown the Percentage of overall Sample Saved (P.O.S.S.) of $\tilde{\theta}_{DS}$ w.r.t. $u$, $n_l$ and $\lambda$ when $\alpha = 0.05$

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Table (12)
Shown the Probability of a Voiding Second Sample \([Av]\) w.r.t. \(u, n_1\) and \(\lambda\) when \(\alpha=0.05\)

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