Atmospheric Turbulences Image Restoration by Using Unconstrained Restoration Filters

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Abstract
Image restoration process or deblurring is the process in which the ideal image is estimated from that having a blur and noise and resulting from different degradation factors such as linear motion between object and camera, atmospheric turbulence and optical aberrations.

In this research, unconstrained linear restoration filters, which give the solution for the optimal image by one step without iteration such as inverse filter and wiener filter restore atmospheric turbulence images that have been photographed by using satellite.

Introduction
When aerial photographs are produced for remote sensing purposes, blurs are introduced by atmospheric turbulence, aberrations in the optical system, and relative motion between the camera and the ground [1].

The atmosphere is turbulent. These turbulence are first characterized by the deformation of the wave fronts seen by one telescope. The origin of these turbulence is that the eventual clouds, the gases composing the atmospheric move, the temperature and the pressure vary constantly, so is the index of refraction of air varying with time and space.

The atmospheric turbulence will affect the path of the light beams. When a star (or another stellar object) produces light, there is an isotropic distribution of the light beams; the wave fronts are spherical (like a stone falling into water produces spherical waves). If we observe these beams far enough from the star, we can locally approximate the spherical wave fronts as plane wave fronts. But when the plane wave fronts penetrate the atmosphere, the perturbations due to the turbulence modify the form of the wave fronts and they are not planar any more. The variation of index of refraction implies different speeds of the light waves for different beams.

The new form of wave fronts is varying continuously depending on the local atmospheric conditions. These effects make the light to be scattered and cause a blur, which distorts the image. This distortion may cause loss of some information contained in the image and increased difficulty of the remaining information by human observer. The lost information is irremovable. However, the image can be processed so that a human viewer more easily interests the remaining information [2].

If an optical transfer function of the turbulence can be associated with the turbulence and if this function is known then one method of processing is to correct by applying the function to the degraded image by this function. In some cases, the effect of the turbulence may result in an optical transfer function, which is not easily represented by an analytical function. An example the case in which the turbulence causes rapid non-symmetric changes in the structure of the image, and the image is re-sampled in a short period of time. To apply image processing techniques to a single image under these conditions require that the optical transfer function be recorded in detail at the same time as the image [3].

Theory
Consider an incoherently illuminated object imaged by an optical system. In the absence of turbulence, the image plane intensity distribution is the ideal image \( g(x,y) \) in the presence of turbulence, the distribution is the input image \( g(x,y) \).

The distribution is characterized by an input point spread function \( h(x,y) \) which is assumed to be independent of its location in the image plane (space-invariant)[3]. Thus, the image can be represented by a convolution of position independent operator, \( h(x,y,y,y) \), with the ideal image \( f(x,y) \), i.e.:\n
\[
g(x,y) = \int f(x_0,y_0) h(x-x_0,y-y_0) dx_0 dy_0\quad (1)
\]

where \((x,y)\) and \((x_0,y_0)\) are the coordinates of object and image, respectively, i.e.:\n
\[
g(x,y) = f(x,y) \otimes h(x,y)\quad (1)
\]

where \(h(x,y)\) is the point spread function of blur function which is invariant-space blur \(x,y\) are spatial coordinates.

This function convolves with the original image \( f(x,y) \) to produce the degraded image [3].

Also, many of physical mechanisms of recording images e.g. photographic film or photo sensor are inherently non-linear and cause the noise which is dependent on the recorded signal [4].

For simplicity, the noise contributions will modeled as an additive zero-mean white Gaussian.
noise process with variance \( \sigma^2 \), which is statistically uncorrelated with the image.

Thus, the image model of the degraded image will be in the following formulation:

\[
g(x, y) = f(x, y) \ast h(x, y) + n(x, y) \quad (2)
\]

where \( n(x, y) \) is the noise term.

For computational simplicity, the eq. (1) should be transformed into frequency domain by using Fourier transform as follow[3]:

\[
G(u, v) = H(u, v) \hat{F}(u, v) + N(u, v)
\]

where \( G(u, v), H(u, v), \hat{F}(u, v) \) and \( N(u, v) \) are the Fourier Transforms of the degraded image, PSF, original image and noise respectively.

Unconstrained restoration techniques

Unconstrained linear techniques can be considered in the absence of knowledge about the noise \( n \) [3,6].

From equation:

\[
g = Hf + n
\]

The noise term is given by:

\[
n = g - Hf \quad \quad \quad \quad (3)
\]

Where \( n, g, H \) and \( f \) are the noise, degraded image, point spread function and ideal image, respectively.

To make \( H\hat{f} \) approximated \( g \), then the noise term is assumed to be as small as possible. In the other words, we wish to find an \( \hat{f} \) such that:

\[
\frac{1}{2} || g - H\hat{f} ||^2 \quad \quad \quad \quad (4)
\]

is minimum.

Where \( \hat{f} \) is the restored image.

From equation (4), we consider problem to minimize the criterion function:

\[
J(\hat{f}) = || g - H\hat{f} ||^2 \quad \quad \quad \quad (5)
\]

with respect to \( \hat{f} \).

This is obtained by taking partial derivative with respect to \( \hat{f} \), then setting the result equal to zero. That is:

\[
\frac{\partial J(\hat{f})}{\partial \hat{f}} = 0 \quad \Rightarrow \quad 2H^* (g - H\hat{f}) = 0 \quad \quad \quad \quad (6)
\]

where \( H^* \) is the complex conjugate of the point spread function.

Then, solving equation (6) yield:

\[
\hat{f} = H^{-1} g \quad \quad \quad \quad (7)
\]

from equation (7), the inverse filter expresses as:

\[
H_i(u, v) = \frac{1}{H(u, v)} \quad \quad \quad \quad (8)
\]

by taking the Fourier transform of equation (7) results:

\[
\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)} \quad \quad \quad \quad (9)
\]

where \( \hat{F}(u, v), G(u, v) \) and \( H(u, v) \) are the Fourier transforms of restored image, degraded image, and point spread function, respectively.

This means that the restored image is obtained by multiplying the degraded image by the inverse filter function \( \left( \frac{1}{H(u, v)} \right) \). Thus, the restored image is given by the following equation:

\[
\hat{f}(x, y) = \hat{F} \cdot \left( \hat{f}(u, v) \right) = F^* \cdot \left[ \frac{G(u, v)}{H(u, v)} \right] \quad \quad (10)
\]

where \( F^* \) represents the inverse Fourier transform.

Equation (10) point out that computational will be encountered in the restoration process. This may be points or regions in the uv-plane where \( H(u, v) = 0 \). In the absence of noise, the transform \( G(u, v) \) of the degraded image would also be zero at those frequencies, leading to indeterminate solutions.

So we see that even in the absence of noise, it is in general impossible to reconstruct \( f(x, y) \) exactly if \( H(u, v) = 0 \).

In the presence of noise, we have:

\[
G(u, v) = H(u, v) \hat{F}(u, v) + N(u, v) \quad \quad \quad (11)
\]

where \( N(u, v) \) is the Fourier transform of noise.

So that applying the restoring filter gives:

\[
\hat{F}(u, v) = \left[ \frac{1}{H(u, v)} \right] \frac{1}{1 + \frac{N(u, v)}{H(u, v)}} \quad \quad (12)
\]

in the neighborhood of zeros of \( H(u, v) \), it may take on values much smaller than those of \( N(u, v) \), thus, the term \( \frac{N(u, v)}{H(u, v)} \) may have larger magnitude than \( F(u, v) \) in such neighborhood. The inverse transform of \( G(u, v) \) \( H(u, v) \) will then be strongly influenced by these large term, and will no longer resemble \( \hat{f}(x, y) \).

Least square (wiener) filter:

One method to avoid the problem of dividing, or as called ill-conditioned problem, is to find \( \hat{f}(x, y) \) of an image that minimizes some measure of difference between the original and restored one [5]. This minimization can be performed by using the least mean squares filter or as called Wiener filter.
This filter minimizes the mean squared error between \( f \) and \( \hat{f} \), i.e., the original image \( f \) and its restored \( \hat{f} \) is minimized.

\[
\text{OR} E[|g - H\hat{f}|^2] = \min
\]

Where \( E \{ \ldots \} \) is the expectation operator and is a parameter depending on the noise quantity.

Noise estimation depends on the assumptions about the spectral densities of the image and noise to determine the signal to noise ratio [11].

We can write the mathematical formulation of Wiener filter in frequency domain as follows [8.5.12]:

\[
\hat{F}(u,v) = \frac{H^*(u,v) \cdot S_n(u,v)}{[H(u,v)]^2 + \gamma \cdot \frac{S_n(u,v)}{S_s(u,v)}} G(u,v) \quad \text{(12)}
\]

where \( S_s, S_n \) are the power spectra of signal and noise, respectively when, \( \gamma = 1 \), then the equation (13) is called Wiener filter with following form:

\[
\hat{F}(u,v) = \frac{H^*(u,v) \cdot S_n(u,v)}{[H(u,v)]^2 + \frac{S_n(u,v)}{S_s(u,v)}} G(u,v) \quad \text{(13)}
\]

The difficulty in implementing Wiener filter comes from obtaining a priori information about the image and noise spectral functions.

The power spectra of object and noise are unknown. Therefore, the ratio of power spectra of noise to the signal mentioned in equation (15) can be substituted by a constant which represents the signal-to-noise ratio as follows [4.10] -

\[
\frac{S_n(u,v)}{S_s(u,v)} = \frac{1}{\text{SNR}^2}
\]

The general formula of Wiener filter can be expressed as follows:

\[
\hat{F}(u,v) = \frac{H^*(u,v)}{[H(u,v)]^2 + \frac{1}{\text{SNR}^2}} G(u,v) \quad \text{(14)}
\]

**Procedure and results**

The image shown in figure (1-a) was photographed by the satellite (Landsat) using the detector TM. The image represents the surface of Iraq, which described as a dusty weather. This adds another degradation factor to the atmospheric turbulences.

The degraded image shown in figure (1-a) seems to be blurred and has a little noise. The histogram of the degraded image was shown in figure (1-a). This degraded image was restored by using unconstrained restoration filters such as inverse filter and wiener filter.

Figure (2-a) represents the restored image by using inverse filter while figure (2-b) represents the histogram of the restored image. Figures (3-a) and (3-b) show the restored image by using wiener filter and its histogram, respectively.

![Figure 1-a](image1.png)

**Figure 1-a** the original image

![Figure 1-b](image2.png)

**Figure 1-b** the histogram of the original image
Figure (2-a) the blurred noisy image

Figure (2-b) the histogram of the blurred noisy image

Figure (3-a) the restored image by using inverse filter

Conclusions

In this research, our goal is to improve the quality of the degraded images affected by atmospheric turbulences. This was achieved by using the restoration algorithms, which give the direct solution, or what is called one-shot solution.
The result obtained by using inverse filter is better than that resulting by using wiener filter. The restored image by using inverse filter is clear because the blur was reduced, while the restored image by using wiener filter remained to be blurred. This can be concluded by using the histogram of each image.

From the histogram, it is noted the following:
1. the histograms of restored images are better than that of the degraded image because they are near to the Gaussian distribution which means that the degraded image have been improved.
2. the histogram of the restored image by using inverse filter is the nearest to the Gaussian distribution which means that the restored image by using inverse filter is the best.

Reference