NUMERICAL MODELING OF MIXED CONVECTION HEAT TRANSFER OF LAMINAR FLUID IN RECTANGULAR ENCLOSURE WITH OPENINGS IN THE SIDE

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ABSTRACT
An analysis of mixed convection heat transfer inside a rectangular enclosure with side openings was numerically examined. The fluid enters the enclosure through an opening in the left vertical wall and exits from another opening placed in the right vertical wall. Two-dimensional Navier-Stokes equations are solved by using control volume based finite element technique. For mixed convection, the significant parameters are Grasfof number, Richardson number, and Reynolds number by which different fluid and heat transfer characteristics inside the enclosure are obtained. Streamlines, isotherms, average Nusselt number, maximum temperature and other parameters of the heated wall are reported in the present study for Ri=0 to 10, Re=50,100,500,1000, Pr=0.7 to 100 and different aspect ratios. The computational results indicate that the heat transfer coefficient is strongly affected by Reynolds number and Richardson number. Higher Nusselt number is observed at large value of Pr. The comparison of the present results has been made are found in good agreement with known results.

Keyword: numerical, Natural convection, heated plate.
النماذج العددية لانتقال الحرارة بالحمل المختلط لمنائج طبقي في حيز مستطييل ذو فتحات في جانبه

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الخليفة:

في هذه الدراسة، تم تحليل انتقال الحرارة بالحمل المختلط في وسط مغلق مستطييل الشكل بوجود فتحات جانبية بصوره عديم.

يدخل المائع عن طريق فتحة موجودة في الجدار العمودي الأيسر للوسط مغلق و يغادر من خلال فتحة أخرى في الجدار العمودي الأيمن. تم استخدام عدارات نتير ستوك ببعدين وباستخدام تقنية الحجوم المحدودة العوامل المؤثرة في الحمل المختلط هي رقم كريشوف، ورقم رينولدز ورقم ريكاردسون. وقد تم الحصول على خواص حرارية مختلفة مانع داخل الوسط من خلال عوامل الانسياب، خطوط درجات الحرارة الثابتة، رقم نسلت، أكبر درجة حرارة وعمولات أخرى للجدار المغلق. وقد تم استخدام مدى واسع لهذه المعادلات (0 ≤ Re ≤ 50,100 ≤ Ri ≤ 0,7) وعدد نسبة عرض على طول مختلفة.

وقد بينت الحسابات أن معالج كان يتأثر بصورة كبيرة بثب رينولدز وثب ريكاردسون، حيث وجد أن أكبر قيمة لثب نسلت تكون عند

 أهم عدد برايتل كبير. وقد قوبلت النتائج لهذه الدراسة مع الدراسات السابقة ووجد تطابق جيد.

الكلمات الدلالية: عديم، حمل طبيعى، صفحتى مسخنة.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>g</td>
<td>gravitational acceleration ((\text{ms}^{-2}))</td>
</tr>
<tr>
<td>A</td>
<td>Aspect ratio ((D/H))</td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number, (g \beta q H^4/\nu^2 k)</td>
</tr>
<tr>
<td>h</td>
<td>convective heat transfer coefficient ((Wm^{-2}K^{-1}))</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity of the fluid ((Wm^{-2}K^{-1}))</td>
</tr>
<tr>
<td>H</td>
<td>Height of the vertical sidewall of the enclosure ((m))</td>
</tr>
<tr>
<td>(h_i)</td>
<td>Height of inflow and outflow openings</td>
</tr>
<tr>
<td>D</td>
<td>Width of the enclosure</td>
</tr>
<tr>
<td>(D_H)</td>
<td>length of the heated wall</td>
</tr>
<tr>
<td>(Nu_d)</td>
<td>Average Nusselt number</td>
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<tr>
<td>N</td>
<td>non-dimensional distance</td>
</tr>
<tr>
<td>P</td>
<td>pressure ((\text{Nm}^{-2}))</td>
</tr>
<tr>
<td>(P)</td>
<td>non-dimensional pressure</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number, (\nu/\alpha)</td>
</tr>
<tr>
<td>q</td>
<td>heat flux ((\text{Wm}^{-2}))</td>
</tr>
<tr>
<td>Ra</td>
<td>Rayleigh number, (Gr\cdot Pr)</td>
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<tr>
<td>Re</td>
<td>Reynolds number, (u_i H/\nu)</td>
</tr>
<tr>
<td>Ri</td>
<td>Richardson number, (Gr/Re)</td>
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</table>
Introduction

In a mixed convection, both natural convection and forced convection participate in the heat transfer process [1]. Thermal buoyancy forces play a significant role in forced convection heat transfer when the velocity is relatively small and the temperature difference between the surface and the free stream is relatively large. The buoyancy force modifies the flow and the temperature fields and hence the heat transfer rate from the surface [2]. Many theoretical as well as experimental studies have been made for either pure forced convection or pure free convection [3]. Very little has been known about combined free and forced convection heat transfer. In reality, pure forced convection heat transfer seldom occurs since density is dependent on temperature. Mixed convection, i.e., combined free and forced convection is the most general heat transfer phenomena. Pure free and pure forced convection are only limiting cases when either type of mixing motion can be neglected in comparison to the other [4].

Arpaci and Larsen [5] have presented an analytical treatment of the mixed convection heat transfer in tall cavities which had one vertical side moving, other vertical boundaries at different temperatures and horizontal boundaries are adiabatic. They showed that in this particular case, the forced and buoyancy-driven parts of the problem could be solved separately and combined to obtain the general mixed convection problem. Manca et al. [6] worked out a numerical investigation of three different cases of mixed convection in a channel with an open cavity for various ratios of channel opening and cavity height and the range of the governing parameters were 0.1 < Ri < 100, and Re = 100 and 1000. Mixed convection in enclosures with one isolated heat source, in which the interaction between an external forced flow and an internal buoyancy flow determines the fluid flow and heat transfer structures, has received considerable attention. Papanicolaou and Jaluria [7] numerically studied two-dimensional laminar mixed convection in a rectangular enclosure with a discrete heat source mounted on the wall. Ahmed M., E. [3], studied mixed convection in vertical stationary eccentric annulus for power law fluids using finite difference analysis. The nonlinear coupled momentum and energy equations have been solved using an iterative technique to obtain the velocity and temperature profiles. The outer surface of the annulus is considered to be adiabatic, while the inner surface has a uniform temperature.

Carios et al. [8] studies combined forced and free convection heat transfer in semi porous two-dimensional rectangular open cavity. The open cavity consist of two vertical walls closed to the bottom by uniform heat flux. One vertical wall is a porous wall and fluid inflows normal to it. The other wall transfers the same uniform heat flux to the cavity. They showed how natural convection effects may improve the forced convection inside the open cavity. Mixed convective is one of the preferred methods for cooling computer systems and other electronic equipments. In this paper, we
investigate the problem of mixed convection in rectangular horizontal vented enclosure of different aspect ratios with a uniform constant flux heat source embedded on the bottom wall, and the other walls are insulated. The objective of this study, is to analyze the effect of Prandl, Richardson, and Reynolds numbers on the heat transfer characteristics of laminar mixed convection.

**MATHEMATICAL ANALYSIS**

We consider a rectangular enclosure of different aspect ratios with uniform constant-flux heat source \( q \) applied on the bottom wall. The inflow opening located on the left vertical wall and the outflow opening at the top of right vertical wall and the size of the inlet port is the same size as the exit port which is equal to \( h_i=0.1H \). The other walls are assumed adiabatic. Details of the geometry and coordinate system are shown in Fig.(1). The flow velocity of the fluid through the inflow opening is assumed to be uniform \( u_i \) at constant temperature \( T_i \). The governing equations are those expressing the conservation of mass, momentum, and energy. The flow is considered to be steady, laminar and two-dimensional. Constant thermal properties are assumed except for the density in the body force term of the \( y \)-momentum equation which is modeled by the Boussinesq approximation.

The governing equations for steady mixed convection flow can be written as [2]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
u \left( \frac{\partial v}{\partial x} \right) + v \left( \frac{\partial u}{\partial y} \right) = -\left( \frac{1}{\rho} \right) \left( \frac{\partial p}{\partial x} \right) + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2}
\]

\[
u \left( \frac{\partial u}{\partial x} \right) + v \left( \frac{\partial v}{\partial y} \right) = \left( \frac{1}{\rho} \right) \left( \frac{\partial p}{\partial y} \right) + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + g\beta(T - T_0) \tag{3}
\]

\[
u \left( \frac{\partial T}{\partial x} \right) + v \left( \frac{\partial T}{\partial y} \right) = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}
\]

Using the following dimensionless variables[8]

\[
\begin{align*}
X &= \frac{x}{H}, & Y &= \frac{y}{H} \\
U &= \frac{u}{u_i}, & V &= \frac{v}{u_i} \\
P &= \frac{P}{\rho u_i^2}, & \theta &= \frac{T - T_i}{qH/K} \tag{5}
\end{align*}
\]
The governing equations (1) to (4) reduce to non-dimensional form:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad \ldots (6)
\]

\[
U \cdot \frac{\partial U}{\partial X} + V \cdot \frac{\partial U}{\partial Y} = -\left(\frac{\partial P}{\partial X}\right) + \frac{1}{Re} \left[\left(\frac{\partial^2 U}{\partial X^2}\right) + \left(\frac{\partial^2 U}{\partial Y^2}\right)\right] \quad \ldots (7)
\]

\[
U \cdot \frac{\partial V}{\partial X} + V \cdot \frac{\partial V}{\partial Y} = -\left(\frac{\partial P}{\partial Y}\right) + \frac{1}{Re} \left[\left(\frac{\partial^2 V}{\partial X^2}\right) + \left(\frac{\partial^2 V}{\partial Y^2}\right)\right] + \frac{Gr}{Re^2} \theta \quad \ldots (8)
\]

\[
U \cdot \frac{\partial \theta}{\partial X} + V \cdot \frac{\partial \theta}{\partial Y} = \frac{1}{Re Pr} \left[\left(\frac{\partial^2 \theta}{\partial X^2}\right) + \left(\frac{\partial^2 \theta}{\partial Y^2}\right)\right] \quad \ldots (9)
\]

The dimensionless parameters are used to describe the performance of fluid flow and heat transfer characteristics are as follows: Reynolds number

\[
Re = \frac{u_i H}{\nu}
\]

Grashof number for constant heat flux(q)source is

\[
Gr = \frac{gbqH^4}{K\nu^2}
\]

And Prandtl number is

\[
Pr = \frac{\nu}{\alpha}
\]

The other two governing parameters, Richardson number and Peclet number are defined as :

\[
Ri = \frac{Gr}{Re^2}
\]

\[
Pec = Re \cdot Pr
\]

The heat transfer calculation within the enclosure is measured in terms of the average Nusselt number at the heated wall as follows[1]:

\[
Nu = \frac{1}{D_H} \int_0^{D_H} \frac{h(x)X}{K} \, dx = \frac{1}{D_H} \int_0^{D_H} \frac{X}{\theta_H} \, dX \quad \ldots (10)
\]

Where \(D_H\) and \(h(x)\) are the length and the local convection heat transfer coefficient of the heated wall respectively, \(\theta_H\) is the local dimensionless temperature.

**BOUNDARY CONDITIONS**

On each of the computational domain boundaries, boundary conditions are required to solve the differential equations. The boundary conditions are:

\[u = u_i \quad , \quad v = 0 \quad , \quad T = T_i \quad \text{at the inlet}\]
\[ u = 0, \quad v = 0, \quad \frac{\partial T}{\partial x} = 0 \text{ at exit} \]

\[ u = 0, \quad v = 0, \quad q = -K \frac{\partial T}{\partial x} \] along the heated bottom wall

\[ u = 0, \quad v = 0, \quad \frac{\partial T}{\partial x} = 0 \] along the vertical insulated walls

\[ u = 0, \quad v = 0, \quad \frac{\partial T}{\partial y} = 0 \] along the horizontal top wall

Using the dimensionless variables, the boundary conditions become:

\[ U = V = \frac{\partial \theta}{\partial N} = 0 \] (at the enclosure walls except the horizontal bottom wall)

\[ U = 1, \quad V = 0, \quad \theta = 0 \] at the inlet

\[ \frac{\partial U}{\partial X} = \frac{\partial V}{\partial X} = \frac{\partial \theta}{\partial X} = 0 \] at the exit

\[ U = V = 0, \quad \frac{\partial \theta}{\partial Y} = -1 \] (At the heated horizontal bottom wall)

**COMPUTATIONAL PROCEDURE**

The governing equations for \((U, V, \theta)\) can be written in a common form for the (convection-diffusion) problem as follow \([9]\):

\[
\frac{\partial}{\partial X_i} \left( \rho u_i \Phi \right) = \frac{\partial}{\partial X_i} \left( \Gamma \frac{\partial \Phi}{\partial X_i} \right) + S \quad \cdots \cdots \ (11)
\]

where the general scalar \(\Phi\) stands for any one of the dependent variables under consideration, the diffusion coefficient \(\Gamma\) and the source term \(S\) in cartesian form are listed below for each governing equation;

**Continuity equation**

\(\Phi = 1, \quad \Gamma = 0, \quad S = 0\)

**Momentum equation in X-direction**

\(\Phi = U, \quad \Gamma = \frac{1}{Re}, \quad S = -\frac{\partial P}{\partial X}\)

**Momentum equation in Y-direction**

\(\Phi = V, \quad \Gamma = \frac{1}{Re}, \quad S = -\frac{\partial P}{\partial X} + \frac{Gr}{Re^2} \theta\)

**Energy equation**
\[ \Phi = \theta, \Gamma = \frac{1}{Re Pr}, S = 0 \]

The numerical solution of the governing equations will be made according to the finite volume method, this method is based on principle of dividing the flow field to a number of volume elements, each one of them is called (control volume), After that a discretization process was carried out by integrating the general conservation equation (11) over a control volume element, where this equation will be as follow [10];

\[
a_p \Phi_p = a_E \Phi_E + a_W \Phi_W + a_N \Phi_N + a_S \Phi_S + S_u \quad \cdots \cdots \quad (12)
\]

\[
a_p = a_E + a_W + a_N + a_S - S_p \quad \cdots \cdots \quad (13)
\]

Where the coefficients \((a_E, a_W, a_N, a_S)\) add the effect of convection between the main node \((P)\) and the neighboring nodes \((E, W, N, S)\) as shown in Fig.(2). After that and by following the (SIMPLE Algorithm) procedure, where the continuity equation was reformulated as a pressure-correction equation[10]. Calculation starts with solving the \((u, v)\) momentum equations, and subsequently this estimated velocity field is corrected by solving the pressure correction equation so that the velocity field fulfils the continuity principle. Finally, the energy equation are solved. This iteration scheme also allows the variables to be updated sequentially. This iteration sequence is repeated until convergence is achieved. Convergence was measured in terms of the maximum change in each variable during an iteration, the maximum change allowed for convergence check was \((10^{-5})\). After getting the final values of all dependent variables in the flow field, calculations will be made for average Nusselt number.

**RESULTS AND DISCUSSION**

The numerical results are carried out for mixed convection in a rectangular vented enclosure. The effect of Re,Pr,Ri numbers, and aspect ratios on heat transfer characteristics have been presented. All these values are varied over wide ranges to study the effects on the thermal transport and fluid flow phenomena. The relative strength of the forced convection over natural convection can be judged on the base of Ri, in the mixed convection. In the case as Ri approaches to unity, the buoyancy effect becomes important. Consequently the natural convection dominates the mixed convection when Ri>1. Figure 3(a1 to a3,b1 to b3) shows streamlines and isotherm plots respectively for different values of Ri(0,1,10) for (A=1) and at (Re=100,Pr=0.7). At low Ri, buoyancy effects are weak and due to hot horizontal wall, fluids move along the heated wall and rise up forming a single cell over the main flow with clockwise rotation inside the enclosure. As, Ri increases buoyancy effects accelerated the fluid near the heated wall causing an asymmetric flow.
pattern producing closer streamlines near the walls. The corresponding isotherm plots for the above cases are presented in Figure 3(b1 to b3). The thermal boundary layer decreases thickness slowly as the Ri increases. This is revealed by the denser concentration of isotherms near the active corner region between the right insulated wall and the wall containing the heat source. Figure 4(a1 to a3) represent streamline plots for different values of Pr(1,50,100) at (Re=500,Ri=1,A=1.5). It can be seen as Pr is increased; the fluid conductivity decreases and limits the acceleration of the near hot-wall fluid to a thinner thermal boundary layer region. Therefore the corresponding decreases in velocity away from the hot wall is also reduced. The corresponding isotherm plots are presented in Figure 4(b1 to b3). As Pr increases, the thermal boundary layer decreases in thickness which can be seen by the denser dustering of isotherms close to the hot wall. The spread of isotherms at low values of Pr is due to a strong stream wise conduction that decreases the stream wise temperature gradient in the fluid. The effect of increase Re(100,500,1000) on the flow and temperature distribution was seen in Figure 5 at (A=2,Pr=10,Ri=3). Figure 5(a1 to a3) represents the streamlines plots and the corresponding temperature distribution can be seen in Figure 5(b1 to b3). It can be seen the increase in Re reduces the thermal boundary thickness and it possible since at large value of Re, the effect of gravitation force becomes negligible. The effect of varying A on the flow pattern can be seen in Figure 6. It can be seen that any increase in aspect ratio due to increase the appearance of convective mode. The heat transfer effectiveness of the enclosure is displayed in terms of average Nusselt number values and the dimensionless maximum fluid temperature. Nusselt number as a function of Richardson number for different Reynolds numbers is shown in Figure 7. When Ri increases from 0 to 3, the average Nusselt number increases moderately at low Reynolds number, but at high Re (Re>100) the average Nusselt number increases very sharply. This behavior results from the onset of thermal instabilities and the probable development of secondary flow due to uniform heating and forced flow causes rapid mixing. For increment of Ri from 3 to 10, causes a slow increase in the average Nusselt number. The effect of varying A on average Nu along the hot wall are illustrated for different Ri in Figure 8 for a Pr of 50. As the flow develops and the bulk fluid gets heated for low Ri, the Nusselt number profiles expectedly increases as the aspect ratio increase because the heat flux wall become bigger. Thus the area for convective contribution, compared to the path for flow. There always exist a maximum Nu at (A=4) for all values of Ri whereas a minimum value is found at (A=0.5). Figure 9 present the overall heat transfer results for different values of Pr for (A=1) and different Ri. Nu increases with increasing Pr, and is due to a decrease in thermal boundary thickness with a consequent increase in the temperature gradient as Pr increase. Figure 10 helps to find the onset of the convection regime. With the increase of the heat transfer rate or fluid mixing, causes increasing of $\theta_m$. In the present analysis for Ri>3, $\theta_m$ depends
strongly on Re, but is nearly invariant with Ri, keeping essentially the levels of pure natural convection. This suggests that in this range of Ri the degree of mixing is either very low or the mixing pattern is such that it does not contribute any significantly to convective heat transfer. A dominance of the mixed convective heat transfer mechanism is therefore inferred for Ri<3. Conversely it can found that for 3 ≤ Ri ≤ 10, the maximum temperature tends to become independent of Ri. Figure (11) illustrate the influence of Pr on the maximum temperature distribution for various Ri. Observing the temperature profile, no significant changes occurs with the increase of Ri for low value of Pr, but for high Pr the maximum temperature increases as Ri <6 and increasing Pr has almost constant effect on θw for all value of Ri>6. This is because Pr is an important parameter in the energy equation and its effects are always present independent of the magnitude of buoyancy forces. Higher Pr implies lower thermal diffusion of heat from heated surface and lower magnitudes of the buoyancy source term and buoyancy induced flow acceleration. Table (1) shows results for the pure forced convection case (Ri=0). U velocities are always higher than Velocities for all Reynolds numbers, showing that the flow is dominated by an intense lateral mixing.

**COMPARISION**

Figure(12 a) compares the present result for Nusselt number with the result by Sumon et al.[1] for mixed convection in rectangular vented enclosure and uniform heat source on the bottom wall. The right, left, and top walls are insulted. Figure(12 b) also shows the comparison of maximum temperature between the present investigation and Summon et al.[1]. The present results has a very good agreement with the results by Sumon et al.[1].

**CONCLUSIONS**

The problem of mixed convection heat transfer of fluid in a two-dimensional horizontal vented rectangular enclosure with a constant heat flux source mounted on the bottom wall has been investigated numerically using a finite element technique. The main conclusions of the present study are:

1. The heat transfer results explain the importance of the non-dimensional parameters like Reynolds number and Richardson number in the natural and mixed convection regime. The effects of these parameters on the flow fields are also investigated.

2. The study encompasses a range of Re from 50 to 1000, a range of Pr from 0.7 to 100 and a range of Ri from 0 to 10.
3-The numerical solutions indicate that increasing the value of Re or Gr leads to higher heat transfer coefficient, higher heat source temperature and higher intensity of recirculation.

3-Adetial investigation of heat transfer in terms of average Nusselt number and fluid temperature has been undertaken for different values of A ,Ri ,Re, and Pr. The average Nu plotted along the variation of Richardson number indicates that the heat transfer from the heated wall increases rapidly up to Ri=3, then it increases at a very slow rate.

4-The maximum fluid temperature tends to be remains constant in the highly buoyancy dominated convection regime.

REFERENCES


Table(1) Absolute maximum values of velocities at Ri=0 as a function of Reynolds number

<table>
<thead>
<tr>
<th>Re</th>
<th>$U_m$</th>
<th>$V_m$</th>
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<tbody>
<tr>
<td>50</td>
<td>0.11794</td>
<td>0.11742</td>
</tr>
<tr>
<td>100</td>
<td>0.21756</td>
<td>0.19878</td>
</tr>
<tr>
<td>500</td>
<td>1.03001</td>
<td>1.02164</td>
</tr>
<tr>
<td>1000</td>
<td>2.02379</td>
<td>2.0109</td>
</tr>
</tbody>
</table>

$q = -k \frac{\partial T}{\partial y}$

Fig.(1) Schematic diagram of the problem and coordinate system

Fig.(2) Mesh distribution for two dimensional flow
Fig. (3) Streamlines and isothermal plot for different values of $R_i$ at $Re=100, Pr=0.7, A=1$
Fig.(4) Streamlines and isothermal plot for different values of Pr at Re=500, Ri=1, A=1.5
Fig.(5) Streamlines and isothermal plot for different values of Re at Ri=3, Pr=10, A=2
Fig. (6) Streamlines and isothermal plot at different $A (A=1,2,4)$ for $Re=50, Pr=50, Ri=1$
Fig. (7) Variation of $\alpha_{Nu}$ with the Ri for different values of Re at $Pr=0.7$

Fig. (8) Variation of $\alpha_{Nu}$ with the A for different values of Ri at Re=100
Fig.(9) Variation of $N_u_a$ with the Ri for different values of Pr

Fig.(10) Variation of $\theta_m$ with the Ri for different values of Re
Fig. (11) Variation of $\theta_m$ with the Ri for different values of Pr

Fig. (12 a) Comparison of $Nu_a$ with results of Ref.[1] at Re=200

Fig. (12 b) Comparison of $\theta_a$ with results of Ref.[1] at Ri=0