Turn-on Dynamic with Nonlinear Carriers
Scattering Rates in InAs/GaAs QD Lasers

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Abstract
Based on the relaxation oscillations theory in semiconductor lasers of Quantum dots (QDs) based on a microscopic approach by K. Lüdge et al. (2008) as a basic model used in this work. We introduce a new expression of nonlinear scattering rates by using the curve fitting functions. We can discuss the influence of different values of the QD density upon the dynamic of laser output in detail of our simulations results. By taking into account, we study the dependence of the carrier-carrier scattering rates on the injection current. We present a theoretical simulation of characteristics and the turn-on dynamics of InAs/GaAs semiconductor QD laser output lasing with pulse wavelength of 1.3µm at room-temperature.

Introduction
The recent research is focused on some of the most important aspects of the QD laser dynamics: Such as, modulation dynamics of QD lasers in the relaxation time approximation, a study of quantum correlations in the optical emission [Christopher Gies et al. (2007)], and a dynamical hierarchy for the population. The interaction of electrons in the injection pumped bulk regions, the quantum-well WL, and QDs in the gain regime is provided by relevant relaxation processes such as electron-electron and electron-phonon interaction [Peter Michler. (2003), Y. Fu et al. (2009) and Massimo Rontani. (1999)]. Above threshold, a microscopic description of the dynamics of QD lasers would be required to describe the polarization and population dynamics of an inhomogeneous distribution of these states [Pter Blood. (2009) and Ian O’Driscoll et al. (2010)]. A fully microscopic approach for all time and length scales of the dynamics of QD lasers is by far numerically too demanding.

In this work, we focus on the turn-on light field dynamics with nonlinear QD carrier scattering rates and the population dynamics induced by the interaction of the QD states with the temporally current modulated population reservoir of WL states. We focus on the dynamics of relaxation oscillations on a nanosecond timescale for current injection well above the laser threshold. In high excitation limit, electron-electron scattering provides the main interaction channel. A detailed comparison between experimental and theoretical data is given for a wide range of different pump currents or other control parameters.

Rate Equations Model
The rate equations models are a most of semiconductor QD laser models which was derived from what is known as semiclassical laser theory [Pierre Meystre et al. (2007)]. This theory incorporates the classical electrodynamics that occurs within the laser as well as the quantum mechanics associated with the active material. The rate equations model (REM) for the electromagnetic fields in matter such, which is derived beginning with Maxwell-Bloch equations (MBE) and using the slowly varying envelope to
express the electric and magnetic fields [Sergei V. Zhukovsky et.al. (2007)]. Here, the material polarization in semiconductors media substituting in the displacement in MBE. It has two components linear and nonlinear parts.

The following nonlinear rate equations for the photon density (PD), $\bar{\tilde{n}}_{ph}$, the charge-carrier densities in the QDs, and the carrier densities in the wetting layer, determine the dynamics [Ermin Malić et.al. (2007) and Kathy Lüdge et.al. (2008)]:

$$\frac{d}{dt} \bar{\tilde{n}}_{e,h} = -\frac{\bar{\tilde{n}}_{e,h}}{\tau_{e,h}} + S_{e,h} \bar{n}_{QD} - \xi_{ind} \left( \bar{\tilde{n}}_{e,h} \bar{n}_{QD} \right) - \xi_{QD} \left( \bar{\tilde{n}}_{e,h} \bar{n}_{QD} \right) \ldots (2)$$

$$\frac{d}{dt} \bar{n}_{w,h} = \frac{j}{e_0} + \frac{\bar{\tilde{n}}_{e,h}}{\tau_{e,h}} \bar{n}_{QD} - S_{e,h} \bar{n}_{WL} - \xi_{WL} \left( \bar{\tilde{n}}_{w,h} \bar{n}_{WL} \right) \ldots (3)$$

where the coefficient $k$ expresses the total cavity loss (TCL) or cavity damping, $j$ is the injection current density pulse, $\beta$ is the spontaneous emission coupling factor (SEC), and $\tau_{e,h} \tau_{e,h}$ are the intraband scattering times of electron / hole where

$$\tau_{e,h} = \frac{1}{S_{e,h} + S_{e,h}} \ldots (4)$$

Here $\xi_{ind}$ is the induced processes rate of absorption and emission, exciting-dominated spontaneous emission in the QD is approximated by bimolecular recombination is modeled by $\xi_{QD}$, and $\xi_{WL}$ is the WL spontaneous recombination rate:

$$\xi_{ind} \left( \bar{\tilde{n}}_{e,h} \bar{n}_{QD} \right) = \beta E A_{WL} \bar{n}_{Dh} \left( \bar{\tilde{n}}_{e,h} + \bar{n}_{QD} \right) \ldots (a)$$

$$\xi_{QD} \bar{n}_{QD} \bar{n}_{QD} = \frac{\beta E}{\Pi_{QD}} \bar{n}_{QD} \bar{n}_{QD} \ldots (b)$$

$$\xi_{WL} \bar{n}_{WL} \bar{n}_{WL} = \frac{\beta E}{\Pi_{WL}} \bar{n}_{WL} \bar{n}_{WL} \ldots (c)$$

Where $\bar{n}_{QD} \bar{n}_{QD}$ denotes the QD density and $\bar{n}_{WL} \bar{n}_{WL}$ is the WL effective density of states, $A_{WL} A_{WL}$ is the WL normalization area and $\beta E$ is the Einstein coefficient. Both spontaneous emission and induced processes are proportional to the Einstein coefficient [Emmanuel Rosencher et.al. (2004)]

$$\beta E = \frac{|\mu|^2}{3 \pi e_0 \hbar} \left( \frac{\omega}{C} \sqrt{\frac{d_b}{d_b}} \right)^3$$

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Where \( \mu \mu \) is the dipole moment, \( d_\text{B}d_\text{B} \) is the background dielectric constant, \( \varepsilon_\text{B} \varepsilon_\text{B} \) is the vacuum permittivity, \( \omega \) is the optical frequency, and \( c \) is the light speed.

The carrier-light interaction is summarized in \( \bar{n}_{\text{ph}} \bar{n}_{\text{ph}} \), which includes all longitudinal modes. The optical confinement factor \( \Gamma \) in Eq. (1) expresses the difference of the optical and electronic active area and is a measure for the radiative processes depends on the QD density \( n_{\text{QD}} \) \( n_{\text{QD}} \) [Gregory Sun et.al. (2004)]

\[
\Gamma = n_{\text{QD}} \alpha_L Y_{xy} \Gamma Z
\]

Where \( Y_{xy} \) \( Y_{xy} \) is the in-plane size of a QD, \( \Gamma Z \) \( \Gamma Z \) is the vertical confinement factor and \( \alpha_L \) \( \alpha_L \) the number of QD layers.

The coefficient \( k \) expresses the total cavity loss [Ermin Mali\'c et.al. (2007)]

\[
k = k_{\text{in}} - \frac{\ln(R_1 R_2)}{2\ell} c \quad \ldots (8)
\]

Where \( k_{\text{in}} \) \( k_{\text{in}} \) is the internal loss, \( \ell \) \( \ell \) is the cavity length, and \( R_1, R_2 \) \( R_1, R_2 \) are the front / rear facet reflection coefficients respectively.

For a better comparison with simplified models choosing \( r_{\text{w}} r_{\text{w}} \) in agreement with corresponding stationary values of \( \bar{n}_{\text{W}} \bar{n}_{\text{W}} \) and \( \bar{n}_{\text{W}} \bar{n}_{\text{W}} \) we represent the numerically evaluated nonlinear scattering rates \( S_{\delta} S_{\delta} \), \( S_{\delta} S_{\delta} \), \( S_{\delta} S_{\delta} \) and \( S_{\delta} S_{\delta} \) by using the Curve Fitting Functions (CFF) [Sigurd A. Nelson II. (1997), Marko Ledvij (2003) and M. Lorke et.al. (2007)] for the nonlinear scattering rates curves in Fig.2 in Ref.[ Kathy Lüdge et.al.(2008)]. Our results for nonlinear scattering rates formalisms are

\[
S_{\delta} = 0.012 \exp \left[ -\frac{\left( \bar{n}_{\text{W}} - 124.5 \right)^2}{29} \right] + \\
g_1 \left( \frac{10}{1 + \exp \left[ \frac{38 - \bar{n}_{\text{W}}}{6} \right]} \right) \left( \frac{\exp \left[ \frac{38 - \bar{n}_{\text{W}}}{g_2} \right]}{1 + \exp \left[ \frac{38 - \bar{n}_{\text{W}}}{g_2} \right]} \right) \quad \ldots (9)
\]

\[
S_{\delta} = \exp \left[ \frac{1.73 - \bar{n}_{\text{W}}}{g_5} \right] + 0.1154 \times \exp \left[ \frac{\left( \bar{n}_{\text{W}} - 27.2 \right)^2}{137.8} \right] \\
\left[ 1 - \exp \left[ \frac{1 - \bar{n}_{\text{W}}}{2} \right] \right] \exp \left[ \frac{\bar{n}_{\text{W}}^2}{g_5} \right] \quad \ldots (11)
\]

\[
S_{\delta} = g_7 \left[ 1 - \exp \left[ -\frac{\bar{n}_{\text{W}}}{1.7} \right] \right] \exp \left[ -\frac{\bar{n}_{\text{W}}^2}{18954} \right] + \exp \left[ -\frac{\bar{n}_{\text{W}} - g_8}{26.4} \right] \quad \ldots (12)
\]
To depict the measured laser output for different pump currents, the carrier injection into the wetting layer is expressed by the injection current density pulse \( j(t) \) divided by the elementary charge \( e_0 \). For the simulation a current pulse, we use the expression [Ermin Malić et al. (2006)]

\[
j = j_0 \exp \left[ -\left( \frac{t-t_0}{2.5 \, \text{ns}} \right)^m \right] \quad \text{(14)}
\]

here \( j_0 \) is the maximum amplitude of current pulse. In REM (1)–(3) it can be seen that injection current density pulse affect the dynamics of the WLs, both for electrons and holes. So that its included in Eqs.(3) since the laser operation occur starting from this region where the carriers be in the QD. The time \( t \) is chosen to assure a constant injection density through pulse time.

To determine the optimum operation conditions for the QD laser, we have solved the REM (1)–(3) making use of Eq.(14) to evaluate the result shown in Fig.1. Two distinct regions separated by the value \( j_{th} = 2.6628 \, \text{mA cm}^{-2} \) can be seen in this figure. On the left hand side of this value, the region of building the proper population inversion in order to reach the threshold population inversion where it can be seen that photon density is almost zero. On the right hand side of injection current density pulse, the photon density grow with time which is the logical behavior of any laser device. Accordingly, we have chosen this value of \( j_{th} \) in this work.
Figure 1: Steady-State input-output characteristic of InAs/GaAs QD laser with a wave-length of 1.3 µm: Simulated PD vs. Pump current \( j \). The threshold pump rate \( j_{th} = 1.662 \times 10^{10} \text{ cm}^{-2} \text{ps}^{-1} \) is determined from the extrapolated laser onset if spontaneous emission is neglected. The other parameters [Kathy Lüdge et. al (2008)]: \( \beta = 1.3 \text{ ns}^{-1} \), \( A_{WL} = 4. \times 10^{-5} \text{ cm}^{-2} \), \( \Pi_{QD} = 1. \times 10^{10} \text{ cm}^{-2} \), \( \Pi_{WL} = 2. \times 10^{13} \text{ cm}^{-2} \), \( a_c = 15 \), \( e = 13.18 \), \( k = 0.12 \text{ ns}^{-1} \), \( k_m = 220 \text{ m}^{-1} \), \( \ell = 1 \text{ mm} \), \( T = 300 \text{ K} \), \( \Gamma = 0.0011 \), \( \Gamma_z = 2.5 \times 10^{-3} \), \( r_w = 3.4 \), and \( r_1, r_2 = 0.32 \).

Returning to the set of REM (1) – (3), one can see there exist a number of parameters, that called order parameters, have different effects on the QD Laser dynamics viz; pump current, normalized WL area, Einstein coefficient, optical confinement factor, QD density, effective WL density state, ratio of carrier WL densities and cavity damping. These order parameters have different effects on the nonlinear scattering rates, densities of carriers in QD and WL regions as well as Laser field. In most of the results given here, we will focus on PD.

**The injection Current effect**

There are clear effects of the injection current on the dynamics of carriers scattering rates, microcavity carrier’s density and QD Laser field. The maximum amplitude of injection current pulse can be expressed as:

\[
\tilde{j}_0 = j_{th} R_{th} \quad \ldots (15)
\]

where \( R_{th} \) is a threshold injection ratio.

The effect of injection current pulse on nonlinear scattering rates in the QD is shown in Fig.2. It can be seen that all the four quantities dynamics, i.e. the electron in/out scattering and hole in/out scattering are affected by varying threshold injection ratio \( R_{th} \).

The microcavity carriers density (in QD and WL) are all plotted as function of \( R_{th} \) in Fig.3. Here, our results are in good agreement with theoretical results in Ref. [Kathy Lüdge et. al (2008)]. The dynamics of carrier densities also shows oscillations in Fig.3, but they are less pronounced due to the capacitive inertia of charge carriers. The electron density has a lower value than hole density in WL (Fig.3 (a) and (b)). Vice versa, it has a higher stationary value in QD (Fig.3 (c) and (d)) due to the larger electron in-scattering rate \( S^e \). The rise time for both \( \tilde{n}^e_D \) are dominated by the corresponding in-scattering terms (ps range) in Eqs.(2), whereas the fall time is much longer due to slower radiative processes (ns range).
Figure 2: Turn-on dynamic of InAs/GaAs QD laser model scattering rates calculated from REM(1)–(3) for different value of the pump current ($P_{th}$ = 1.4, 1.9, 2.7 and 4.1 respectively): (a) and (b) the in-scattering rate (electron/hole respectively), (c) and (d) the out-scattering rate (electron/hole respectively).
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Figure 3: Turn-on dynamic of InAs/GaAs QD laser model carriers density calculated from REM(1)–(3) for different value of the pump current ($J_{th}$): (a) and (b) the QD carrier density (electron/hole respectively), (c) and (d) the WL carrier density (electron/hole respectively).

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A description of QD laser operation can be achieved using rate equations for WL/QD carriers (electrons and holes) and photons (Eqs. (1)–(3)), which describe the interaction of electrons and photons by stimulated and spontaneous emissions mechanisms as well as the pump process (presented her by $T_{th}T_{th}$). Plotting the PD with time is of great importance to determine the features of QD laser output. Fig. 4 presents a theoretical simulation of the turn-on dynamics of InAs /GaAs QD lasers driven by electrical current pulse. It’s clear enough the way by which QD laser field is increases with increasing $T_{th}T_{th}$, the time arrival of QD laser output signal in start of the transient region, the time needed to reach the first maximum value in the transient region, length of the transient region, and the start of steady state output.

**Figure 4:** Turn-on dynamic of InAs/GaAs QD laser model of PD for different value of the pump current ($P_0 = 1.4, 1.9, 2.7$ and $4.1$ respectively).

**Optical Confinement Factor**

There is a competition between radiative recombination and non-radiative scattering events for the electrons in the QDs. One parameter (optical confinement factor) that affects the ratio of both processes [Kathy Lüdge et. al (2008), Paul Harrison. (2005), F.F. Schrey et. al. (2004) and F.F. Schrey et. al. (2004)]: For stronger radiative recombination the electron-electron scattering processes lose importance, and thus the strong damping of the relaxation oscillations disappears. The parameter is the OCF it is a measure for the relative importance of the radiative processes. Increasing OCF changes the dynamics completely and results in pronounced weakly damped oscillations, as shown below.

The optical confinement factor is a ratio of the effective region length of a laser (i.e. length of active area of the QD laser medium) to the microcavity length. Any increment increase in OCF means an increase in the length of the action region. This leads to the increase of stimulated emission. As can be seen in Fig. 5 its increase leads to improvement in the laser output characteristics. Fig. 6 is a characteristics summary of the whole effects of OCF on the InAs/GaAs QD laser.
Figure 5: Turn-on dynamic of InAs/GaAs QD laser model of PD for different value of the OCF ($\Gamma=0.0001,0.001,0.01$, and $0.3$ respectively).

Figure 6: Characteristics of InAs/GaAs QD laser PD vs. the OCF: (a) The Delay time, (b) The Rise time, and (c) PD.
Conclusion

Based on new modified expressions of nonlinear carrier-carrier scattering, we have a theoretical results appeared in this work which are in agreement with others theoretically and experimentally data. We studied the turn-on dynamics of InAs/GaAs QD lasers with pulse wavelength of 1.3 µm at room-temperature. The results show the variation and characteristics of nonlinear scattering rates and the output laser with the variation of the spatial control parameters (injection current effect and optical confinement factor).

References


Massimo Rontani. (1999), PhD. thesis, Università degli Studi di Modena e Reggio Emilia, Italy.