Design of Dielectric Bragg Mirror Consisting Quarter-Wave Stacks Using Transfer-Matrix Method

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ABSTRACT

All recent researches in Iraq deal with Nanotechnology. Our research deals with Femtotechnology, which is very advanced in laser and fiber optics technology. Where: Nanotechnology = 9 × 10^{-9} seconds. Nevertheless:
Femtotechnology $= 10^{-15}$ seconds. Where in femtosecond lasers: lasers emitting pulses with durations between a few femtoseconds and hundreds of femtoseconds. The generation of ultrashort pulses, that is pulses in the order of picoseconds and femtosecond lasers involves optical coatings as important functional elements, e.g. high reflectors (HR), output couplers (OC) and antireflection (AR) coatings. These optical elements are based on the interference phenomenon of light. Their theoretical analysis generally relies on the well-known scattering matrix formalism derived from the Maxwell equations. Laser performance strongly depends on the quality of optical coatings: reflectance of high reflectors should approach the ideal 100% value at the operation wavelengths in order to minimize laser inter cavity losses and output coupling has to be set to specific values to ensure optimal operation. This paper reports a theoretical design of dielectric Bragg mirror (DBM) to achieve high reflectivity and dispersion compensation over a broad bandwidth. Analytic expressions for reflectivity (R), group delay (GD) and group delay dispersion (GDD) are used. Dielectric materials TiO$_2$/SiO$_2$ arranged as a periodic stacks have been used to design Bragg mirrors using the Fusedsilica as a substrate. In this paper we demonstrate a dielectric multilayer mirror with a controlled reflectivity and dispersion in the wavelength range $650–900\text{nm}$, it exhibits a reflectivity of $>99.999999\%$. These stacks have a maximum reflectivity at 800nm. The group delay and group delay dispersion also have shown a low oscillation.

**INTRODUCTION**

In this paper, we study the theoretical design of Dielectric Bragg Mirror (BM). In the design of dielectric mirrors, an optical transfer-matrix method can be used. The most general method of calculating the reflectance and the transmittance of a multilayer is based on a matrix formulation, of the boundary conditions at the film surfaces derived from Maxwell's equations.

A Bragg Mirror is a periodic structure composed of pairs of layers of dielectric or semiconductor materials characterized by different refractive indices. A Bragg mirror (also called distributed Bragg reflector) is a structure, which consists of an alternating sequence of layers of two different optical materials [1-3].
The most frequently used design is that of a quarter-wave mirror, where each optical layer thickness corresponding to one quarter of the wavelength for which the mirror is designed. The latter condition holds for normal incidence; if the mirror is designed for larger angles of incidence, accordingly thicker layers are needed [2].

Bragg mirrors can be fabricated with different technologies:
(a) - Dielectric Bragg mirrors, based on thin-film coating technology, fabricated for example with electron beam evaporation or with ion beam sputtering, are used as laser mirrors in solid-state bulk lasers. The mirror structure then consists of amorphous materials [4].
(b) - Fiber Bragg gratings, including long-period fiber gratings, are often used in fiber lasers and other fiber devices. They can be fabricated by irradiating a fiber with spatially patterned ultraviolet light [4]. Similarly, volume Bragg gratings like (BragGrate™ Mirror) can be made in photosensitive bulk glass. The BragGrate™ Mirror is a reflecting volume Bragg grating (RBG) recorded in a bulk of photosensitive silicate glass. BragGrate™ Mirror placed in a laser resonator enables spectral and thermal management of the laser radiation and can withstand high optical densities up to 5 J/cm². The laser modal structure is controlled by the longitudinal mode selection with the bandwidth down to 20 pm and the customized central wavelengths with accuracy of 0.1-0.5 nm. BragGrate™ Mirrors have record low absorption and allow thermal laser wavelength shift reduction to 0.005 nm/K [5,6].
(c) - Semiconductor Bragg mirrors, can be produced with lithographic methods. They are used, for example, in laser diodes, particularly in surface-emitting lasers.

There are various types of Bragg reflectors used in other waveguides, based on, e.g., corrugated waveguide structures which can be fabricated via lithography. Such kind of gratings are used in some distributed Bragg reflector or distributed feedback laser diodes [7,8].

**PRINCIPLE OF OPERATION**

The principle of operation and the theory model for dielectric Bragg mirror can be understood as follows: Each interface between the two dielectric materials contributes a Fresnel reflection [4]:

\[ r_F = \frac{n_b - n_i}{n_b + n_i} \]  

\[ \ldots \ldots \text{_(1)} \]
Where \( r_F \) Fresnel reflectivity, \( n_h \) high refractive index and \( n_l \) low refractive index.

For the design wavelength, the optical path length difference between reflections from subsequent interfaces is half the wavelength; in addition, the reflection coefficients for the interfaces have alternating signs. Therefore, all reflected components from the interfaces interfere constructively, which results in a strong reflection. The reflectivity achieved is determined by the number of layer pairs and by the refractive index contrast between the layer materials. The reflection bandwidth is determined mainly by the index contrast [4].

Ultrashort pulse generation has advanced to a level where the bandwidth of standard Bragg mirrors, composed of SiO\(_2\) and TiO\(_2\) quarter-wave layers, limits the pulse width (Figure 1).

![Fig. 1: Standard dielectric quarter-wave Bragg mirror [5]](image)

The limitation is two fold. First, due to the limited difference in refractive index of both materials, SiO\(_2\)=1.45 and TiO\(_2\)=2.4, the high-reflectivity bandwidth of a standard quarter-wave Bragg mirror at 800 nm is only about 200 nm. Second, the higher order group delay dispersion (GDD) produced by quarter-wave Bragg mirrors further limits the useful bandwidth to about 100 nm for 10-fs pulses. The effect of the dispersion from quarter-wave Bragg mirrors on short pulse generation has already been investigated with CPM-dye lasers [5].

In this paper, we present the transfer matrix method allowing solving Maxwell equations in multilayer dielectric structures. We shall consider an example of a periodical structure (Bragg mirror) and derive general equations in planar structures. In the beginning, we consider propagation of light in the normal to layer planes direction. We shall generalize the transfer matrix approach for TE and TM linear polarizations of light. By definition, TE-polarized (also referred to as s-polarized) light has the electric field vector parallel to the layer planes, TM-polarized light (also referred to as p-polarized) has the magnetic field vector parallel to the planes (see Figure 2).
Fig. 2: Orientation of electric and magnetic fields in TE- and TM-polarized incident on a planar boundary [9].

What happens to the electromagnetic field at the planar interface between two dielectric media with different refractive indices? The answer can be found by resolving the system of Maxwell equations independently in the two media and then matching of the solutions for electric and magnetic fields by the Maxwell boundary conditions at the interface. These conditions require continuity of the tangential components of both fields. They can be microscopically justified for any abrupt interface in the absence of free charges and free currents.

Consider a transverse light-wave propagating along the $z$-direction in a medium characterized by a refractive index $n$ that is homogeneous in the $xy$ plane but possibly $z$-dependent. The wave equation in this case becomes [1,9]:

$$\frac{\partial^2 E}{\partial z^2} = -k_0^2 n^2 E \quad \text{.....(2)}$$

where $k_0$ is the wave-vector of light in a vacuum. The general form of the solution of Eq. (1) writes:

$$E = A^+ \exp(ikz) + A^- \exp(-ikz) \quad \text{.....(3)}$$

where $k = k_0 n$, $A^+$, $A^-$ are coefficients. Using the Maxwell equation one can easily obtain the general form of the magnetic field amplitude

$$B = A^+ n \exp(ikz) - A^- n \exp(-ikz) \quad \text{.....(4)}$$

If we consider reflection of light incident from the left side to the boundary ($z = 0$) between two semi-infinite media characterized by refractive indices $n_1$ (left) and $n_2$ (right), the matching of the tangential components of electric and magnetic fields would give

$$A^+_1 + A^-_1 = A^+_2 \quad \text{.....(5)}$$

$$(A^+_1 - A^-_1) n_1 = A^+_2 n_2 \quad \text{.....(6)}$$
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where $A_1^+$, $A_1^-$ and $A_2^+$ are the amplitudes of incident, reflected and transmitted light, respectively. One can easily obtain the amplitude reflection coefficient

$$ r = \frac{A_1^-}{A_1^+} = \frac{n_1 - n_2}{n_1 + n_2} \quad \text{.....(7)} $$

and the amplitude transmission coefficient

$$ t = \frac{A_2^+}{A_1^+} = \frac{2n_1}{n_1 + n_2} \quad \text{.....(8)} $$

The ratio of reflected to incident energy flux (reflectivity) is given by

$$ R = |r|^2 \quad \text{.....(9)} $$

and the ratio of transmitted to incident energy flux (transmittance) is

$$ T = \frac{n_2}{n_1} |t|^2 \quad \text{.....(10)} $$

In the last formula, the factor $\frac{n_2}{n_1}$ comes from the ratio of light velocities in two media.

In multilayer structures, direct application of Maxwell boundary conditions at each interface leads to the necessity to resolve a substantial number of algebraic equations (two per interface). A convenient method allowing reducing the number of equations to be resolved to a strict minimum (four in general case) is the transfer matrix method, which we are going to describe briefly here.

Let us introduce the vector

$$ \Phi(z) = \begin{bmatrix} E(z) \\ B(z) \end{bmatrix} = \begin{bmatrix} E(z) \\ i \frac{\partial E(z)}{\partial z} \end{bmatrix} \quad \text{.....(11)} $$

where $E(z)$, $B(z)$ are the amplitudes of the electric and magnetic field of any light wave propagating in the $z$ direction in the structure under study. Note that $\Phi(z)$ is continuous at any point in the structure due to the Maxwell’s boundary conditions. In particular, it is continuous at all interfaces where $n$ changes abruptly.

By our definition, the transfer matrix $\hat{T}_a$ across the layer of width $a$ is such a $2 \times 2$ matrix that:

$$ \hat{T}_a \Phi|_{z=a} = \Phi|_{z=a} \quad \text{.....(12)} $$
It is easy to verify by substitution into Eq. (11) of the electric and magnetic amplitudes (2), (3) that if \( n \) is homogeneous across the layer,

\[
\hat{T}_a = \begin{bmatrix}
\cos ka & \frac{i}{n} \sin ka \\
\sin ka & \cos ka
\end{bmatrix}
\]

…..(13)

The transfer matrix across a structure composed of \( m \) layers can be found as

\[
\hat{T} = \prod_{i=1}^{m} \hat{T}_i
\]

…..(14)

Where \( \hat{T}_i \) is the transfer matrix across \( i \)-th layer. The order of multiplication in Eq. (13) is essential. The amplitude reflection and transmission coefficients \( (r_s, t_s) \) of a structure containing \( m \) layers, and sandwiched between two semi-infinite media with refractive indices \( n_{\text{left}}, n_{\text{right}} \) before and after the structure, respectively, can be found from the relation

\[
\hat{T} \begin{bmatrix}
1 + r_s \\
\frac{n_{\text{left}} - n_{\text{left}} r_s}{n_{\text{right}} r_s}
\end{bmatrix} = \begin{bmatrix}
t_s \\
\frac{n_{\text{right}} t_s}{n_{\text{left}} t_s}
\end{bmatrix}
\]

…..(15)

One can easily obtain

\[
r_s = \frac{n_{\text{right}} t_{11} + n_{\text{left}} n_{\text{right}} t_{12} - t_{21} - n_{\text{left}} t_{22}}{t_{21} - n_{\text{left}} t_{22} - n_{\text{right}} t_{11} + n_{\text{left}} n_{\text{right}} t_{12}}
\]

…..(16)

\[
t_s = \frac{2n_{\text{left}} t_{12} t_{21} - t_{11} t_{22}}{t_{21} - n_{\text{left}} t_{22} - n_{\text{right}} t_{11} + n_{\text{left}} n_{\text{right}} t_{12}}
\]

…..(17)

The intensities of reflected and transmitted light normalized by the intensity of the incident light are given by

\[
R = |r_s|^2, \quad T = |t_s|^2 \frac{n_{\text{right}}}{n_{\text{left}}}
\]

…..(18)

respectively.

In its turn, the transfer matrix across a layer can be expressed via reflection and transmission coefficients of this layer. If the reflection and transmission coefficients for light incident from the right-hand side and left-hand side of the layer are the same, and \( n_{\text{left}} = n_{\text{right}} = n \) (the symmetric case realized, in particular, in a quantum well well embedded in a cavity), the Maxwell boundary conditions for light incident from the left and right sides of the structure yield:

\[
\hat{T} \begin{bmatrix}
1 + r_s \\
\frac{n - n r_s}{-n}
\end{bmatrix} = \begin{bmatrix}
t_s \\
\frac{n t_s}{-n}
\end{bmatrix},
\]

…..(19)
This allows the matrix $\mathbf{T}$ to be expressed as:

$$\mathbf{T} = \frac{1}{2t} \begin{bmatrix} t_s^2 - r_s^2 + 1 & -(1+r_s)^2 - t_s^2 \\ n(r_s - 1)^2 - t_s^2 & t_s^2 - r_s^2 + 1 \end{bmatrix} \quad \text{......(20)}$$

For a quantum well, $t_s = 1 + r_s$, and Eq. (19) becomes

$$\mathbf{T}_{qw} = \begin{bmatrix} 1 & 0 \\ -2n r_s^2 & 1 \end{bmatrix} \quad \text{......(21)}$$

In the oblique incidence case, in the TE-polarization, one can use the basis $[E_r(z) \ B_r(z)]$, where $E_r, B_r$ are the tangential (in-plane) components of the electric and magnetic fields of the light wave. In this case, the transfer matrix (4) keeps its form provided that the following substitutions are made:

$$k_z = k \cos \varphi, \quad n \rightarrow n \cos \varphi \quad \text{......(22)}$$

where $\varphi$ is the propagation angle in the corresponding medium ($\varphi = 0$ at normal incidence).

In the TM-polarization, following Born and Wolf [1] we use the basis $[B_r(z) \ E_r(z)]$, which still allows the transfer matrix (12) to be used provided that the substitutions are done:

$$k_z = k \cos \varphi \quad \text{......(23)}$$

$$n \rightarrow \frac{\cos \varphi}{n}$$

Note that the transfer matrices across the interfaces are still identity matrices, and Eq. (13) for the transfer matrix across the entire structure is valid.

In the formulas for reflection and transmission coefficients (15-18) one should replace, in the TE-polarization

$$n_{left} \rightarrow n_{left} \cos \varphi_{left}, \quad n_{right} \rightarrow n_{right} \cos \varphi_{right} \quad \text{......(24)}$$

and in the TM-polarization

$$n_{left} \rightarrow \frac{\cos \varphi_{left}}{n_{left}}, \quad n_{right} \rightarrow \frac{\cos \varphi_{right}}{n_{right}} \quad \text{......(25)}$$

where $\varphi_{left}, \varphi_{right}$ are the propagation angles in the first and last media, respectively. The same transformations would be applied to the transfer matrices
Note that any two propagation angles \( \varphi_i, \varphi_j \) in the layers with refractive indices \( n_i, n_j \) are linked by the Snell-Descartes law:

\[
n_i \sin \varphi_i = n_j \sin \varphi_j \quad \ldots (26)
\]

which is also valid in the case of complex refractive indices, when the propagation angles formally become complex as well.

The group delay is defined as the negative of the derivative of the phase response with respect to frequency, GD, also known as "Envelope Delay" [16→20]. In physics and in particular in optics, the study of waves and digital signal processing, the term group delay has the following meaning: The rate of change of the total phase shift with respect to angular frequency [21,22]:

\[
GD = -\frac{d\phi}{d\omega} \quad \ldots (27)
\]

Through a device or transmission medium, where \( \phi \) is the total phase shift in radians, and \( \omega \) is the angular frequency in radians per unit time, equal to \( 2\pi f \), where \( f \) is the frequency (hertz if group delay is measured in seconds).

Group delay dispersion is a ubiquitous, and often irritating, phenomenon in ultrafast laser labs. When ultrashort pulses propagate through dispersive media, their frequency components emerge at different times due to GDD, causing the resulting pulse to be chirped and stretched and reducing the pulse’s peak power [23].

\[
GDD = -\frac{d^2\phi}{d\omega^2} \quad \ldots (28)
\]

This effect can be compensated by using a pulse compressor, which can introduce negative GDD [20]. The standard method for computing the GDD is to compute complex reflection coefficients using the transfer matrix technique and then take successive finite difference over frequency [21,22]. Group-delay dispersion of optical elements is a critical parameter for the generation and control of femtosecond laser pulses. GDD can either increase or decrease then pulse duration by modulating the spectral phase of the femtosecond laser pulses. The effect of GDD becomes more significant as the laser pulse duration gets shorter. Ideally, a femtosecond dielectric mirror should not only have high reflectance but also low dispersion over a sufficiently broad spectral bandwidth [24].
RESULTS AND DISCUSSION

For the visible region and near infrared region, the most common coating materials are titanium dioxide TiO2 and the silicon dioxide SiO2. The properties of dielectric optical materials used in this paper, are shown in table-1.

<table>
<thead>
<tr>
<th>Materials Name</th>
<th>Materials Symbol</th>
<th>Materials Index of Refraction</th>
<th>Physical Thickness</th>
<th>Optical Region</th>
<th>Method of Deposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Titanium Dioxide</td>
<td>TiO2</td>
<td>2.2505</td>
<td>88.86914</td>
<td>Visible</td>
<td>Electron beam evaporation/ Suttering</td>
</tr>
<tr>
<td>Silicon Dioxide</td>
<td>SiO2</td>
<td>1.4716</td>
<td>135.9065</td>
<td>Infrared</td>
<td>Electron beam evaporation/ Suttering</td>
</tr>
</tbody>
</table>

In this simulation, we used a quarter wave Bragg mirror. The structure of Bragg mirror design consists of 18 stack layers and in the wavelength range 650 – 900\(nm\) for normal incidence of light from air, with \(n_b = 2.2505, \quad n_i = 1.4716\), for the refractive indices of TiO2 and SiO2 respectively, this results in a Fresnel reflectivity \(r = 0.209\) and optical thicknesses=200nm. The Bragg wavelength \(\lambda_B = 800nm\) in the rang 650 – 900\(nm\).

1- Figure-3 shows the relationship between the refractive index and the wavelength for (the Fusedsilica substrate, TiO2 material and SiO2 material). This determines the dispersion properties of the structure.
Fig. -3: The relationship between the reflective index and the wavelength for: a-(Fused silica) substrate. b- TiO2 material. c- SiO2 material

2- Figures-4 The relationship between the refractive index and the distance from substrate.

Fig.-4 : The relationship between the refractive index and the distance from substrate

3- Figure-5 the relationship between the phase and the wavelength we see the properties of phase shift in the range between 650-900nm for DGMs.
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Fig. 5: The relationship between the phase and the wavelength.

4- Results given in Figure 6 reveal almost high reflector at design wavelength ($\lambda_b = 800\text{nm}$) when the distribution Bragg mirror consist of alternating periodic quarter-wavelength stack of low and high refractive index. It exhibits a reflectivity of $>99.999999\%$.

Fig. 6: The relationship between the reflectance and the wavelength.

5- Figure 7 shows the characteristics of GD as a function of wavelength, where the group delay is nearly constant over a bandwidth of about 740-850nm.

Fig. 7: The relationship between the group delay and the wavelength.
6- Figure-8 shows the dependence of GDD on the optical wavelength. It is clearly seen that the decrease of the oscillation in GDD in the range wavelength from 770-840nm.

![GDD vs Wavelength Graph]

Fig.-8 : The relationship between the group delay dispersion and the wavelength

Furthermore, group delay dispersion shows monotonic behavior within wavelength range. In the case of high reflectors, a combination of materials with the highest refractive-index ratios $n_h/n_i$ is usually preferred since the higher the ratio, the higher the theoretical reflectance and bandwidth of standard quarter wave stacks. Among its competitors, the TiO2/SiO2 pair has the highest ratio. The results obtained gives a relatively high density optical coatings of the TiO2/SiO2 material pairs with low absorption and scattering losses. These stacks have a peak in the reflectivity at 800nm (Bragg wavelength) and the group delay, group delay dispersion have low oscillation.

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