Factor Affecting The Determination Of Wrinkling Limit Diagram For Metal Sheets

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Abstract:

Wrinkling is one of defects in sheet metal forming operations, it is produced by a compressive stress field. The prediction of wrinkling is important for the design of stamping and deep-drawing processes. Wrinkling is unacceptable in the outer skin panels where the final part appearance is crucial. Wrinkling on the mating surfaces can adversely affect the part assembly and part functions, such as sealing and welding. In addition, severe wrinkles may damage or even destroy dies. Therefore, the prediction and prevention of wrinkling are extremely important in sheet metal forming.

In this paper the wrinkling limit diagrams for steel, aluminum and aluminum alloy (AA3103, AA5182) sheets have been determined theoretically using Marciniak-Kuczynski analysis with Hosford anisotropic yield function.

The effect of strain hardening exponent, normal anisotropic ratio, Strain rate sensitivity exponent and imperfection factors could be determined theoretically, it is shown that the highest wrinkling limit curve appeared in steel sheet and the lowest curve in aluminum alloy (AA 3103)sheet (low resistance of wrinkling). The increase in each of the value of factors (strain hardening exponent, normal anisotropic ratio, Strain rate sensitivity exponent and imperfection factor) improve the resistance sheet against wrinkling.

Keywords: Wrinkling limit diagram(WLD), anisotropic yield criterion, MK analysis.

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Keywords: Wrinkling limit diagram(WLD), anisotropic yield criterion, MK analysis.
في هذه الدراسة تم تعيين مخطط حد التجعد للصفائح الصلب والألمنيوم وسبائك الألمنيوم (AA3103, AA5182) بناءً على نظرية الخضوع (Hosford). 

تم تعيين نظرياً تأثير كل من معامل الأصلاد الإتفاعي ومعامل تباين الخواص ومعامل حساسية معدل الانفعال ومعامل عدم التجانس. وقد تم اشتمال التجد في صفـيحة الصلب وإدنى منحنى في صفـيحة سبيكة الألمنيوم. (AA3103) أقل مقاومة لتـجعد وعند زيادة قيم كل من العوامل (معامل الأصلاد الإتفاعي ومعامل تباين الخواص ومعامل حساسية معدل الانفعال ومعامل عدم التجانس) ادى إلى تحسين مقاومة الصفـيحة ضد التجعد.

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### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ₁, σ₂, σ₃</td>
<td>Principal stresses</td>
<td>MPa</td>
</tr>
<tr>
<td>ε₁, ε₂, ε₃</td>
<td>Principal strains</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>Strain rate sensitivity</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>Strain hardening exponent</td>
<td></td>
</tr>
<tr>
<td>σ'</td>
<td>Effective stress</td>
<td>MPa</td>
</tr>
<tr>
<td>ε</td>
<td>Effective strain</td>
<td></td>
</tr>
<tr>
<td>ε'</td>
<td>Strain rate</td>
<td>1/sec</td>
</tr>
<tr>
<td>ε''</td>
<td>Effective strain rate</td>
<td>1/sec</td>
</tr>
<tr>
<td>ρ</td>
<td>Ratio of minor strain to major strain</td>
<td></td>
</tr>
<tr>
<td>ta</td>
<td>Thickness of the sheet</td>
<td></td>
</tr>
<tr>
<td>tb</td>
<td>Thickness of groove</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>Principle stress ratio</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>Imperfection factor</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>Yield criterion index</td>
<td></td>
</tr>
<tr>
<td>K</td>
<td>Strength coefficient</td>
<td>MPa</td>
</tr>
<tr>
<td>R'</td>
<td>Normal plastic anisotropic ratio</td>
<td></td>
</tr>
<tr>
<td>φ</td>
<td>Ratio of principal stress to effective stress</td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>Ratio of effective strain to principal strain</td>
<td></td>
</tr>
<tr>
<td>M-K</td>
<td>Marciniak-Kuczynski analysis</td>
<td></td>
</tr>
<tr>
<td>F₁a</td>
<td>Principal Force in region (a) in M.K analysis</td>
<td>N</td>
</tr>
<tr>
<td>F₁b</td>
<td>Principal Force in region (b) in M.K analysis</td>
<td>N</td>
</tr>
<tr>
<td>WLC</td>
<td>Wrinkling limit curve</td>
<td></td>
</tr>
</tbody>
</table>
1. Introduction:

Production engineers and researchers have paid much attention to the behavior of wrinkling in sheet metal forming operation over the last decade. It is difficult to define criteria for describing the stability in metal forming. In general, wrinkling may be affected by various factors including material property, punch and die geometry, blank geometry and holding conditions, interface friction and lubrication state. Numerous studies have been carried out on the relationship between wrinkling and material characteristics[1-3].

One of the most representative works was performed by Yoshida[4] and was generally directed towards investigation of the conditions under which wrinkling would occur in a large shallow pressing, characterised by the well known Yoshida buckling test. Karima and Sowerby [5] have attempted a bifurcation approach to the study of wrinkling during deep drawing. They treated flange wrinkling during deep drawing without a blank holder as a problem of elastic–plastic buckling of an annular plate. Their results indicate that a high rate of hardening has favorable effect on the prevention of buckling when deep drawing through conical and tapered dies. However, Narayanasamy and Sowerby [6] has showed that the stainless steel 304 sheets which have a low value of normal anisotropy and a high value of normalized hardening rate, have better resistance to the formation of wrinkles. The wrinkling behavior of cold rolled and annealed sheet metals with the aim of prediction the onset of wrinkling during drawing through tractrix and conical dies, has been studied in addition [7]. Di and Thomson [8] have used the neural network principle for the prediction of strain at the onset of wrinkling based on the Yoshida material parameters.

The effect of geometrical variables that affect the onset of wrinkling during deep drawing has been investigated by Wang et al. [9], using a neural network approach. Kim and Son [10] studied wrinkling behavior of sheet metals using a numerical analysis for evaluating a wrinkling limit diagram (WLD) for an anisotropic sheet subjected to biaxial plane stress.

Optical measurements of wrinkles on thin polymer films might provide a simple and accurate alternative to more traditional techniques used to determine thicknesses, mechanical properties (Stafford, Harrison, Beers, Karim, Amis, Vanlandingham, Kim, Volksen, Miller and Simonyi [11]), and residual stresses (Chung, Chastek, Fasolka, Ro and Stafford [12]). Similarly, measuring wrinkles on cell membranes might provide important insight regarding cell locomotion (Burton and Taylor [13], Harris, Wild and Stopak [14]). Applications that exploit wrinkling to tune the optical properties of cavities (Kolaric, Vandeparre, Desprez, Vallee and Damman [15]) and to shape capillaries for micro-fluidic purposes (Ohzono, Monobe, Shiokawa, Fujiwara and Shimizu [16]) also seem relevant.

The purpose of this paper is to determine the limit of wrinkling for all sheets(steel ,aluminum and aluminum alloy (AA3103 , AA5182)) by using wrinkling limit curve theoretically and compare. And determine the effect of strain hardening exponent, normal anisotropic ratio, Strain rate sensitivity exponent and imperfection factors in wrinkling limit curve.
2. Chemical composition and Mechanical properties:

The chemical composition of the aluminum, mild steel, aluminum alloy for AA5182 and AA3103 sheets were shown in Table (1, 2, 3 and 4).

### Table (1) Chemical analysis of Aluminum

<table>
<thead>
<tr>
<th>Material</th>
<th>Ti%</th>
<th>Mg%</th>
<th>Si%</th>
<th>Mn%</th>
<th>Cu%</th>
<th>Fe%</th>
<th>Zn%</th>
<th>Cr%</th>
<th>Al%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.017</td>
<td>0.015</td>
<td>0.1</td>
<td>0.02</td>
<td>0.2</td>
<td>0.21</td>
<td>0.03</td>
<td>0.004</td>
<td>Rem.</td>
</tr>
</tbody>
</table>

### Table (2) Chemical analysis of Mild steel

<table>
<thead>
<tr>
<th>Material</th>
<th>C%</th>
<th>P%</th>
<th>Mn%</th>
<th>S%</th>
<th>Si%</th>
<th>Mo%</th>
<th>Cr%</th>
<th>Cu%</th>
<th>Fe%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild steel</td>
<td>0.2</td>
<td>0.003</td>
<td>0.2</td>
<td>0.02</td>
<td>0.05</td>
<td>0.005</td>
<td>0.04</td>
<td>0.02</td>
<td>Rem.</td>
</tr>
</tbody>
</table>

### Table (3) Chemical analysis of AA5182 sheet

<table>
<thead>
<tr>
<th>Material</th>
<th>Mg%</th>
<th>Mn%</th>
<th>Fe%</th>
<th>Si%</th>
<th>Cu%</th>
<th>Cr%</th>
<th>Ti%</th>
<th>Al%</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA 5182</td>
<td>4.51</td>
<td>0.34</td>
<td>0.18</td>
<td>0.08</td>
<td>0.05</td>
<td>0.02</td>
<td>0.02</td>
<td>Rem.</td>
</tr>
</tbody>
</table>

### Table (4) Chemical analysis of AA3103 sheet

<table>
<thead>
<tr>
<th>Material</th>
<th>Mn%</th>
<th>Fe%</th>
<th>Mg%</th>
<th>Si%</th>
<th>Cu%</th>
<th>Zn%</th>
<th>Ti%</th>
<th>Al%</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA 3103</td>
<td>1.16</td>
<td>0.46</td>
<td>0.009</td>
<td>0.07</td>
<td>0.004</td>
<td>0.003</td>
<td>0.006</td>
<td>Rem.</td>
</tr>
</tbody>
</table>

The mechanical properties of all sheets were obtained from tensile test. The values of Strain Hardening exponent (n), Strain rate sensivity (m), Yield stress 0.2% offset (YS) and Strength coefficient (K),which were used in the theoretical determination of WLD, are shown in Table (5).

### Table (5) Mechanical properties of sheets

<table>
<thead>
<tr>
<th>Material</th>
<th>Strain Hardening exponent (n)</th>
<th>Strain rate sensivity (m)</th>
<th>Strength coefficient (K)[MPa]</th>
<th>0.2% Proof stress [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.21</td>
<td>0.001</td>
<td>280</td>
<td>82</td>
</tr>
<tr>
<td>Mild steel</td>
<td>0.27</td>
<td>0.018</td>
<td>940</td>
<td>410</td>
</tr>
<tr>
<td>AA3103 Sheet</td>
<td>0.226</td>
<td>0.002</td>
<td>180</td>
<td>55</td>
</tr>
<tr>
<td>AA5182 Sheet</td>
<td>0.3232</td>
<td>0.0001</td>
<td>585.2</td>
<td>143.5</td>
</tr>
</tbody>
</table>
The plastic anisotropic ratio \( (R) \) were obtained from tensile test with different specimen angle\( (0^\circ,45^\circ,90^\circ) \) and after determined Normal plastic anisotropic ratio \( (R') \) by using eq.(2.1), which were used in the theoretical determination of WLD, are shown in Table (6).

\[
R' = \frac{R_0 + 2R_{45} + R_{90}}{4}
\]  

(2.1)

<table>
<thead>
<tr>
<th>Material</th>
<th>Normal plastic Anisotropic ratio ( (R') )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.83</td>
</tr>
<tr>
<td>Mild Steel</td>
<td>1.51</td>
</tr>
<tr>
<td>AA3103</td>
<td>0.5675</td>
</tr>
<tr>
<td>AA5182</td>
<td>0.8872</td>
</tr>
</tbody>
</table>

3. Theoretical Analysis:

The theoretical wrinkling limit diagrams presented in this work were calculated using a more general code(M-K analysis and Theory of plasticity) for predicting the wrinkling limits under linear strain paths. The code consists of a main part and several subroutines allowing the implementation of hardening law, yield function.

The geometry of neck formation and the element of sheet undergoing plastic deformation are shown in Figure.(1). Following the M-K analysis, based on a simplified model with assumed pre-existing thickness imperfection in the form of a groove perpendicular to the principal stress directions, The sheet is composed of the nominal area and weak groove area, which are denoted by ‘a’ and ‘b’, respectively. The initial imperfection factor of the groove, \( f0 \), is defined as the thickness ratio \( (f0=t0b/t0a) \); where \( (t) \) denotes the thickness and subscript \( (0) \) denotes the initial state. A biaxial stress state is imposed on the nominal area and causes the development of strain increments in both the nominal (a) and the weak area (b).

The yield criterion proposed by Hosford\(^{[17]}\) was used in the calculation in the plane stress state, this criterion is obtained as follows:

\[
(\sigma)' = \frac{1}{(R'+1)}\left[(\sigma_1)' + (\sigma_2)' + R'(\sigma_1 - \sigma_2)'</right]........................................................................(3.1)
\]
using $e = 8$
from eq.(3.1)

\[
\varphi = \frac{\sigma_1}{\sigma^*} = \left[ \frac{(R' + 1)}{1 + (\alpha)^{-1} + R'(1 - \alpha)^{-1}} \right]^{\frac{1}{\epsilon}}
\] .............................................................. (3.2)

The behavior of material can be represented in the form of Power law

\[
\sigma^* = K\varepsilon^{n} \sigma
\] .............................................................. (3.3)

The ratio of the principal stress and strain are defined as follows:

\[
\alpha = \frac{\sigma_2}{\sigma_1}, \rho = \frac{\varepsilon_2}{\varepsilon_1} = \frac{d\varepsilon_2}{d\varepsilon_1}
\] .............................................................. (3.4)

The associated flow rule is expressed by

\[
d\varepsilon_{ij} = d\kappa \frac{\partial \sigma^*}{\partial \sigma_{ij}}
\] .............................................................. (3.5)

and

\[
d\varepsilon_1 = \frac{d\varphi}{(R' + 1)} \left[ 1 + R'(1 - \alpha)^{-1} \right]
\] .............................................................. (3.6)

\[
d\varepsilon_2 = \frac{d\varphi}{(R' + 1)} \left[ (\alpha)^{-1} - R'(1 - \alpha)^{-1} \right]
\] .............................................................. (3.7)

using condition of constant volume in plastic deformation

\[
d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0
\] .............................................................. (3.8)

from eq.(3.8)

\[
d\varepsilon_3 = -\frac{d\varphi}{(R' + 1)} \left[ 1 + (\alpha)^{-1} \right]
\] .............................................................. (3.9)

then, by applying the principle of equivalence of plastic work

\[
\sigma^* d\varepsilon' = \sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2
\] .............................................................. (3.10)
the compatibility condition is given by

\[ \epsilon_{2a} = \epsilon_{2b} \] \hspace{1cm} (3.11)

from Marciniak-Kuczynski analysis\textsuperscript{[18]}.

\[ f = \frac{t_b}{t_a} \] \hspace{1cm} (3.12)

\[ f = f_o \exp (\epsilon_{3b} - \epsilon_{3a}) \] \hspace{1cm} (3.13)

the equilibrium condition requires that the applied load remains constant along the specimen

\[ F_{1a} = F_{1b} \] \hspace{1cm} (3.14)

from eq.(3.3 - 3.14)

\[ \varphi (\epsilon' + \epsilon) = f \varphi (\epsilon' + d\epsilon) \] \hspace{1cm} (3.15)

**Fig. (1) Marciniak-Kuczynski analysis**

Equilibrium equation (3.15), is an equation that can be found and solved numerically. Imposing a loading path (\(\rho_a\)), a finite increment of strain is also imposed in region (a), and numerical computation is performed by using computer program (Fortran power Station) to determine the limit strain of a strain path in the WLD, and the limit strain is determined when \([(d\epsilon_{a1}/d\epsilon_{a1}) > 10]\) in the range of strain ratios from (-2 to -0.5)\textsuperscript{[19]}. 

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4. Results and Discussion:

The method of solving equations is numerically by using finite increment of strain in multiple steps of computer program *Fortran power station 4* to determine the limit strain of a strain path (-2 to -0.5), the input of the program is mechanical property of sheet (strain hardening exponent, strain rate sensitivity, imperfection factor, and normal plastic Anisotropic ratio), and the output of the program are major strain and minor strain.

Figure(2) shows the theoretical wrinkling limit diagram of Steel sheet using Hosford yield criterion.

![Wrinkling limit diagram of Steel sheet](image)

**Fig .(2) Wrinkling limit diagram of Steel sheet**

Figure(3) shows the theoretical wrinkling limit diagram of Aluminum sheet using Hosford yield criterion.
Fig. (3) Wrinkling limit diagram of Aluminum sheet

**Figure (4)** shows the theoretical wrinkling limit diagram of Aluminum alloy AA3103 sheet using Hosford yield criterion.
Figure(5) shows the theoretical wrinkling limit diagram of Aluminum alloy AA5182 sheet using Hosford yield criterion.

![Figure 5](image1.png)

**Fig .(5) Wrinkling limit diagram of Aluminum alloy AA5182 sheet**

Figure(6) shows the effect of strain hardening exponent (n value) on the wrinkling limit diagram. It can be seen the limits of wrinkling limit curve increase while increase the strain hardening exponent(n value).

![Figure 6](image2.png)

**Fig .(6) Effect of strain hardening exponent on WLC**
Figure 7 shows the effect of normal anisotropic ratio ($R'$ value) on the wrinkling limit diagram. It can be seen the limits of wrinkling limit curve increase while increase the normal anisotropic ratio.

Figure 8 shows the effect of imperfection factor ($f$ value) on the wrinkling limit diagram. It can be seen the limits of wrinkling limit curve increase while increase the imperfection factor.
Figure (9) shows the effect of Strain rate sensitivity exponent (m value) on the wrinkling limit curve. It can be seen that the limits of the wrinkling limit curve increase while increasing the Strain rate sensitivity exponent.

Fig. (9) Effect of Strain rate sensitivity exponent on WLC

Figure (10) shows the comparison of wrinkling limit curve between the aluminum and aluminum alloy sheets. It is shown that the highest wrinkling limit curve appeared in AA 5182 sheet and the lowest curve in aluminum alloy (AA 3103) sheet. AA 5182 sheet more resistance to wrinkling than all aluminum alloy sheets, because the AA 5182 sheet having high strain hardening exponent (n value) comparing with aluminum sheet.

Fig. (10) Comparison of wrinkling limit curve between the aluminum and aluminum alloy (AA 3103, AA 5182) sheets
Figure (11) shows the comparison of wrinkling limit curve between the Steel, aluminum and aluminum alloy sheets. It is shown that the steel sheet more resistance to wrinkling than all sheets. because the steel sheet having high normal plastic Anisotropic ratio ($R'$) comparing with all aluminum alloy sheet.

![Comparison of wrinkling limit curve between the steel, aluminum and aluminum alloy (AA 3103, AA 5182) sheets](image)

**Fig. (11) Comparison of wrinkling limit curve between the steel, aluminum and aluminum alloy (AA 3103, AA 5182) sheets**

5. **Conclusions:**

The major conclusions are listed below:

1) Wrinkling limit diagrams drawn in terms of strain with strain ratio (-0.5, -2) is highly suitable for the theoretical study of wrinkling behavior of sheet metals.

2) The onset of wrinkling depends on the factors, the strain hardening exponent, normal anisotropic ratio, Strain rate sensitivity exponent and imperfection factor improve the resistance sheet against wrinkling.

3) Steel sheet more resistance to wrinkling than aluminum and aluminum alloy sheets.

6. **References:**

