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THE αĝ-KERNEL AND WEAKLY ULTRA αĝ-SEPARATION AXIOMS VIA αĝ-OPEN SETS

النواة لمجموعات αĝ و بديهية الفصل فوق الضعيف بين تلك المجموعات باستخدام مجموعات αĝ المفتوحة

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Abstract:
The aim of this paper to introduce the concept of αĝ-kernel also we introduced the concept of the weakly ultra αĝ-separation of two sets in a topological space using αĝ-open sets. The αĝ-closure is defined in terms of this weakly ultra αĝ-separation. We also investigate some properties of weak separation axioms like αĝ-R_i-spaces, i = 0,1 and αĝ-T_i-spaces, i = 0,1,2.
Keywords: αĝ-open sets,αĝ-kernel,αĝ-closure,weakly ultra αĝ-separation,αĝ-R_i-spaces,i= 0,1 and αĝ-T_i-spaces, i = 0,1,2.

الخلاصة :
يهدف البحث لدراسة مفهوم النواة لمجموعات αĝ و بديهية الفصل فوق الضعيف بين تلك المجموعات باستخدام مجموعات αĝ المفتوحة في الفضاء التبولوجي. ومن ثم استخدمنا هذين المفهومين لتعريف الانغلاق لهذه المجموعات ودراسة بعض خواص بديهيات الفصل αĝ بين مجموعات αĝ مثل αĝ-R_i, i=0,1 و αĝ-T_i, i=0,1,2 و العلاقة بينهم.

1. Introduction:

In 1970, Levine [2],[3], introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. This concept was found to be useful and many results in general topology were improved. In 2014, V. Senthilkumaran, R. Krishnakumar and Y. Palaniappan [5], introduced αĝ-closed set. In this paper, we introduced some properties of αĝ-separation axioms by using some definition of new concept via αĝ-open sets. Throughout this paper, the closure and the interior of A are denoted by cl(A) and int(A), respectively.

2. Preliminaries:

Before entering to our work, we recall the following definitions, which are useful in the sequel.

Definition 2.1:[4] A subset A of a topological space X is called a α-open set if A⊆int(cl(int(A))) and a α-closed set if cl(int(cl(A)))⊆A.

Definition 2.2:[2] A subset A of a topological space X is called a generalized closed set (briefly g-closed set) if cl(A)⊆U whenever A⊆U and U is open in X.

Definition 2.3:[1] A subset A of a topological space X is called α generalized closed set (briefly αg-closed set) if cl(A)⊆U whenever A⊆U and U is open in X.

Definition 2.4:[5] A subset A of a topological space X is called $\hat{\alpha}$ generalized closed set (briefly $\hat{\alpha}$g-closed set) if int(cl(int(A)))⊆U whenever A⊆U and U is open in X. The complement of $\hat{\alpha}$g-closed set in X is $\hat{\alpha}$g-open in X.

Definition 2.5:[6] The intersection of all $\hat{\alpha}$g-closed sets in X containing A is called $\hat{\alpha}$ generalized closure of A and is denoted by $\hat{\alpha}$g-cl(A).

3. $\hat{\alpha}$g-Kernel and $\hat{\alpha}$g-R_i-Spaces, i = 0, 1:

Definition 3.1: The intersection of all $\hat{\alpha}$g-open subset of X containing A is called the $\hat{\alpha}$g-kernel of A (briefly $\hat{\alpha}$g-ker(A)), this means $\hat{\alpha}$g-ker(A)=⋂{G∈ $\hat{\alpha}$g-O(X):A⊆G}, where O(X) is an open sets in X

Example 3.2: Let X={a,b,c} , τ ={φ,{a},{a,b},{a,c,d},X}
$\hat{\alpha}$g-open sets ={φ,{a},{c},{d}, {a,b},{c,d}, {a,d},{a,c},{a,b,d},{a,c,d},X}
$\hat{\alpha}$g-ker{a}={a}⋂{a,b}⋂{a,d}⋂{a,c}⋂{a,b,d}⋂{a,c,d}={a}
$\hat{\alpha}$g-ker{b}={a,b}⋂{a,b,d}={a,b}.

Definition 3.3: In a space (X,τ), a set A is said to be weakly ultra $\hat{\alpha}$g-separated from B if there exists an $\hat{\alpha}$g-open set G such that G∩B=∅ or A∩$\hat{\alpha}$g-cl{B}=∅.

By the definition 3.2, we have the following for x, y∈X of a topological space,
(i) $\hat{\alpha}$g- cl{x} = {y :{y}is not weakly ultra $\hat{\alpha}$g-separated from{x}}
(ii) $\hat{\alpha}$g-ker{x} = {y :{x}is not weakly ultra $\hat{\alpha}$g-separated from{y}}.

Example 3.4: Let X ={a,b,c} , τ ={φ,{a},{a,b},{a,c,d},X}
$\hat{\alpha}$g-open sets ={φ,{a},{c},{d},{a,b},{c,d},{a,d},{a,c},{a,b,d},{a,c,d},X}
{a}is weakly ultra $\hat{\alpha}$g-separated from{b}, but {b}is not weakly ultra $\hat{\alpha}$g-separated from{a}.

Theorem 3.5: Let (X,τ) be a topological space then x∈$\hat{\alpha}$g-cl{y} iff y∈$\hat{\alpha}$g-ker{x}for each x ≠ y ∈ X.

Proof: Let (X,τ) be a topological space. And let x∈$\hat{\alpha}$g-cl{y}, then for each U is an $\hat{\alpha}$g-open set such that x∈U implies y∈U this means y∈$\hat{\alpha}$g-ker{x}. Let y∈$\hat{\alpha}$g-ker{x}, then for each U is an $\hat{\alpha}$g-open set such that x∈U implies y∈U this means U∩{y}≠∅. Hence x∈$\hat{\alpha}$g-cl{y}.

Definition 3.6: A topological space (X,τ) is called an $\hat{\alpha}$g-R_0-space if for each x ∈X and U $\hat{\alpha}$g-open set containing x, then $\hat{\alpha}$g-cl{x}⊆U.

Example 3.7: Let X ={a,b,c} , τ ={φ,{a},{b,c},X}
$\hat{\alpha}$g-open sets ={φ,{a},{b},{c},{a,b},{a,c},{b,c},X}.

Definition 3.8: A topological space (X,τ) is called an $\hat{\alpha}$g-R_1-space if for each two distinct point x , y of X with $\hat{\alpha}$g-cl{x} ≠ $\hat{\alpha}$g-cl{y}, there exist disjoint $\hat{\alpha}$g-open sets U,V such that $\hat{\alpha}$g-cl{x} ⊆ U and $\hat{\alpha}$g-cl{y}⊆ V.

Example 3.9: Let X ={a,b,c} , τ ={φ,{a},X}
$\hat{\alpha}$g-open sets ={φ,{a},{b},{c},{a,b},{a,c},X}.

Theorem 3.10: Let (X,τ) be a topological space. Then (X,τ) is $\hat{\alpha}$g-R_0-space if and only if, $\hat{\alpha}$g-cl{x} = $\hat{\alpha}$g-ker{x}, for each x∈X.

Proof: Let (X,τ) be an $\hat{\alpha}$g-R_0-space, and let y be another point such that y∉$\hat{\alpha}$g-cl{x}, implies y∈($\hat{\alpha}$g-cl{x})^c is an $\hat{\alpha}$g-open set. By assumption (X,τ) be an $\hat{\alpha}$g-R_0-space, then $\hat{\alpha}$g-cl{y}⊆($\hat{\alpha}$g-cl{x})^c [By Definition 3.6], there for x∉$\hat{\alpha}$g-cl{y} and y∉$\hat{\alpha}$g-ker{x} [By Theorem 3.5]. So we get $\hat{\alpha}$g-ker{x}⊆$\hat{\alpha}$g-cl{x}.Also since (X,τ) be an $\hat{\alpha}$g-R_0-space, then $\hat{\alpha}$g-cl{x}⊂U for each U $\hat{\alpha}$g-open set

containing x [By Definition 3.6], implies α̂g-cl{x}⊆⋂{U:x∈U}. So we get α̂g-cl{x}⊆α̂g-ker{x}[By Definition 3.1]. Thus α̂g-cl{x} = α̂g-ker{x}.
Conversely, let α̂g-cl{x} = α̂g-ker{x}, for each α̂g-open set U and x ∈U, then α̂g-ker{x}= α̂g-cl{x}⊆U [By Definition 3.1]. Hence by Definition 3.6,(X,τ) is an α̂g-R_0-space.

Theorem 3.11: A topological space (X,τ) is an α̂g-R_0-space if and only if for each F α̂g-closed set and x∈F then α̂g-ker{x}⊆F.

Proof: Let for each F α̂g-closed set and x∈F then α̂g-ker{x}⊆F and let U be an α̂g-open set, x∈U then for each y∉U implies y∈U^c is an α̂g-closed set implies α̂g-ker{y}⊆U^c [By assumption]. Therefore x∉α̂g-ker{y} implies y∉α̂g-cl{x}[By corollary 3.5]. So α̂g-cl{x} ⊆ U. Thus (X,τ) is an α̂g-R_0-space.
Conversely, let a topological space (X,τ) be a α̂g-R_0-space and F be α̂g-closed set and x∈F. Then for each y∉F implies y∈F^c is α̂g-open set, then α̂g-cl{y}⊆F^c [since (X,τ) is α̂g-R_0-space], so α̂g-ker{x}= α̂g-cl{x}. Thus α̂g-ker{x}⊆F.

Corollary 3.12: A topological space (X,τ) is an α̂g-R_0-space if and only if for each U α̂g-open set and x∈U then α̂g-cl(α̂g-ker{x})⊆U.

Proof: Clearly.

Theorem 3.13: Every α̂g-R_1-space is an α̂g-R_0-space.

Proof: Let (X,τ) be an α̂g-R_1-space and let U be an α̂g-open set, x∈U, then for each y∉U implies y∈U^c is an α̂g-closed set and α̂g-cl{y}⊆U^c implies α̂g-cl{x} ≠ α̂g-cl{y}. Hence by definition 3.8, α̂g-cl{x}⊆ U. Thus(X,τ) is an α̂g-R_0-space.

Theorem 3.14: A topological space (X,τ) is an α̂g-R_1-space if and only if for each $x \neq y \in X$ with α̂g-$ker\{x\}$ ≠ α̂g-$ker\{y\}$ then there exist α̂g-closed sets F_1, F_2 such that α̂g-$ker\{x\}\subseteq F_1$,α̂g-$ker\{x\}\cap F_2 = \emptyset$ and α̂g-$ker\{y\}\subseteq F_2$, α̂g-$ker\{y\}\cap F_1 = \emptyset$ and $F_1 \cup F_2 = X$.

Proof: Let a topological space (X,τ) be an α̂g-R_1-space. Then for each $x \neq y \in X$ with α̂g-$ker\{x\}$≠α̂g-$ker\{y\}$. Since every α̂g-R_1-space is an α̂g-R_0-space [by theorem 3.13], and by theorem 3.10,α̂g-$cl\{x\}$ ≠ α̂g-$cl\{y\}$, then there exist α̂g-open sets G_1,G_2 such that α̂g-$cl\{x\} \subseteq G_1$ and α̂g-$cl\{y\} \subseteq G_2$ and $G_1 \cap G_2 = \emptyset$ [Since (X,τ) is α̂g-R_1-space], then G_1^c and G_2^c are α̂g-closed sets such that $G_1^c \cup G_2^c = X$. Put $F_1 = G_1^c$ and $F_2 = G_2^c$. Thus, $x \in G_1 \subseteq F_2$ and $y \in G_2 \subseteq F_1$ so that α̂g-$ker\{x\} \subseteq G_1 \subseteq F_2$ and α̂g-$ker\{y\} \subseteq G_2 \subseteq F_1$.
Conversely, let for each $x \neq y \in$ X with α̂g-$ker\{x\}$≠ α̂g-$ker\{y\}$, there exist α̂g-closed sets F_1,F_2 such that α̂g-$ker\{x\}\subseteq F_1$,α̂g-$ker\{x\}\cap F_2 = \emptyset$ and α̂g-$ker\{y\}\subseteq F_2$, α̂g-$ker\{y\}\cap F_1 = \emptyset$ and $F_1 \cup F_2 = X$, then F_1^c and F_2^c are α̂g-open sets such that $F_1^c \cap F_2^c = \emptyset$. Put $F_1^c = G_2$ and $F_2^c = G_1$. Thus α̂g-$ker\{x\} \subseteq G_1$ and α̂g-$ker\{y\} \subseteq G_2$ and $G_1 \cap G_2 = \emptyset$, so that $x \in G_1$ and $y \in G_2$ implies $x \notin$α̂g-$cl\{y\}$ and $y \notin$α̂g-$cl\{x\}$, then α̂g-$cl\{x\} \subseteq G_1$ and α̂g-$cl\{y\} \subseteq G_2$. Thus,(X,τ) is an α̂g-R_1-space.

Corollary 3.15: A topological space (X,τ) is an α̂g-R_1-space if and only if for each x ≠ y ∈ X with α̂g-cl{x}≠ α̂g-cl{y} there exist disjoint α̂g-open sets U,V such that α̂g-cl(α̂g-ker{x}) ⊆U and α̂g-cl(α̂g-ker{y}) ⊆V.

Proof: Let (X,τ) be an α̂g-R_1-space and let x ≠ y∈X with α̂g-cl{x}≠ α̂g-cl{y}, then there exist disjoint α̂g-open sets U, V such that α̂g-cl{x}⊆ U and α̂g-cl{y}⊆ V. Also (X,τ) is α̂g-R_0-spece [by Theorem 3.13] implies for each x∈X, then α̂g-cl{x}= α̂g-ker{x} [By Theorem 3.10], but α̂g-cl{x}= α̂g-cl(α̂g-cl{x}) = α̂g-cl(α̂g-ker{x}). Thus α̂g-cl(α̂g-ker{x}) ⊆ U and α̂g-cl(α̂g-ker{y}) ⊆ V.
Conversely, let for each x ≠ y∈X with α̂g-cl{x}≠ α̂g-cl{y} there exist disjoint α̂g-open sets U,V such that α̂g-cl(α̂g-ker{x})⊆U and α̂g-cl(α̂g-ker{y})⊆V. Since {x}⊆ α̂g-ker{x} then α̂g-cl{x}⊆ α̂g-cl(α̂g-ker{x}) for each x∈X. So we get α̂g-cl{x}⊆U and α̂g-cl{y}⊆V. Thus, (X,τ) be an α̂g-R_1-space.

4. $\alpha\hat{g}$-T_i-Spaces, i = 0, 1, 2:

Definition 4.1: Let (X,τ) be a topological space. Then X is called $\alpha\hat{g}$-T_0-space iff for each pair of distinct points in X, there exists a $\alpha\hat{g}$-open set in X containing one and not the other.

Example 4.2: Let X={a,b,c} , τ ={φ,{a},{a,b},{a,c},X}
$\alpha\hat{g}$-open sets ={φ,{a},{a,b},{a,c},X}.

Definition 4.3: Let (X,τ) be a topological space. Then X is called $\alpha\hat{g}$-T_1-space iff for each pair of distinct points x and y of X, there exists $\alpha\hat{g}$-open sets G,H containing x and y respectively such that y∉G and x∉H.

Example 4.4: Let X={a,b,c} , τ ={φ,{a},{a,b},X}
$\alpha\hat{g}$-open sets ={φ,{b},{a,b},{a,c},X}.

Definition 4.5: Let (X,τ) be a topological space. Then X is called $\alpha\hat{g}$-T_2-space iff for each pair of distinct points x and y of X, there exist disjoint $\alpha\hat{g}$-open sets G,H in X such that x∈G and y∈H.

Example 4.6: Let X={a,b,c} , τ ={φ,{a},{b,c},X}
$\alpha\hat{g}$-open sets ={φ,{a},{b},{c},{a,b},{a,c},{b,c},X}.

Theorem 4.7: A topological space (X,τ) is an $\alpha\hat{g}$-T_0-space if and only if either y∉$\alpha\hat{g}$-ker{x} or x∉$\alpha\hat{g}$-ker{y}, for each x ≠ y∈ X.

Proof: Let (X,τ) be an $\alpha\hat{g}$-T_0-space then for each x ≠ y ∈ X, there exists an $\alpha\hat{g}$-open set G such that either x ∈ G, y ∉ G or x ∉ G, y ∈ G. Thus either x ∈ G, y ∉ G implies y ∉ $\alpha\hat{g}$-ker{x} or x ∉ G, y∈G implies x ∉$\alpha\hat{g}$-ker{y}.
Conversely, let either y ∉$\alpha\hat{g}$-ker{x} or x ∉$\alpha\hat{g}$-ker{y}, for each x ≠ y∈ X. Then there exists an $\alpha\hat{g}$-open set G such that either x∈ G, y ∉ G or x ∉ G, y ∈ G. Thus (X,τ) is a $\alpha\hat{g}$-T_0-space.

Theorem 4.8: A topological space (X,τ) is an $\alpha\hat{g}$-T_0-space if and only if either $\alpha\hat{g}$-ker{x} is weakly ultra $\alpha\hat{g}$-separated from {y} or $\alpha\hat{g}$-ker{y} is weakly ultra $\alpha\hat{g}$-separated from {x} for each x≠y∈X.

Proof: Let (X,τ) be an $\alpha\hat{g}$-T_0-space then for each x≠y∈X, there exists an $\alpha\hat{g}$-open set G such that x∈G, y∉G or x∉G, y∈G. Now if x∈G, y∉G implies $\alpha\hat{g}$-ker{x} is weakly ultra $\alpha\hat{g}$-separated from {y}. Or if x∉G, y∈G implies $\alpha\hat{g}$-ker{y} is weakly ultra $\alpha\hat{g}$-separated from {x}.
Conversely, let either $\alpha\hat{g}$-ker{x} be weakly ultra $\alpha\hat{g}$-separated from {y} or $\alpha\hat{g}$-ker{y}be weakly ultra $\alpha\hat{g}$-separated from {x}. Then there exists an $\alpha\hat{g}$-open set G such that $\alpha\hat{g}$-ker{x}⊆G and y∉G or $\alpha\hat{g}$-ker{y}⊆G, x∉G implies x∈G, y∉G or x∉G, y∈G. Thus, (X,τ) is a $\alpha\hat{g}$-T_0-space.

Theorem 4.9: A topological space (X,τ) is an $\alpha\hat{g}$-T_1-space if and only if for each x≠y∈X, $\alpha\hat{g}$-ker{x} is weakly ultra $\alpha\hat{g}$-separated from {y} and $\alpha\hat{g}$-ker{y} is weakly ultra $\alpha\hat{g}$-separated from {x}.

Proof: Let (X,τ) be an $\alpha\hat{g}$-T_1-space then for each x≠y∈X, there exists an $\alpha\hat{g}$-open sets U,V such that x∈U, y∉U and x∉V, y∈V. Implies $\alpha\hat{g}$-ker{x} is weakly ultra $\alpha\hat{g}$-separated from {y} and $\alpha\hat{g}$-ker{y} is weakly ultra $\alpha\hat{g}$-separated from {x}.
Conversely, let $\alpha\hat{g}$-ker{x} be weakly ultra $\alpha\hat{g}$-separated from {y} and $\alpha\hat{g}$-ker{y} be weakly ultra $\alpha\hat{g}$-separated from {x}. Then there exists an $\alpha\hat{g}$-open sets U,V such that $\alpha\hat{g}$-ker{x}⊆U, y∉U and $\alpha\hat{g}$-ker{y}⊆V, x∉V implies x∈U, y∉U and x∉V, y∈V. Thus, (X,τ) is a $\alpha\hat{g}$-T_1-space.

Theorem 4.10: A topological space (X,τ) is an $\widehat{\alpha g}$-T_1-space if and only if for each $x\in X$, $\widehat{\alpha g}$-ker$\{x\}=\{x\}$.

Proof: Let (X,τ) be an $\widehat{\alpha g}$-T_1-space and let $\widehat{\alpha g}$-ker$\{x\}\neq\{x\}$. Then $\widehat{\alpha g}$-ker$\{x\}$ contains another point distinct from x say y. So $y\in\widehat{\alpha g}$-ker$\{x\}$ implies $\widehat{\alpha g}$-ker$\{x\}$ is not weakly ultra $\widehat{\alpha g}$-separated from $\{y\}$. Hence by Theorem 4.9, (X,τ) is not an $\widehat{\alpha g}$-T_1-space this is a contradiction. Thus $\widehat{\alpha g}$-ker$\{x\}=\{x\}$.
Conversely, let $\widehat{\alpha g}$-ker$\{x\}=\{x\}$, for each $x\in X$ and let (X,τ) be not an $\widehat{\alpha g}$-T_1-space. Then by theorem 4.9, $\widehat{\alpha g}$-ker$\{x\}$ is not weakly ultra $\widehat{\alpha g}$-separated from $\{y\}$, this means that for every $\widehat{\alpha g}$-open set G contains $\widehat{\alpha g}$-ker$\{x\}$ then $y\in G$ implies $y\in\bigcap\{G\in\widehat{\alpha g}\text{-}O(X): x\in G\}$ implies $y\in\widehat{\alpha g}$-ker$\{x\}$, this is a contradiction. Thus,(X,τ) is a $\widehat{\alpha g}$-T_1-space.

Theorem 4.11: A topological space (X,τ) is an $\widehat{\alpha g}$-T_1-space if and only if for each $x\neq y\in X$, $y\notin\widehat{\alpha g}$-ker$\{x\}$ and $x\notin\widehat{\alpha g}$-ker$\{y\}$.

Proof: Let (X,τ) be a $\widehat{\alpha g}$-T_1-space then for each $x\neq y\in X$, there exists an $\widehat{\alpha g}$-open sets U,V such that $x\in U$, $y\notin U$ or $y\in V$, $x\notin V$. Implies $y\notin\widehat{\alpha g}$-ker$\{x\}$ and $x\notin\widehat{\alpha g}$-ker$\{y\}$.
Conversely, let $y\notin\widehat{\alpha g}$-ker$\{x\}$ and $x\notin\widehat{\alpha g}$-ker$\{y\}$, for each $x\neq y\in X$. Then there exists an $\widehat{\alpha g}$-open sets U,V such that $x\in U$, $y\notin U$ and $y\in V$, $x\notin V$. Thus (X,τ) is a $\widehat{\alpha g}$-T_1-space.

Theorem 4.12: A topological space (X,τ) is an $\widehat{\alpha g}$-T_1-space if and only if for each $x\neq y\in X$ implies $\widehat{\alpha g}$-ker$\{x\}\cap\widehat{\alpha g}$-ker$\{y\}=\emptyset$.

Proof: Let a topological space (X,τ) be $\widehat{\alpha g}$-T_1-space. Then $\widehat{\alpha g}$-ker$\{x\}=\{x\}$ and $\widehat{\alpha g}$- ker$\{y\}=\{y\}$ [By Theorem 4.10]. Thus $\widehat{\alpha g}$-ker$\{x\}\cap\widehat{\alpha g}$-ker$\{y\}=\emptyset$.
Conversely, let for each $x\neq y\in X$ implies $\widehat{\alpha g}$-ker$\{x\}\cap\widehat{\alpha g}$-ker$\{y\}=\emptyset$ and let (X,τ) be not $\widehat{\alpha g}$-T_1-space then for each $x\neq y\in X$ implies $y\in\widehat{\alpha g}$-ker$\{x\}$ or $x\in\widehat{\alpha g}$-ker$\{y\}$[by theorem 4.10], then $\widehat{\alpha g}$-ker$\{x\}\cap\widehat{\alpha g}$-ker$\{y\}\neq\emptyset$, this is a contradiction. Thus, (X,τ) is a $\widehat{\alpha g}$-T_1-space.

Corollary 4.13: Every $\widehat{\alpha g}$-T_2-space is $\widehat{\alpha g}$-T_1-space and every $\widehat{\alpha g}$-T_1-space is an $\widehat{\alpha g}$-T_0-space.

Proof: Clearly.

Theorem 4.14: A topological space (X,τ) is an $\widehat{\alpha g}$-T_1-space if and only if (X,τ) is $\widehat{\alpha g}$-T_0-space and $\widehat{\alpha g}$-R_0-space.

Proof: Let (X,τ) be $\widehat{\alpha g}$-T_1-space and let $x\in U$ be and $\widehat{\alpha g}$-open set, then for each $x\neq y\in X$, $\widehat{\alpha g}$-ker$\{x\}\cap\widehat{\alpha g}$-ker$\{y\}=\emptyset$ [by theorem 4.12] implies $x\notin\widehat{\alpha g}$-ker$\{y\}$ and $y\notin\widehat{\alpha g}$-cl$\{x\}$ this means $\widehat{\alpha g}$-cl$\{x\}=\{x\}$, hence $\widehat{\alpha g}$-cl$\{x\}\subseteq U$. Thus, (X,τ) is a $\widehat{\alpha g}$-R_0-space.
Conversely, let (X,τ) be $\widehat{\alpha g}$-T_0-space and $\widehat{\alpha g}$-R_0-space, then for each $x\neq y\in X$ there exists an $\widehat{\alpha g}$-open set U such that $x\in U$, $y\notin U$ or $x\notin U$, $y\in U$. Say $x\in U$, $y\notin U$ since (X,τ) is a $\widehat{\alpha g}$-R_0-space, then $\widehat{\alpha g}$-cl$\{x\}\subseteq U$, this means there exists an $\widehat{\alpha g}$-open set V such that $y\in V$, $x\notin V$. Thus, (X,τ) is a $\widehat{\alpha g}$-T_1-space.

Theorem 4.15: A topological space (X,τ) is an $\widehat{\alpha g}$-T_2-space if and only if
(1) (X,τ) is an $\widehat{\alpha g}$-T_0-space and $\widehat{\alpha g}$-R_1-space.
(2) (X,τ) is an $\widehat{\alpha g}$-T_1-space and $\widehat{\alpha g}$-R_1-space.

Proof (1): Let (X,τ) be an $\widehat{\alpha g}$-T_2-space then it is an $\widehat{\alpha g}$-T_0-space. Now since (X,τ) be an $\widehat{\alpha g}$-T_2-space then for each $x\neq y\in X$, there exist disjoint $\widehat{\alpha g}$-open sets U,V such that $x\in U$ and $y\in V$ implies $x\notin\widehat{\alpha g}$-cl$\{y\}$ and $y\notin\widehat{\alpha g}$-cl$\{x\}$, therefore $\widehat{\alpha g}$-cl$\{x\}=\{x\}\subseteq U$ and $\widehat{\alpha g}$-cl$\{y\}=\{y\}\subseteq V$. Thus, (X,τ) is a $\widehat{\alpha g}$-R_1-space.
Conversely, let (X,τ) be an $\widehat{\alpha g}$-T_0-space and $\widehat{\alpha g}$-R_1-space, then for each $x\neq y\in X$, there exists $\widehat{\alpha g}$-open set U such that $x\in U$, $y\notin U$ or $y\in U$, $x\notin U$, implies $\widehat{\alpha g}$-cl$\{x\}\neq\widehat{\alpha g}$-cl$\{y\}$, since (X,τ) is an $\widehat{\alpha g}$-R_1-space [By assumption], then there exist disjoint $\widehat{\alpha g}$-open sets G,H such that $x\in G$ and $y\in H$ [Def. 3.8]. Thus, (X,τ) is a $\widehat{\alpha g}$-T_2-space.

Proof (2): By the same way of part (1) an $\hat{\alpha g}$-T_2-space is an $\hat{\alpha g}$-T_1-space and $\hat{\alpha g}$-R_1-space.
Conversely, let (X,τ) be $\hat{\alpha g}$-T_1-space and $\hat{\alpha g}$-R_1-space, then for each $x \neq y \in X$, there exists $\hat{\alpha g}$-open sets U,V such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$, implies $\hat{\alpha g}$-cl$\{x\} \neq \hat{\alpha g}$-cl$\{y\}$, since (X,τ) is an $\hat{\alpha g}$-R_1-space, then there exist disjoint $\hat{\alpha g}$-open sets G,H such that $x \in G$ and $y \in H$. Thus, (X,τ) is a $\hat{\alpha g}$-T_2-space.

Corollary 4.16: A topological $\hat{\alpha g}$-T_0-space is an $\hat{\alpha g}$-T_2-space if and only if for each $x \neq y \in X$ with $\hat{\alpha g}$-ker$\{x\} \neq \hat{\alpha g}$-ker$\{y\}$ then there exist $\hat{\alpha g}$-closed sets F_1, F_2 such that $\hat{\alpha g}$-ker$\{x\} \subseteq F_1$, $\hat{\alpha g}$-ker$\{x\} \cap F_2 = \emptyset$ and $\hat{\alpha g}$-ker$\{y\} \subseteq F_2$, $\hat{\alpha g}$-ker$\{y\} \cap F_1 = \emptyset$ and $F_1 \cup F_2 = X$.

Proof: By Theorem 3.14 and Theorem 4.15.

Corollary 4.17: A topological $\hat{\alpha g}$-T_1-space is an $\hat{\alpha g}$-T_2-space if and only if one of the following conditions holds:
(1) For each $x \neq y \in X$ with $\hat{\alpha g}$-cl$\{x\} \neq \hat{\alpha g}$-cl$\{y\}$ then there exist an $\hat{\alpha g}$-open sets U,V such that $\hat{\alpha g}$-cl($\hat{\alpha g}$-ker$\{x\}$) $\subseteq$ U and $\hat{\alpha g}$-cl($\hat{\alpha g}$-ker$\{y\}$) $\subseteq$ V.
(2) For each $x \neq y \in X$ with $\hat{\alpha g}$-ker$\{x\} \neq \hat{\alpha g}$-ker$\{y\}$ then there exist $\hat{\alpha g}$-closed sets F_1,F_2 such that $\hat{\alpha g}$-ker$\{x\} \subseteq F_1$, $\hat{\alpha g}$-ker$\{x\} \cap F_2 = \emptyset$ and $\hat{\alpha g}$-ker$\{y\} \subseteq F_2$, $\hat{\alpha g}$-ker$\{y\} \cap F_1 = \emptyset$ and $F_1 \cup F_2 = X$.

Proof (1): By Corollary 3.15 and Theorem 4.15.

Proof (2): By Theorem 3.14 and Theorem 4.13.

Theorem 4.18: A topological $\hat{\alpha g}$-R_1-space is an $\hat{\alpha g}$-T_2-space if and only if one of the following conditions holds:
(1) For each $x \in X$, $\hat{\alpha g}$-ker$\{x\} = \{x\}$.
(2) For each $x \neq y \in X$, $\hat{\alpha g}$-ker$\{x\} \neq \hat{\alpha g}$-ker$\{y\}$ implies $\hat{\alpha g}$-ker$\{x\} \cap \hat{\alpha g}$-ker$\{y\} = \emptyset$.
(3) For each $x \neq y \in X$, either $x \notin \hat{\alpha g}$-ker$\{y\}$ or $y \notin \hat{\alpha g}$-ker$\{x\}$.
(4) For each $x \neq y \in X$ then $x \notin \hat{\alpha g}$-ker$\{y\}$ and $y \notin \hat{\alpha g}$-ker$\{x\}$.

Proof (1): Let (X,τ) be an $\hat{\alpha g}$-T_2-space. Then (X,τ) is a $\hat{\alpha g}$-T_1-space and $\hat{\alpha g}$-R_1-space [By theorem 4.15]. Hence by Theorem 4.10, $\hat{\alpha g}$-ker$\{x\} = \{x\}$ for each $x \in X$.
Conversely, let for each $x \in X$, $\hat{\alpha g}$-ker$\{x\} = \{x\}$, then by Theorem 4.10,(X,τ) is a $\hat{\alpha g}$-T_1-space. Also (X,τ) is an $\hat{\alpha g}$-R_1-space by assumption. Hence by Theorem 4.15,(X,τ) is an $\hat{\alpha g}$-T_2- space.

Proof (2): Let (X,τ) be an $\hat{\alpha g}$-T_2-space. Then (X,τ) is an $\hat{\alpha g}$-T_1-space [By theorem 4.3]. Hence by theorem 4.12, $\hat{\alpha g}$-ker$\{x\} \cap \hat{\alpha g}$-ker$\{y\} = \emptyset$ for each $x \neq y \in X$.
Conversely, assume that for each $x \neq y \in X$, $\hat{\alpha g}$-ker$\{x\} \neq \hat{\alpha g}$-ker$\{y\}$ implies $\hat{\alpha g}$-ker$\{x\} \cap \hat{\alpha g}$-ker$\{y\} = \emptyset$, so by Theorem 4.12, (X,τ) is an $\hat{\alpha g}$-T_1-space, also (X,τ) is an $\hat{\alpha g}$-R_1-space by assumption. Hence by Theorem 4.15, (X,τ) is an $\hat{\alpha g}$-T_2-space.

Proof (3): Let (X,τ) be an $\hat{\alpha g}$-T_2-space. Then (X,τ) is an $\hat{\alpha g}$-T_0-space [By Theorem 4.13]. Hence by theorem 4.3, either $x \notin \hat{\alpha g}$-ker$\{y\}$ or $y \notin \hat{\alpha g}$-ker$\{x\}$ for each $x \neq y \in X$.
Conversely, assume that for each $x \neq y \in X$, either $x \notin \hat{\alpha g}$-ker$\{y\}$ or $y \notin \hat{\alpha g}$-ker$\{x\}$ for each $x \neq y \in X$, so by theorem 4.3,(X,τ) is an $\hat{\alpha g}$-T_0-space also (X,τ) is an $\hat{\alpha g}$-R_1-space by assumption. Thus, (X,τ) is an $\hat{\alpha g}$-T_2-space [By Theorem 4.15].

Proof (4): Let (X,τ) be an $\hat{\alpha g}$-T_2-space. Then (X,τ) is an $\hat{\alpha g}$-T_1-space and an $\hat{\alpha g}$-R_1-space [By theorem 4.15]. Hence by Theorem 4.11, $x \notin \hat{\alpha g}$-ker$\{y\}$ and $y \notin \hat{\alpha g}$-ker$\{x\}$.
Conversely, let for each $x \neq y \in X$ then $x \notin \hat{\alpha g}$-ker$\{y\}$ and $y \notin \hat{\alpha g}$-ker$\{x\}$. Then by Theorem 4.11, (X,τ) is an $\hat{\alpha g}$-T_1-space. Also (X,τ) is an $\hat{\alpha g}$-R_1-space by assumption. Hence by Theorem 4.15,(X,τ) is an $\hat{\alpha g}$-T_2-space.

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