Degradation Improvement in BER Due to Various Sources of Noise in Optical Transceivers

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Abstract

The quality of digital communications whether at radio or optical frequencies depends primarily on the Bit Error Rate (BER). Reducing the BER improves the communication system behavior. At optical frequencies the digital signal is subjected to various types of noise that tends to degrade the BER. This paper describes the dependence of BER on noise and how these effects can be encountered by using longer wavelengths (1.5-1.6 um) at different bit rates.

الخلاصـة

إن جودة الاتصالات الرقمية سواء للترددات الراديوية والضوئية تعتمد بشكل رئيسي على نسبة الخطأ في البت المرسلة، وإن تقليل هذه النسبة من الأمور المهمة في أنظمة الاتصالات في الترددات الضوئية تعرض الإشارة الرقمية لأنواع مختلفة من الضوضاء التي تسبب زيادة في احتمالية الخطأ في هذا البحث تم توضيح اعتماد نسبة الخطأ في البت على الضوضاء، وكذلك بين البحث أن استخدام الأطوال الموجية الطويلة (الفترة الثالثة التي تمتذج 1.6 -- 1.5 um) يؤدي إلى تقليل تأثير الضوضاء على نسب الخطأ في البت ولعدة نسب مختلفة من ضغ المعلومات.
1. Introduction

In optical communications, the light signal is generated, transmitted and detected at the receiver. In parallel with this, various kinds of noise are also generated, transmitted and added to the final detected photocurrent. Added to this the distortion of the transmitted signal due to the transmission channel if the transmission channel is not ideal. This means that the transmitted signal can not be perfectly recovered. Therefore it is important to minimize the effects of noise and distortion at the receiver. In digital communications this means minimizing the Bit Error Rate (BER).

Noise sources in optical communications can be considered as waveform domain noise, (i.e. they cause random distortion of the signal waveform). In digital communications, there can also be time domain noise called jitter which is the timing error of the recovered bit clock with respect to the received data sequence that is used to sample the received signal for detection. Unlike Thermal noise, most noise sources in optical communications are signaling dependent. Some of these noise sources can be theoretically reduced to zero, for example the use of single mode laser eliminates the Mode Partition Noise (MPN), and the use of non-coherent detection eliminates Phase Noise. Avoiding the use of optical amplifiers eliminates the Amplified Spontaneous Emission noise (ASE). This leaves the signal with the basic noise sources such as InterSymbol Interference (ISI) which is due to channel effects, jitter, and shot noise which is intrinsic in photocurrent generation and can never be reduced to zero.

In a previous paper \[1\], it has been shown that bit error rate can be improved by using longer wavelengths. This paper describes the dependence of BER on various effects of noise. The results obtained show that the BER is degraded due to these effects, at different bit rates. But the degradation can be improved by using longer wavelengths.

2. Theoretical Analysis

2-1 BER Calculation Assuming Gaussian Distribution

The optical signal power bearing the digital information and incident on the photo detector \[2\]:

\[
p(t) = \sum_{k=0}^{K} A_k p(t - KT) + n(t) \]

This equation indicates that the received signal is a Gaussian shape pulse, added to it the InterSymbol Interference ISI, and \(n(t)\) which comprises the effects of thermal noise and shot noise.
where: \( A_k \) is the amplitude of the \( K_{th} \) pulse, \( p(t) \) is the received pulse, and \( T \) is the interval between two consecutive pulse. To detect the transmitted amplitude \( A_k \), the received signal is first sampled at \( KT + \tau \) for certain \( \tau \) within \((0, T)\). From equation (1), the sampled output is

\[
p_k = p(KT + \tau) = \sum_i A_i p[(k-i)T + \tau] + n_k \text{~~~~~~~~...........~~~~~~~~~~~~~~~~} (2)
\]

\[
= \sum_i A_i p_{k-i} + n_k = A_k + \sum_{i=k} A_i p_{k-i} + n_k
\]

\[
= A_k + \text{ISI}_k + n_k
\]

where: \( p_i = p(iT + \tau) \) and \( n_k = n(kT + \tau) \).

Equation (2) shows that the sampled output consists of three terms: signal (\( A_k \)), noise (\( n_k \)), and distortion (\( \sum A_i p_{k-i} \)).

In digital communications, the last distortion term is called the InterSymbol Interference (ISI) because it is caused by adjacent symbols and pulses.

In a previous paper \([1]\), it has been shown the Q-factor for a Gaussian shape pulse is given by:

\[
Q = \frac{m_1 - m_0}{\sqrt{m_1 + m_0}} \text{................................................................. (3)}
\]

where: \( m_0 = m_{b0} + m_d \), The average number of electrons representing the ‘zero’ symbol \( m_1 = m_{b1} + m_d \), The average number of electrons representing the ‘one’ symbol \( m_d = \frac{i_d T}{q} \), The average numbers of electrons correspond to the dark current \( i_d \) during the symbol interval \([0,T]\).

\[
m_{b0} = \frac{\eta \lambda}{hc} p_0 T, \quad m_{b1} = \frac{\eta \lambda}{hc} p_1 T
\]

where:

\( \eta \) efficiency of photo detector
\( \lambda \) wavelength of photo transmitter
\( h \) Blank constant \((6.625*10^{-34} \text{ J.S})\)
\( c \) light speed \((3*10^8 \text{ m/s})\)
\( q \) electron charge \((1.602*10^{-19} \text{ C})\)
\( i_d \) dark current
\( p_0 \) optical power representing the ‘zero’ symbol
\( p_1 \) optical power representing the ‘one’ symbol
BER = \frac{1}{2} \text{erfc}\left(\frac{Q}{\sqrt{2}}\right) \hspace{1cm} \text{(4)}

Where: \text{erfc}(x) is the complementary error function, defined as:
\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-y^2} dy

2-2 BER Calculation Taking into Consideration the Effect of InterSymbol Interference (ISI)

In this section we consider an optical system where the received pulses may overlap resulting in intersymbol interference (ISI). From equation (1), the number of electrons occurring in the \(K_{th}\) symbol interval has a poisson distribution with mean value \(^{[3]}\).

\[ m(k) = m_d + \int_{0}^{(k+1)T} P(t)dt \] \hspace{1cm} \text{(5)}

where: \(m_d\) represents the dark current electron intensity of the photoelectrons. The total optical energy expressed as the average number of photoelectrons is

\[ m = \int_{-\infty}^{\infty} P(t)dt. \] \hspace{1cm} \text{(6)}

Let \(\delta\) be the relative optical energy outside the signaling interval \([0,T]\).

\[ 1 - \delta = \frac{1}{m} \int_{0}^{T} P(t)dt. \] \hspace{1cm} \text{(7)}

The average number of received photoelectrons corresponding to a transmitted ‘one’ during the signal interval at minimal ISI is obtained from (5), with \(A_k = 0\), \(k=0\) and \(A_k = 1\), for \(k \neq 0\).

\[ m_1 = m_d + m_{b0} + (1-\delta)m \] \hspace{1cm} \text{(8)}

For detection of a binary ‘zero’ the least favorable situation is when \(A_0=0\) and it is surrounded by \(A_k = 1\), for \(k \neq 0\).

\[ m_0 = m_d + m_{b0} + \int_{0}^{T} \sum_{k=0}^{T} i(t-kT)dt \]
\[ m_0 = m_d + m_{b0} + \delta m \] ................................................................. (9)

The error probability, with equal probabilities for ‘ones’ and ‘zeros’;

\[ BER = \frac{1}{2} \text{erfc} \left( \frac{Q}{\sqrt{2}} \right) \] ................................................................. (10)

where:

\[ Q = \sqrt{\frac{i_d T}{q}} \frac{\eta \lambda}{\hbar c} \left( p_0 T + (1 - \delta)E \right) - \sqrt{\frac{i_d T}{q}} \frac{\eta \lambda}{\hbar c} \left( p_0 T + \delta E \right) \] ................................ (11)

\[ E = (p_1 - p_0) T \]

\[ p_1 \] the optical power representing the ‘one’ symbol
\[ p_0 \] the optical power representing the ‘zero’ symbol

2-3 BER Calculation Taking into Consideration the Effect of ISI, Gaussian, and Thermal Noise

Taking the internal resistance of the photodiode, the input resistance of the front-end amplifier, the transconductance of the input transistor and using the Gaussian approximation to estimate the error probability, the mean of the decision variable depends on the transmitted binary symbol. When 'zero' is transmitted it is. [4]

\[ E_0 = m_0 = m_d \] ................................................................. (12)

\[ m_d = \frac{i_d T}{q} \] ................................................................. (13)

When ‘one’ is transmitted

\[ E_1 = m_1 = \frac{\eta \lambda}{\hbar c} \int_0^T P(t) dt + m_d = m + m_d \] ................................................................. (14)
The variances are:

\[ \sigma_0^2 = m_0 + \sigma_x^2 \]  \hspace{1cm} (15)

\[ \sigma_1^2 = m_1 + \sigma_x^2 \]  \hspace{1cm} (16)

\[ \sigma_x^2 = \sigma_0^2 - m_0 \]

\[ \sigma_1^2 = m_1 + \sigma_x^2 - m_0 = m + \sigma_0^2 \]

\[ m = m_1 - m_0 \]

Respectively, with \( \sigma_x^2 \) equal to the variance of the thermal noise

\[ \sigma_x^2 = \frac{2k_B T_n T}{q^2 R} \left( 1 + \frac{1}{g_m R} \right) \]  \hspace{1cm} (17)

with, \( R = \frac{R_a R_d}{R_a + R_d} \)

where:

- \( R_a \) = input resistance of the front end amplifier
- \( R_d \) = internal resistance of the photodiode.
- \( g_m \) = the transconductance of the input transistor.
- \( k_B \) = Boltzmann constant \((1.38 \times 10^{-23} \text{ J/K})\)
- \( T_n \) = absolute value of temperature \((\text{K})\)
- \( q \) = electron charge \((1.602 \times 10^{-19} \text{ C})\)

From before \([1]\), the Q-factor is given by:

\[ Q = \frac{E_1 - E_0}{\sigma_1 + \sigma_0} \]  \hspace{1cm} (18)

The Q-factor for a system with a Gaussian shape pulse and thermal noise is:

\[ Q = \frac{m}{\sqrt{m + m_0 + \sigma_x^2} + \sqrt{m_0 + \sigma_x^2}} \]  \hspace{1cm} (19)

\[ Q = \frac{m}{\sqrt{m + \sigma_0^2} + \sigma_0} \]  \hspace{1cm} (20)
A system with Intersymbol Interference is easily analyzed using the analysis given in section (2-3). The mean values eq.(8) and eq.(9) are:

\[ m_0 = m_d + m_{b0} + \delta m \]

and

\[ m_1 = m_d + m_{b0} + (1 - \delta)m \]

\( \delta \) is the ISI parameter which is given by \[8\].

\[
1 - \delta = \frac{\int_{-\infty}^{\infty} p(t)dt}{\int_{0}^{\infty} p(t)dt} \quad \text{...(21)}
\]

The Q-factor for a system with Intersymbol interference, Gaussian, and thermal noise is:

\[
Q = \frac{m(1 - 2\delta)}{\sqrt{m_d + (1 - \delta)m + \sigma_x^2} + \sqrt{m_d + \delta m + \sigma_x^2}} \quad \text{...(22)}
\]

where: \( m = m_1 - m_0 \)

**2-4 BER Calculation Taking into Consideration the Effect of Shot Noise**

From the Gaussian approximation discussed in \[1\], the BER is

\[
p(e) = \frac{1}{2} \text{Er} \left( \frac{V_{s1} - \gamma}{\sigma_H} \right) + \frac{1}{2} \text{Er} \left( \frac{\gamma - V_{s0}}{\sigma_L} \right) \quad \text{...(23)}
\]

where: \( E_r \) is the error function, \( V_{s1} \) and \( V_{s0} \) are the mean values of the decision variable, and \( \sigma_H \) and \( \sigma_L \) are the standard deviations when the symbols ‘one’ and ‘zero’, are transmitted, respectively, \( \gamma \) is the threshold level.

At optimum threshold, the two right terms in equation (23), must be equal

\[
\frac{V_{s1} - \gamma}{\sigma_H} = \frac{\gamma - V_{s0}}{\sigma_L}
\]

This results in:

\[
\gamma g = \frac{\sigma_L V_{s1} + \sigma_H V_{s0}}{\sigma_H + \sigma_L} \quad \text{...(24)}
\]
where: \( \gamma_g \) is the optimum threshold \(^6\).

Substitution of (24) into (23) gives:

\[
p(e) = Er(Q) \quad \text{.......................................................... (25)}
\]

where,

\[
Q = \frac{V_{s1} - V_{s0}}{\sigma_H + \sigma_L} \quad \text{.......................................................... (26)}
\]

Consider an optical receiver in which a PIN diode is used for photo detection and the receiver filter is a rectangle function, at a load resistance \( R_L \) \(^7,8\).

\[
V_{s1} - V_{s0} = (AH - AL)TR_L = \rho p_{in} TR_L \quad \text{.......................................................... (27)}
\]

where: \( p_{in} \) is the peak received power due to bit ‘one’ and the extinction ratio is assumed to be ‘zero’, and \( \rho \) is the responsivity of the photo detection at the receiver \(^9,10\).

\[
\sigma_H + \sigma_L = \left( \sqrt{2q(\rho p_{in} + i_d)T} + \sqrt{2q i_d T} \right) R_L \quad \text{.......................................................... (28)}
\]

\[
\therefore Q = \frac{\rho p_{in} \sqrt{T}}{\sqrt{2q(\rho p_{in} + i_d) + \sqrt{2q i_d}}} \quad \text{.......................................................... (29)}
\]

3. Simulation Results

Figure (1) shows the received power required to achieve the same bit error rate is less at the longer wavelength. This reduction in power can be utilized in many ways such as increasing the number of users, increasing the transmission distance, or reducing the number of repeaters.

Figure (2) shows that the use of longer wavelength decreases the effects of ISI on high bit rates. This means that for the same transmitted power and the same BER, the bit rate can be increased as shown in Table (1). Or it can be utilized for more users.

Figure (3) shows the improvement obtained in BER for the same power and the same bit rate for different types of noise.

Figures (4, 5, 6) show the effects of the different type of noise on BER and they also indicate how these effects can be reduced by using the third window wavelengths as shown in Table (2).
Figure (1) Average power received Vs. BER, with and without ISI

Figure (2) Average power received Vs. BER, with ISI
Table (1) Bit Rates can be transmitted using wavelengths (0.8 um, 1.5 um) at the same power received when the effect of ISI Noise is considered

<table>
<thead>
<tr>
<th>Bit rate at (0.8 um)</th>
<th>Bit rate at (1.5 um)</th>
<th>Power received (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100Mbps</td>
<td>198Mbps</td>
<td>-46.79</td>
</tr>
<tr>
<td>200Mbps</td>
<td>380Mbps</td>
<td>-44.02</td>
</tr>
<tr>
<td>500Mbps</td>
<td>950Mbps</td>
<td>-40.20</td>
</tr>
<tr>
<td>1000Mbps</td>
<td>1900Mbps</td>
<td>-37.25</td>
</tr>
</tbody>
</table>

Figure (3) Average power received Vs. BER, at different type of noise
Figure (4) Average power received Vs. BER, at Bit rate (100Mbps)

Figure (5) Average power received Vs. BER, at 100Mbps, 0.8um
Figure (6) Average power received Vs. BER, at 100Mbps, 1.5um

Table (2) minimum power received at wavelengths (0.8 um, 1.5 um) at 100Mbps and different types of noise to get BER (10^-10)

<table>
<thead>
<tr>
<th>Noise</th>
<th>Gaussian</th>
<th>ISI</th>
<th>Shot</th>
<th>ISI+Gaussian+Thermal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. Power Received (dBm) at 0.8um</td>
<td>-54</td>
<td>-46.8</td>
<td>-51.52</td>
<td>-38.79</td>
</tr>
<tr>
<td>Min. Power Received (dBm) at 1.5um</td>
<td>-56.74</td>
<td>-49.52</td>
<td>-54.25</td>
<td>-41.52</td>
</tr>
</tbody>
</table>
Number of users can be calculating as follows:

\[
\text{Power budget} = \frac{P_{tx}}{P_{\text{min}}} \hspace{1cm} \text{......................................................... (30)}
\]

\[
\text{Power budget [dB]} = P_{tx} \text{[dB]} - P_{\text{min}} \text{[dB]} \hspace{1cm} \text{......................................................... (31)}
\]

\[
\alpha_{\text{fiber}}L + \alpha_{\text{coupling}}N + \text{connection loss} \leq \text{Power budget [dB]} \hspace{1cm} \text{................. (32)}
\]

where:

\(P_{tx}\) = optical power transmitted.

\(P_{\text{min}}\) = average minimum power received.

\(\alpha_{\text{fiber}}\) = loss in fiber optic.

\(\alpha_{\text{coupling}}\) = loss in coupling.

\(L\) = length of fiber optic.

4. Conclusion

In this paper, the results obtained indicate that using longer wavelength (1.5-1.6 um) transceivers improves the BER and reduces the transmitted power required, about (3dB) to get the same BER (1e-10), at different kinds of noise sources, and different bit rates for transmission in optical network.

The results also indicate that noise effects can be encountered by using longer wavelength at different bit rates.

The power margin obtained can be used to increase the number of users in fiber optic LAN.

Depending on the line parameters, the percentage increase in the number of users was found to be 12%.

5. References


