The Effect of Quantization Process on FIC for Gray Image Using Traditional Method

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Abstract:
In this work we study the effect of quantization process on the reconstructed image in fractal image compression (FIC) method. This method is tested on 8 bits/pixel gray image. The test results conducted on Lena image indicated that the quantization increases compression ratio about (65.8%) and encoding time about (17.1%) but the decreases of PSNR around (1.73%). The quality of the reconstructed image is good either we use the quantization process or not.

Key Words: Image Compression, Quantization, fractal image compression.
1- Introduction

Quantization is the process of reducing the number of bits needed to store coefficient values by reducing its precision (i.e., rounding from float type to integer). The quantization process may be performed either by using uniform or non-uniform method. At this research project, the uniform quantization was adopted, because it can be run faster than the other method.

The Iterated function system (IFS) coefficients are real values, so in order to increase the compression it must be quantized before storage. The quality of the reconstructed image depends on the quantization process. The change of numbers from real to integer leads to loss of more information. If this information loss is big, the quality of the reconstructed image will be worse. While, if the lost information is small, the quality of the reconstructed image will be better and the compression ratio will be little. [Geor06]

2- Fractal Image coding

As the compression method, the implemented fractal compression Scheme consists of two major units, the first unit is the encoding unit and the second one is the decoding unit. Each of these two units consists of many modules, as shown in Figure (1) [Hilo07].

For a range block with pixel values ($r_0, r_1, ..., r_{n-1}$), and the domain block ($d_0, d_1, ..., d_{n-1}$) the contractive affine approximation is [GeHi11]:

\[ \hat{r}_i \approx sd_i + o \]

Where, $\hat{r}_i$ is the optimally approximated i\textsuperscript{th} pixel value in the range block. $d_i$ is the corresponding pixel value in the domain block. The symbols $s,o$ represent the scaling and offset coefficients, respectively.

These parameters ($s$) and ($o$) are determined by applying the least sum $\epsilon^2$ of square errors between ($\hat{r}_i$) and ($r_i$) according to following equation [Fish95]:

\[ \epsilon^2 = \sum_{i=0}^{n-1} (\hat{r}_i - r_i)^2 \]

The minimum of $\epsilon^2$ occurs when:

\[ \frac{\partial \epsilon^2}{\partial s} = 0 \quad \text{and} \quad \frac{\partial \epsilon^2}{\partial o} = 0 \]

The straightforward manipulation of the above equation leads to:
3. Quantization

Quantization is the process of reducing the number of possible values of a quantity, thereby reducing the number of bits needed to represent it. Quantization is a lossy process and implies in a reduction of the color information associated with each pixel in the image.[Khal10]

The quantization of IFS coefficients is done by assigning number of bits for both, scale and offset, to store their quantization indices. The quantized scale and offset values have been computed by using the following equations [Geor06]:

\[
Q_s = \begin{cases} \frac{s_{\text{Max}}}{2^{b_0-1}} - 1 & \text{if } s_{\text{Max}} = -s_{\text{Min}} \\ \frac{s_{\text{Max}} - s_{\text{Min}}}{2^{b_0-1}} & \text{if } s_{\text{Max}} \neq -s_{\text{Min}} \end{cases} \quad \ldots (7)
\]

\[
i_o = \text{round} \left( \frac{O}{Q_o} \right), \ldots \ldots \ldots \ldots (11)
\]

\[
o_q = i_o Q_o, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (12)
\]

\(s_{\text{Max}}\) is the highest permissible value of scale coefficients, \(s_{\text{Min}}\) is the lowest permissible value of scale coefficients, \(b_0\) is the number of allocated bits to represent the quantization index of the scale coefficients, \(Q_s\) is the quantization step of the scale coefficients, \(i_o\) is the quantization index of the offset coefficients and \(o_q\) is the quantized value of the offset coefficients.

- The quantized values of scale (s) and offset (o) parameters should be used to calculate the sum of square error \(\epsilon^2\) by using equation (6).
- After the computation of the IFS parameters and the sum of error \(\epsilon^2\) for any matching instance between the range and each domain block listed in the domain pool, then the value of \(\epsilon^2\) is compared with the minimum registered error \(\epsilon_{\text{min}}^2\):

\[
\text{If } \epsilon^2 < (\epsilon_{\text{min}}^2) \text{ then } \quad \text{End if}
\]

4. Encoding Unit

The implementation encoding method could be summarized by the following steps:

A. Load BMP image and put it in (2D arrays).
B. Establish the range image (array).
C. Down sample the range image to produce the domain array.
D. Great range and domain pool by partitioning:

1. The range array must be partitioned into non-overlapping fixed blocks, to generate the range blocks \((r_1, ..., r_n)\).

2. The domain must be partitioned into overlapping blocks, using specific step size, to generate the domain blocks \((d_1, ..., d_n)\). They should have the same size of range blocks.

3. Searching: For each range block do the following:

   1. Pick up a domain block from the domain pool.
   2. Perform one of the isometric mappings.
   3. Calculate the scale \((s)\) and offset \((o)\) coefficient using equations (4,5).
   4. Apply the following condition to bound the value of \((s)\) and offset \((o)\) coefficient:

      \[
      \begin{align*}
      \text{If } s &< s_{\text{min}} \text{ then } s = s_{\text{min}} \\
      \text{Else if } s &> s_{\text{max}} \text{ then } s = s_{\text{max}} \\
      \text{If } o &< o_{\text{min}} \text{ then } o = o_{\text{min}} \\
      \text{Else if } o &> o_{\text{max}} \text{ then } o = o_{\text{max}}
      \end{align*}
      \]

   5. Quantize the value \((s)\) and offset \((o)\).

   6. Compute the approximation error \((\epsilon_{\text{min}}^2)\) using equation (6).

   7. After the computation of IFS code and the sum of error \((\epsilon^2)\) of the matching between the range and the tested domain block, the \((\epsilon^2)\) is compared with registered minimum error \((\epsilon_{\text{min}}^2)\); such that:

      \[
      \text{If } \epsilon^2 < (\epsilon_{\text{min}}^2) \text{ then } \\
      s_{\text{Opt}} = i_s; \quad o_{\text{Opt}} = i_o; \quad \epsilon_{\text{min}}^2 = \epsilon^2 \\
      \text{PosI}=\text{domain block index} \quad \text{Iso}=\text{isometric index}
      \]

8. If \(\epsilon_{\text{min}}^2 < \epsilon\) then the search across the domain blocks is stopped, and the registered domain block is considered as the best matched block.

9. Repeat steps (4) to (10) for all isometric states of the tested domain block.

10. Repeat steps (3) to (11) for all the domain blocks listed in the domain pool.

11. The output is the set of IFS parameters \((i_s, i_o, \text{PosI}, \text{Iso})\) which should be registered as a set of fractal coding parameters for the tested range block.

12. Repeat steps (1) to (12) for all range blocks listed in the range pool.

13. Store all IFS mapping parameters as an array of record. The length of this array is equal to the number of range blocks in the range pool.

5. Decoding Unit

This unit consists of two modules, as shown in Figure (2) [Hilo07]

![Decoding Unit Diagram](image)

The decoding process can be summarized by the following steps:

1. Generate arbitrary domain pool, the domain pool could be initialized as a
blank image or a piece of image extracted from any available image.

2. The values of the indices of \((i_s)\) and \((i_o)\) for each range block should be mapped to reconstruct the quantized values of the scale \((s_q)\) and offset \((o_q)\) coefficients.

3. Choose the number of possible iterations, and the threshold value of the mean square error (TMSE). At each
   a. For each range block determine the coordinates \((x_d,y_d)\), of the best matched domain, from the IFS parameters \((posI)\), in order to extract the domain block \((d)\) from the arbitrary domain image.
   b. For each range block, its approximation \(r'_i\) is obtained by multiplying the corresponding best matched domain block \((d)\) by the scale value \((s_q)\) and adding to the result the offset value \((o_q)\), according to equation (1).
   c. The generated \(r'_i\) block is transformed (rotated, reflected, or both) according to its corresponding IFS isometric parameter value \((Iso)\).
   d. Put the generated \(r'_i\) block in its position in the decoded image array (i.e., range image).
   e. Check whether there is another range block, if yes then repeat steps (b,c,d)
   f. Down sample the reconstructed image (range pool) in order to produce the domain pool using the averaging sampling.
   g. Calculate the mean square error MSE between the reconstructed range and the previous reconstructed range image. If the MSE is greater than TMSE value then the iteration continues and the above steps (a-f) should repeated; this iteration is continued till reaching the attractor state (i.e., the newly reconstructed range image is very similar to the previous reconstructed image). Otherwise the iteration continues till reaching the predefined maximum number of iterations.

4. Tests Results

This work is carried out in Visual Basic 6.0 version on Acer laptop with (2.30GHZ, RMA 2 GB) Windows 7. To evaluate the performance of the established gray FIC system by Zero-Mean method, the proposed system has been tested using lena image (256x256 pixel, 8bits) as test image.

This set of tests was conducted to study the effect of quantization on the reconstructed image. Quantization tests include the effect of the ScaleBits and OffsetBits on the reconstructed image. In this set of tests the values of other coding parameters were taken as shown in table (1).

The listed results showed that quantization increases the compression ratio and causing an increase in PSNR. Table (2) shows the effects of the ScaleBits parameter on the compression performance parameters. Figure (3) shows the reconstructed images for the cases of using and not using quantization. Figures (4-7) show the effect of the OffsetBits parameter on BitRate, PSNR, CR, and CF, respectively.

In these tests the value of compression parameters are set in table (1)

| Table (1) The values of all parameters that are fixed in Quantization Tests |
|-----------------------------|-----------------|-----------------|-----------------|
| Image Size                  | 256x256         | TMSE            | 0.05            |
| Min Scale                   | -1.5            | Min Offset      | -255            |
| Max Scale                   | 3               | Max Offset      | 255             |
| Step Size                   | 2               | Block Size      | 4x4             |
| Dom Size                    | 128x128         | Error           | 0.4             |
Table (2) Effects of the Scale Bits parameter on compression performance parameters

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<th>Scale Bit</th>
<th>Offset Bit</th>
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<th>MAD</th>
<th>MSE</th>
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<th>CF</th>
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</table>

Fig (3) The reconstructed images for the cases of no quantization and quantization
1. The case of ScaleBits 6 when OffsetBits 6 leads to high PSNR with appropriate CR.
2. The case of ScaleBits 3 and OffsetBits 4 leads to high CR = (5.82).
3. Quantization increases CR and ET but decreases PSNR and BitRate.
4. MAD and MSE are inversely proportional to OffsetBits.
5. PSNR is directly proportional to OffsetBits.
6. CR is inversely proportional to OffsetBits.
7. BitRate is directly proportional to OffsetBits.
8. CF and BitRate increase with the increase of OffsetBits and ScaleBits.

8. Reference


7. Conclusions

The following remarks summarize the noticed behaviour in the above listed results:-