Symmetric Bi-Centralizers on Semiprime Rings

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Received 14, May, 2014
Accepted 5, February, 2015

Abstract:

Let R be a 2-torsion free semiprime ring and $F: R\times R\rightarrow R$ be a symmetric Bi-additive mapping. The purpose of this paper is to prove the following results:
(1) If $F(x^2, y)= F(x, y)x$ fulfilled for all $x,y \in R$, then F is a symmetric left Bi-centralizer. (2) If $F(x\omega x, y) = x\, F(\omega, y)\, x$ fulfilled for all $x,y, \omega \in R$, then F is a symmetric Bi-centralizer (3) Let R be a 2-torsion free semiprime ring with an identity element and $F:R\times R\rightarrow R$ be a symmetric Bi-additive mapping such that $F(x^3, y) = x\, F(x, y)x$ fulfilled for all $x,y \in R$, then F is a symmetric Bi-centralizer

Key words: Semiprime ring, A symmetric al left (right) Bi-Centralizer, A symmetric al left (right) Jordan Bi-Centralizer, A symmetric al Jordan Bi-Centralizer.

Introduction:

This note motivated by the work of J. Vokman [1] and B. Zalar [2]. Throughout, R will represent an associative ring with the center $Z(R)$. A ring R is said be n-torsion free if $nx=0$, $x\in R$ implies that $x=0$ [3]. Recall that R is prime if $aRb=(0)$ implies $a=0$ or $b=0$, and semiprime if $aRa=(0)$ implies $a=0$ [4]. We write $[x, y]$ for the commutator $xy - yx$ and make extensive use of the commutator identities $[xz, y] = [x, y]z + x[z, y]$, $[x, yz] = [x, y]z + y[x, z]$. An additive mapping $T:R\rightarrow R$ is called a left (right) centralizer in the case $T(xy)= T(x)\, y$ **(**$T(xy) = xT(y)$**)** fulfilled for all $x,y \in R$. We follow Zalar [5] and call T centralizer in case T is both a left and right centralizer. An additive mapping is called a left (right) Jordan centralizer in case $T(x^2)= T(x)\, x$ **(**$T(x^2) =x\, T(x)$**)** for all $x \in R$. Zalar in [2] has proved that every left (right) Jordan centralizer on a semiprime ring of characteristic not two is a left (right) centralizer. In [6] Vokman proved that if R is a 2-torsion free semiprime ring and $T:R\rightarrow R$ is an additive mapping such that $2T(x^2) = T(x)x + x\, T(x)$ fulfilled for all $x \in R$, then T is centralizer.

In [5] Vokman and Kosi proved that if R is a 2-torsion free semiprime ring and $T:R\rightarrow R$ is an additive mapping such that $2T(xyx) = T(x)yx + xy\, T(x)$ fulfilled for all $x,y \in R$, then T is centralizer. In case R has an identity element then an additive mapping $T: R\rightarrow R$ is a left (right) centralizer if and only if T is of the form $T(x)=ax$ **(**$T(x)=xa$**)** for some $a\in R$. In this paper we generalize this result to a left (right) Bi-Centralizer.

A Bi-additive mapping $B:R\times R\rightarrow R$ is called symmetric if $B(x, y) = B(y, x)$ for all pairs $x,y \in R$ [7]. We introduce the following concept: A symmetric Bi-additive mapping $F: R\times R\rightarrow R$ is called a symmetric left (right) Bi-Centralizer in case $F(xz, y) = F(x, y)z$ **(**$F(xz, y) = xF(z, y)$**)** fulfilled for all $x,y,z \in R$, while F is called symmetric left (right) Jordan Bi-Centralizer in case $F(x^2, y) = F(x, y)x$ **(**$F(x^2, y) = xF(x, y)$**)** fulfilled for all $x,y \in R$. The symmetric Bi-additive mapping F is called a symmetric Bi-Centralizer in case F is both a left and right symmetric Bi-Centralizer. Similarly we define the symmetric Jordan Bi-Centralizer. Every a symmetric Bi-Centralizer is a symmetric Jordan Bi-Centralizer. The converse is in general not true.

1. Preliminaries

Following Lemmas are essential for developing the proofs of our main results.

Lemma 2.1: [8]

Let R be a semiprime ring. If $a,b \in R$ are such that $a x b =0$**,** for all $x \in R$, then $ab=ba=0$.

Lemma 2.2: [2]

Let R be a semiprime ring and G, $F: R\times R\rightarrow R$ be a Bi-additive mappings. If $G(x, y)\, \omega\, F(x, y) =0$, for all $x,y, \omega \in R$, then $G(x, y)\, \omega\, F(u, v)=0$, for all $x,y,u,v, \omega \in R$.

Lemma 2.3: [2]

Let R be a seiprime ring and $a \in R$ some fixed element. If a $[x, y] =0$ for all $x,y \in R$, then there exists an ideal U of R such that $a \in U\subset Z$.

Lemma 2.4: [9], [1]

Let R semiprime ring, suppose that $axb + bxc$ holds all $x \in R$ and some $a,b,c \in R$. In this case $(a + c)x\, b =0$ is satisfied for all x $\in R$.

Also, we see it be useful to introduce the following Lemma.

Lemma 2.5:

Let R be a ring with identity. then a symmetric Bi-additive mapping $F:R\times R\rightarrow R$ is a symmetric left (right) Bi-centralizer if and only if F is of the form $F(x, y) = ayx$ ($F(x, y)= xya$) for some fixed element a $\in R$.

Proof: Suppose T is a symmetric left Bi-centralizer:

$F(xz, y)= F(x, y)\, z$
$= F(1.x, y)z = F(1, y)\, xz$
$= F(1.y, 1)\, xz = F(1, 1)yxz$
$= ayxz$, where a stands for $F(1, 1)$

Hence $F(xz, y) = ayxz$ for all $x,y,z \in R$.

Taking $z =1$ leads to:

$F(x, y) =ayx$, for all $x,y \in R$.

Conversely, suppose $F(x, y) = ayx$ for all $x,y \in R$ then

$F(xz, y)= ayxz = (ayx)z = F(x, y)\, z$

Hence F is a symmetric left Bi-centralizer.

In similar arguments as above, we can prove F is a symmetric right Bi-centralizer if and only if $F(x, y)=xya$.■

2. Main results

Theorem 3.1:

Let R be a 2-torsion free semiprime ring and $F :R\times R\rightarrow R$ be a symmetric Bi-additive mappings. If $F(x^2, y) = F(x, y)x$ fulfilled for all $x,y \in R$ then F is a symmetric left Bi-Centralizer

Proof: We have

$F(x^2 ,y)= F(x,y)x$ for all $x,y \in R$. (1)

Replacing x by $x+\omega$ in (1) we get:

$F(x\omega+\omega x , y) = F(x,y)\, \omega + F(\omega,y)\, x$ (2)

The substitution x^2instead of x in (2), and using (1) on the relation so obtained gives:

$F(x^2\omega+\omega x^2 , y) = F(x,y)x\omega + F(\omega,y)\, x^2$ for all $x,y, \omega \in R$. (3)

Putting $x\omega+\omega x$ for ω in (2), and using (2) we arrive at:

$F(x(x\omega+\omega x) + (x\omega+\omega x)x, y)= F(x,y)x\omega +2\, F(x,y)\omega x+ F(\omega,y)x^2$ for all $x,y,\omega \in R$. (4)

This can be written as:

$F(x^2\omega+\omega x^2 , y) +2F(x\omega x, y)= F(x, y)x\omega + 2\, F(x, y)\omega x +F(\omega, y)\, x^2$ (5)

Comparing (3) and (5) leads to:

$F(x\omega x, y)= F(x, y)\, \omega x$ for $x,y,\omega \in R$. (6)

The linearization of (6) with respect to x gives:

$F(x\omega z+z\omega x, y)= F(x ,y)\omega z + F(z, y)\omega x$ for all $x,y,\omega \in R$. (7)

Now we shall compute $F(x\omega z\omega x + \omega xzx\omega, y)$ in two different ways. The first one by using (6), we see:

$F(x\omega z\omega x + \omega xzx\omega, y) = F(x, y)\omega z\omega x + F(\omega, y)xzx\omega$ (8)

Also using (7) leads to:

$F(x\omega z\omega x + \omega xzx\omega, y) = F(x\omega, y)z\omega x + F(\omega x, y)zx\omega$ (9)

Comparing (8) and (9) we have:

$(F(x\omega, y) – F(x, y)\omega) z\omega x +(F(\omega x, y) + F(\omega, y)x) zx\omega =0$, for all $x,y,\omega \in R$. (10)

According to (2) one can replace $(F(\omega x, y) + F(\omega, y)x)$ by $-(F(x\omega, y) - F(x, y)\omega)$ in (10), so we have:

$(F(x\omega, y) – F(x, y)\omega) z [x, \omega] =0$, for all $x,y,\omega \in R$.

Without lose the generality we fix some $y \in R$ and define $M(x, \omega)=F(x\omega,y) – F(x, y)\omega$, then the above relation reduces to:

$M(x, \omega) z [x, \omega] =0$, for all $x,y,\omega \in R$.

Using Lemma (2.2), the above relation can be given as:

$M(x, \omega) z [u, v] =0$, for all $x,y,\omega \in R$. (11)

Now, fix some $x, \omega \in R$ and let M represent to $M(x, \omega)$, then by Lemma (2.1) the relation (11) becomes:

$M [u, v] =0$, for all $x,y,\omega \in R$.

An application of Lemma (2.3) on the above relation we see that there exist an ideal U of R satisfies $M \in U \subset Z(R)$. In particular $rM , Mr \in Z(R)$ for $r \in R$. This gives us:

$x.M^2\omega = M^2\omega.x = \omega M^2.x = \omega.M^2x$

This gives that $4F(x.M^2\omega, y) = 4F(\omega.M^2x, y)$, both sides of this equality will be computed in few steps using (2) and the above remarks.

$2F(x M^2\omega + M^2\omega x, y) = 2F(\omega M^2x + M^2x\omega , y)$

$2F(x, y)M^2\omega + 2F(M^2\omega, y)x = 2F(\omega, y)M^2x + 2F(M^2x, y)\omega$

$2F(x, y)M^2\omega + F(M^2\omega + \omega M^2, y)x = 2F(\omega, y)M^2x + F(M^2x + xM^2, y)\omega$

$2F(x, y)M^2\omega +F(M, y)M\omega x +F(\omega, y)M^2x = 2F(\omega, y)M^2x +F(M, y)M x\omega +F(x, y)M^2\omega$

$F(x, y)M^2\omega + F(M, y)M\omega x = F(\omega, y)M^2x +F(M, y)M x\omega$

But $M\omega x = M\omega.x = x.M\omega = xM\omega = M x\omega$, therefore we arrive at:

$F(x, y)M^2\omega = F(\omega, y)M^2x$ (12)

On the other hand we have:

$4F(x\omega M^2, y) = 4F(xM.\omega M, y)$

$2F(x\omega M^2 + M^2x\omega, y) = 2F(xM.\omega M + \omega M xM., y)$

$2F(x\omega, y)M^2 + 2F(M, y)M x\omega = 2F(Mx, y)M\omega + 2F(M\omega, y)Mx$

$2F(x\omega, y)M^2 + 2F(M, y)M x\omega = F(xM + Mx, y)M\omega + F(\omega M + M\omega, y)Mx$

$2F(x\omega, y)M^2+2F(M, y)M x\omega =F(x,y)M^2\omega+F(M, y)M x\omega +F(\omega, y)M^2x +F(M, y)M x\omega$

$2F(x\omega, y)M^2 = F(x,y)\omega M^2 + F(\omega, y)xM^2$

In view of (12) the above relation reduces to $F(x\omega, y)M^2= F(x, y)\omega M^2$, consequently we conclude that $M^3 =0$. The fact that R is semiprime ring leads to $M^2RM^2 = M^4R =0$, which means $M^2 =0$.

Also, $MRM = M^2R =0$ implies that $M =0$ and hence:

$F(x\omega, y) = F(x, y)\omega$, for all $x,y,\omega \in R$

■

<u>Theorem 3.2</u>:

let R be a 2-torsion free semiprime ring and $F: R\times R\rightarrow R$ be a symmetric Bi-additive mappings. If $F(x^2 ,y)= x F(x, y)$ holds for all $x,y \in R$, then F is a symmetric right Bi-Centralizer .

<u>Theorem 3.3</u>:

Let R be a 2-torsion free semiprime ring and $F: R\times R\rightarrow R$ be a symmetric Bi-additive mapping. Suppose $F(x\omega x, y) = x F(\omega, y) x$ holds for all $x,y, \omega \in R$, then $[F(x ,y), x]=0$.

Proof: For any $x,y, \omega \in R$, we have:

$F(x\omega x, y) = x F(\omega,y) x$ holds for all $x,y, \omega \in R$. (1)

Putting $x + u$ for x in (1) and using (1), we obtain:

$F(x\omega u+u\omega x, y) = x F(\omega, y) u + u F(\omega, y) x$ (2)
Setting $\omega= x$ and $u= \omega$ in (2) we get:
$F(x^2\omega+\omega x^2, y) = x F(x, y) \omega + \omega F(x, y) x$ for all $x,y, \omega \in R$ (3)
For $u=x^3$ the relation (2) reduces to:
$F(x\omega x^3+x^3\omega x , y) = x F(\omega, y) x^3 + x^3 F(\omega, y) x$ for all $x,y, \omega \in R$ (4)
Putting $x\omega x$ for ω in (3), we see:
$F(x\omega x^3+x^3\omega x , y) = x F(x, y)x\omega x + x\omega x F(x, y) x$ for all $x,y, \omega \in R$ (5)
The substitution $x^2\omega+\omega x^2$ for ω in (1) gives:
$F(x\omega x^3+x^3\omega x , y) = x F(x^2\omega+\omega x^2 , y) x$ for all $x,y, \omega \in R$
In view of (3) the above relation gives:
$F(x\omega x^3+x^3\omega x , y) = x^2 F(x, y)\omega x + x\omega F(x, y) x^2$ for all $x,y, \omega \in R$ (6)
Combining (5) with (6) we get:
$x[F(x, y) , x] \omega x - x\omega[F(x, y), x]x=0$, for all $x,y, \omega \in R$ (7)
The application of lemma (2.4) on (7) gives:
$[[F(x, y), x], x] \omega x =0$ for all $x,y, \omega \in R$ (8)
Replacing ω by $\omega[F(x, y) , x]$ in (8) gives:
$[[F(x ,y) , x] , x] \omega[F(x, y), x] x =0$, for all $x,y, \omega \in R$ (9)
Right multiplication of (8) by $[F(x, y), x]$ implies that:
$[[F(x, y) ,x] ,x] \omega x [F(x, y) ,x] =0$, for all $x,y, \omega \in R$ (10)
Subtracting (10) from (9) we arrive:
$[[F(x, y), x], x] \omega [[F(x, y), x], x] =0$, for all $x,y, \omega \in R$.
By semiprimness property of R we conclude:
$[[F(x, y), x], x] =0$, for all $x,y \in R$. (11)
The next our task is to prove:
$x [F(x, y),x] x =0$, for all $x,y \in R$. (12)
The linearization of (11) with respect to x gives:
$[[F(x, y), x], \omega]+[[F(x, y), \omega], x] +[[F(\omega, y), x], x] +[[F(\omega, y), x], \omega] + [[F(\omega, y), \omega], x] +[[F(x, y), \omega], \omega] =0$, for $x,y, \omega \in R$.

Putting $-x$ instead of x in the last relation and comparing the relation so obtained with it gives:
$[[F(x, y), x], \omega] + [[F(x, y), \omega], x] +[[F(\omega, y), x] , x] =0$, for all $x,y, \omega \in R$. (13)
Putting $x\omega x$ instead of ω in (13) and using (1) ,(11) and (13) we see:
$0= x[[F(x, y), x], \omega]x + [[F(x, y), x] \omega x + x[F(x, y), \omega]x + \omega x[F(x, y), x], x] +$
$[x[F(\omega, y), x] x, x]$
$= x[[F(x, y),x], \omega]x +[F(x, y), x][\omega, x]x + x[[F(x, y) , \omega], x]x+ x[\omega, x][F(x, y), x]+$
$x[[F(\omega, y), x], x]x$
$= [F(x, y) ,x][\omega, x]x + x[\omega, x][F(x, y), x]$
$= [F(x, y), x]\omega x^2 - x^2\omega[F(x, y),x]+ x\omega x[F(x, y), x]- [F(x, y), x]x\omega x$
Hence $[F(x, y), x]\omega x^2 - x^2\omega[F(x, y), x]+ x\omega x[F(x, y), x]- [F(x, y), x] x\omega x =0$
Now, using (7) and (11) we have:
$x\omega x[F(x, y), x]- [F(x, y), x] x\omega x = [[F(x, y),x], x] \omega x = 0$
Therefore the last relation reduces to:
$[F(x,y), x]\omega x^2 - x^2\omega[F(x, y), x] =0$, for all $x,y, \omega \in R$.
Left multiplication of the above relation by x gives:
$x[F(x, y), x]\omega x^2 - x^3\omega[F(x, y), x] =0$, for all $x,y, \omega \in R$. (14)
According to (7) ,one can replace $x[F(x, y), x]\omega x$ by $x\omega[F(x, y), x]x$, so relation (14) can be given by:
$x\omega[F(x, y) ,x] x^2 - x^3\omega[F(x, y), x] =0$, for all $x,y, \omega \in R$. (15)
The substitution $F(x,y)\omega$ for ω in (15) leads to:
$x F(x, y)\omega [F(x, y), x] x^2 - x^3 F(x, y)\omega [F(x, y), x] =0$, for all $x,y, \omega \in R$. (16)
Now, left multiplication of (15) by $F(x, y)$ and subtracting (16) from the relation so obtained gives:
$[F(x, y) ,x] \omega[F(x, y), x] x^2-[F(x, y), x^3] \omega[F(x, y), x]=0$, for all $x,y, \omega \in R$.

Application of lemma (2.4) on the above relation leads to:

$([F(x, y), x^3]- [F(x, y), x]\, x^2)\omega[F(x, y), x]=0$, for all $x,y, \omega \in R$.

By using the identity $[x, yz]=y[x, z]+[x, y]z$ the last relation reduces to:

$([x^2[F(x, y), x] +x[F(x, y), x]x)\, \omega[F(x, y),x]=0$, for all $x,y, \omega \in R$.

But the relation (11) means that $x[F(x, y) ,x]= [F(x, y), x]x$.

So we can replace $x^2[F(x, y) ,x]$ *by* $x[F(x, y) ,x]x$ and the above relation becomes:

$x[F(x, y), x]x\, \omega\, [F(x, y), x] =0$, for all $x,y, \omega \in R$.

Right multiplication of the above relation by x and substitution ωx for ω leads to:

$x[F(x, y), x]\, x\omega x\, [F(x, y), x]x =0$, for all $x,y, \omega \in R$.

Since R is semeiprime ring, hence the relation (12) follows.

The next step is to prove the relation

$x\ [F(x, y), x]\ =0$, for all $x,y, \omega \in R$. (17)

The substitution ωx instead of ω in (7) gives in view of (12)

$x\ [F(x, y), x]\ \omega x^2 =0$, for all $x,y, \omega \in R$. (18)

Putting $\omega[F(x, y), x]$ for ω in (18) leads to:

$x[F(x, y), x]\, \omega\, F(x, y)x^2 =0$, for all $x,y, \omega \in R$. (19)

Right multiplication of (18) by $F(x, y)$ and subtracting the relation so obtained from (19) gives:

$x[F(x, y), x]\ \omega[F(x, y), x^2]$ for all $x,y, \omega \in R$.

That is $x\ [F(x, y),x]\ \omega\ ([F(x, y), x]x + x[F(x, y), x]) =0$, for all $x,y, \omega \in R$.

Again, according to (11) one can replace $[F(x, y), x]x$ in the last relation by $x[F(x, y), x]$ which leads to:

$x[F(x, y), x]\ \omega x\ [F(x, y), x] =0$, for all $x,y, \omega \in R$.

Hence $x\ [F(x, y), x] =0$, for $x,y \in R$ and consequently from (11) we conclude that

$[F(x, y), x]\ x =0$, for all $x,y \in R$

Now, using similar techniques on the above relation as used to get (13) form (11) we arrive:

$[F(x, y), x]\ \omega + [F(x, y), \omega]x + [F(\omega, y), x]x =0$, for all $x,y, \omega \in R$.

Right multiplication of the above relation by $[F(x,y), x]$ gives in view of (17)

$[F(x, y), x]\ \omega\ [F(x ,y), x] =0$, for all $x,y, \omega \in R$.

Since R is a semiprime ring, then the proof of the theorem is complete. ■

<u>Theorem 3.4</u>:

Let R be a 2-torsion free semiprime ring and $F:R\times R\rightarrow R$ be a symmetric Bi-additive mapping. Suppose $F(x\omega x,y) = xF(\omega,y)x$ holds for all $x,y, \omega \in R$, then F is a symmetric Bi-Centralizer .

Proof: For any $x,y, \omega \in R$, we have:

$F(x\omega x,\ y) = x\ F(\omega,\ y)\ x$ (1)

The linearization of (1) with respect x gives:

$F(x\omega u+u\omega x, y)=x\ F(\omega, y)u + u\ F(\omega, y)x$ for all $x,y,u,\omega \in R$. (2)

Taking $u=x^2$ in the above relation leads to:

$F(x\omega x^2+x^2\omega x, y) = x\ F(\omega, y)\ x^2 + x^2 F(\omega, y)\ x$ for all $x,y, \omega \in R$. (3)

The substitution $x\omega+\omega x$ for ω in (1) gives:

$F(x^2\omega x+x\omega x^2 , y) = x\ F(x\omega+\omega x, y)\ x$ for all $x,y, \omega \in R$. (4)

Comparing (3) and (4), we arrive at:

$x\ \mu(x, \omega, y)\ x =0$ for all $x,y, \omega \in R$. (5)

Where $\mu(x, \omega,y)$ stands for $F(x\omega+\omega x , y) - F(\omega, y)\ x - x\ F(\omega, y)$.

The linearization of (5) with respect x gives:

$x\ \mu(x ,\omega, y)\ u+ x\ \mu(u ,\omega, y)x+ u\ \mu(x ,\omega, y)x + x\ \mu(u ,\omega, y)u + u\ \mu(x ,\omega, y)\ u + u\ \mu(u ,\omega, y)x =0$, for all $x,y,u, \omega \in R$. (6)

Putting $-x$ instead of x in (6) and comparing the relation so obtained with it, we arrive at:

$x\mu(x,\omega,y)\ u + x\ \mu(u,\omega,y)\ x + u\ \mu(x,\omega,y)x\ =0,$ for all $x,y,u,\omega \in R$. (7)

Right multiplication of the above relation by $\mu(x,\omega,y)x$ gives because of (5):

$x\ \mu(x,\omega,y)\ u\ \ \mu(x,\omega,y)x\ =0,$ for all $x,y,\omega \in R$. (8)

The next our task is to prove:

$[\mu(x,\omega,y), x]\ =0,$ for all $x,y,\omega \in R$. (9)

Now, by Theorem (3.3) we have:

$[F(x,y), x]\ =0,$ for all $x,y \in R$. (10)

Linearization of the above relation with respect to x gives:

$[F(x,y),\omega] + [F(\omega,y), x]\ =0$ for all $x,y,\omega \in R$. (11)

Putting $x\omega+\omega x$ for ω in (11), we get:

$[F(x,y), x\omega+\omega x)] + [F(x\omega+\omega x, y), x]=0,$ for all $x,y,\omega \in R$.

That is

$x[F(x,y),\omega]+[F(x,y),\omega]x+[F(x\omega+\omega x, y), x]=0,$ for all $x,y,\omega \in R$.

According to (11) one can replace $[F(x,y),\omega]$ by $-[F(\omega,y), x]=0,$ so the last relation gives:

$[F(x\omega+\omega x, y), x] - x[F(\omega,y), x] -[F(\omega,y),x]x =0,$ for all $x,y,\omega \in R$.

This can be written as:

$[F(x\omega+\omega x, y) - x\,F(\omega,y) - F(\omega,y)\,x\ , x] =0,$ for all $x,y,\omega \in R$.

Hence the relation (9) follows.

Now, in view of (9) the relation (8) can be given by:

$\mu(x,\omega,y)x\ \ u\,\mu(x,\omega,y)x\ =0$ for all $x,y,u,\omega \in R$.

The semiprime property of R leads to:

$\mu(x,\omega,y)\ x\ =0,$ for all $x,y,\omega \in R$. (12)

Also, according to (9) we arrive at:

$x\mu(x,\omega,y)\ =0$ for all $x,y,\omega \in R$. (13)

The linearization of (12) with respect to x gives:

$\mu(x,\omega,y)\ u + \mu(x,\omega,y)\ x\ =0$ for all $x,y,u,\omega \in R$.

Right multiplication of the above relation by $\mu(u,\omega,y)$ and using (13) on the relation so obtained yields:

$\mu(x,\omega,y)\ u\ \ \mu(x,\omega,y)\ =0$ for all $x,y,\omega \in R$.

Therefore $\mu(x,\omega,y)\ =0$ for all $x,y,\omega \in R$.

That is $F(x\omega+\omega x\ , y) = F(\omega,y)\,x + x\,F(\omega,y)$ for all $x,y,\omega \in R$.

As particular for $\omega = x$ the above relation gives:

$2F(x^2, y)= F(x,y)\,x + x\,F(x,y)$ for all $x,y \in R$.

In view of (10) the above relation reduces to :

$F(x^2, y) = F(x,y)\,x$ **and** $F(x^2, y) = x\,F(x,y)$ for $x,y \in R$

Using Theorems (3.1) and (3.2) it follows that F is symmetric Bi-Centralizer. ■

Now, if we taking $\omega=x$ in the relation (1), we obtain:

$F(x^3, y) = x\,F(x,y)\,x$, for all $x,y, \in R$.

The question is whether in a 2-torsion free semiprime ring the above relation implies that F is a symmetric Bi-Centralizer. The answer, it's not true in general unless R be a ring with an identity element. In order to prove this fact, we introduce the following result.

<u>Theorem 3.5</u>:

Let R be a 2-torsion free semiprime ring with identity, and let $F:R\times R\rightarrow R$ be a symmetric Bi-additive mapping such that $F(x^3, y)= x\,F(x,y)\,x$ holds for all $x,y \in R$, then F is a symmetric Bi-Centralizer.

Proof: we have

$F(x^3, y)= x\,F(x,y)\,x$, for all $x,y \in R$. (1)

Putting $x+1$ for x in (1), where 1 is the identity element, we get:

$3F(x^2, y) +2F(x,y) = F(x,y)x +x\,F(x,y) + x\,F(1,y)+ F(1,y)x +x\,F(1,y)x$ (2)

Replacing x by $-x$ in (2) gives:

$3F(x^2, y) - 2F(x,y) = F(x,y)x +x\,F(x,y) - x\,F(1,y)- F(1,y)x +x\,F(1,y)x$ for $x,y \in R$.

Comparing (2) with the above relation, we arrive at:

$6F(x^2, y) = 2F(x, y)x + 2xF(x, y) + 2x F(1, y)x$ for all $x,y \in R$. (3)

Also, comparing (2) with (3) implies that :

$2F(x, y) = F(1, y) x + x F(1, y)$ for all $x,y \in R$. (4)

The substitution x^2 for x in (4) leads to:

$2F(x^2, y) = F(1, y) x^2 + x^2 F(1, y)$, for all $x,y \in R$. (5)

In view of (4), (5) and the fact that R is a 2-torsion free the relation (3) reduces to:

$F(1, y) x^2 + x^2 F(1, y) - 2x F(1, y)x = 0$, for $x,y \in R$.

This relation can be written as:

$[[F(1, y), x] , x] = 0$, for all $x,y \in R$.

The linearization of the above relation with respect to x gives:

$[[F(1, y), x] , z] + [[F(1, y), z] , x] = 0$, for all $x,y,z \in R$. (6)

Putting xz instead of z in (6) leads to:

$[[F(1, y), x] , xz]+[[F(1, y), xz], x] = 0$

$x [[F(1, y), x] , z] + [[F(1, y), x]z + x[F(1, y),z] , x] = 0.$

$x [[F(1, y), x] , z] + [[F(1, y), x]z, x] + [x[F(1, y),z] , x] = 0$

$x [F(1, y), x] , z] + [F(1, y), x][z, x] + x[F(1, y),z] , x] = 0$, for all $x,y,z \in R$.

According to (6), the above relation reduces to:

$[F(1, y), x][z, x] = 0$, for all $x,y,z \in R$.

The substitution $zF(1, y)$ for z in the above relation gives:

$[F(1, y), x] z [F(1, y), x] = 0$, for all $x,y,z \in R$.

Using the semiprimeness property of R implies that:

$[F(1, y), x] = 0$, for all $x,y, \in R$

That is $F(1, y) \in Z$ for all $y \in R$, hence the relation (4) reduces to:

$F(x, y) = F(1, y) x = x F(1, y)$ for $x,y \in R$. (7)

On the other hand, in view of (1) and the symmetry of F we have:

$F(x, y^3) = y F(x, y) y$ for all $x,y \in R$.

Using the similar techniques as used on (1) to get the relations (7) we arrive at:

$F(x, y) = F(x, 1) y = y F(x, 1)$, for $x,y \in R$. (8)

Taking $x = 1$ in (8), we get:

$F(1, y) = ay = ya$, where a stands for $F(1, 1)$. (9)

Combining the relations (7) and (9) leads to:

$F(x, y) = ayx = xya$, for $x,y \in R$.

Using Lemma (4.5) we obtain the required results. ■

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التطبيقات المتناظرة ثنائية التمركز على الحلقات شبه الأولية

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الخلاصة:

قدمنا في هذا البحث بعض النتائج الخاصة بالتطبيقات المتناظرة ثنائية التمركز المعرفة على الحلقة شبه الأولية R مميزها لا يساوي 2. إضافة لذالك قدمنا الصيغة العامة لهذه التطبيقات عندما تكون معرفة على حلقة ذات عنصر محايد وبعض النتائج الخاصة بهذا النوع من التطبيقات

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