Abstract. The frequency analysis of bones is a new tool to assess bone quality or integrity to characterize osteoporosis. The modal analysis can also be used to determine failure characteristics of remodeled bone in the fractured model. This study describes the numerical characterization of the modal analysis of the standardized femur model. The objective of the numerical procedure is to identify the natural frequencies and mode shapes of an unconstrained femur. The vibration modes of the human femur are studied by digital modal analysis and finite element simulation using ANSYS version 10 programs, with respect to femur dimensions and mechanical properties. The changing of the values of free vibration natural frequencies and mode shapes of the femur due to changing of the femur densities are studied. The results are compared to those obtained experimentally. The comparison of the results shows a good agreement, which indicates that the used model can be utilized in vibration analysis of bones.

I. Introduction

Vibration characteristics of any object are directly dependent on its physical properties. Therefore, changing the physical properties will change the natural frequencies of that object. A fracture in a bone will change the physical characteristics of it and this will change the natural frequencies and mode shape of that bone. [17]

Only a few studies have been reported on the vibration analysis of human femur. Campbell et al. (1970) reported an experimental method to measure the in vitro impedance of the femur. Their results showed a capability to estimate the mechanical integrity of the bone. Saha et al. (1977) , as cited in [2], recorded vibration response of various long bones to the impact of an instrumented hammer. Experiments with volunteers indicated the difficulty to overcome experiment without involving pain.

In 1998, Dimarogonas [3] showed that the density of a bone in a patient may be determined by either of two methods. In the first method, an impulse of energy is introduced into the tissue, and the resulting vibration in the bone is sensed and analyzed to compute the modal damping factor of the tissue, which is directly related to the density. In the second method, a continuous energy input is introduced into the bone and the resulting vibration is measured with a mechanical-electrical vibration transducer and a modal damping factor is calculated.

Resonance frequency analysis has been used, in orthopedics for monitoring fracture healing and osteoporosis evaluation, to identify the in vivo vibration modes to determine the torsional stiffness of long bones. Vibration analysis was used to assess the stability of fixation of hip stems in vitro as well as in vivo. Moreover, numerical modal analyses were performed to indicate the hip prosthesis loosening in a quantitative manner. [4]

In 2006 Yoon et al,[5], studied the changing in the natural frequencies in mouse femurs with osteoporosis based on a vibration test and a finite element calcuation. Three groups of the femurs were included, the osteoporotic group, the treated group and the normal group. In the vibration test, the natural frequencies were measured by the mobility test. For the finite element analysis, the micro finite element model of the femur was reconstructed using the Micro-CT images and the Voxel mesh generation algorithm. From the results, the averaged natural frequencies in the osteoporotic group were the highest, followed by those in the treated group.

The bone density is important because it can help to predict the risk of getting a fracture. The value of the density changes with the age. It is usually hard to determine so it normally use bone mineral density (BMD) (g/cm²).

BMD is a test that measures the amount of calcium in a special region in the bones. From this information, an estimate of the strength of the bones can be made. BMD helps predict the risk of a future fracture of the bone, measures the amount of bone mass and also monitors the effectiveness of treatment. BMD can be measured using a special x-ray technique called a quantitative computed tomography (QCT) or dual energy x-ray absorptiometry (DEXA). A range for the mineral density of the bone of the spine region is from 1000 to 1200 g/cm2. The range of bone mineral density for the forearm is from 700 to 800 g/cm2 Bone mineral density is depend upon sex, age, and the part of the body[15]. Fig. (9) Shows how that value of average BMD changes for male and female depending on the age.

Jurist (1970) obtained correlations between natural frequencies and the degree of osteoporosis, as sited by [7]. In Rabia study (2004) [7] resonant frequency analysis was used to determine BMD differences in healthy and osteopenia participants in terms of natural frequencies. RABIA's results showed that vibration analysis measurement technique may as reliable as DEXA in monitoring BMD loss in distal radius. The difference in the magnitude of correlation between natural frequency and bone mineral density measured by DEXA and QCT stated that average BMD does not provide enough precise information about bone strength and type of bone mineral loss.

This work will focus on the vibration frequency analysis of long bones in human body which have great benefits in many applications such as, assessing bone quality or integrity, characterizing clinically osteoporosis, healing of fractured bones and the process of osteoid-integration for plates or screws and determining failure characteristics of
remodeled bone in the fractured model. The objectives of the present work are:
- To calculate the natural frequencies of human femur and draw the mode shapes, and
- To study the changing of the natural frequencies due to changing of femur’s density.

ANSYS program is used to achieve these objectives.

2. Vibration of bone

The study of vibration considered the main important parts in motion analysis, especially in calculating natural frequencies and mode shapes which are prime characteristics of any material. Today, motion analysis finds particular use in the medical diagnostics.

The Biological systems may be influenced by vibration at all frequencies if the amplitude is sufficiently great. Most quantitative investigations of the effects of shock and vibration on human are conducted in the laboratory in controlled and simulated environments. Meaningful results can be obtained from such tests only if measurement methods and instrumentation are adapted to the particular properties of the biological system under investigation to ensure noninterference of the measurement with the system’s behavior. This behavior may be physical, physiological, and psychological, although these parameters should be studied separately if possible.

The complexity of a living organism makes such separation, even assuming independent parameters, only an approximation at best. In many cases if extreme care is not exercised in planning and conducting the experiment, uncontrolled interaction between these parameters can lead to completely erroneous results. For example, the dynamic elasticity of tissue of a certain area of the body may depend on, the simultaneous vibration excitation of other parts of the body; the duration of the measurement since the subject’s physiological response or reaction to the test or to the measurement equipment may varies.

Free vibration of bones can occur in an infinite number of modal shapes and that each modal shape has a discrete frequency associated with it. Fortunately, the second, third, and higher modes of vibration can often be neglected by the analyst. This is primarily because the number of nodal point’s increases directly with the mode number and much more energy is required to excite the higher modes to appreciable amplitude. There may be cases, however, in which external driving forces are available at a frequency close to the higher mode frequencies and knowledge of these frequencies becomes essential.

Both experimental and theoretical analysis can show that even small time-varying concentrated or distributed external forces can excite large amplitude vibration of bones if certain conditions are fulfilled. The most important condition is that the time-varying forces, such as the force exerted on the femur by muscles through walking and running process when a man change his speed or they may vary randomly with time. In the specific case of concentrated forces, the force must be located at points other than nodal points for the associated free vibration frequency and modal shape. [1]

It is also true that in many developed problems it is desired to measure dynamic response by means of accelerometers, and other devices. The measuring devices must be located where they will give maximum response.

3. Mechanical representation of bone

Bone is an anisotropic, heterogeneous, inhomogeneous, nonlinear, complex viscoelastic material. It exhibits electromechanical effects, presumed to be due to streaming potentials, both in vivo and in vitro when wet. In the dry state, bone exhibits piezoelectric properties. Because of the complexity of the structure-property relationships in bone, it is necessary to concentrate on one aspect of the mechanics. Currey [1984], as cited in [18], states that, “the most important feature of bone material is its stiffness.” This is, of course, the premiere consideration for the weight-bearing long bones.

The mechanical properties of cortical and cancellous bone are direction-dependant, time dependant, and it varies with anatomical site. The major difference between cortical and trabecular bone is the difference in relative densities, which has profound influence on the elastic modulus. [6].

Depending on the type of testing, the elastic modulus and strength of the cortical bone range from 14.7 to 34.3 GPa and from 133 to 295 Mpa, respectively. The mechanical properties of a long bone are determined by its tubular shape and bone densities, while that of a cortical bone is determined by osteonal direction. The apparent and material densities are mostly equal in cortical bone since it does not contain space. It is nearly equal to 1.9 g/cm³. [7].

The proximal femur consists of cortical and trabecular regions. The literature reports experimentally derived homogenized mechanical properties of both regions as well as isotropic Young’s modulus E and other elastic constants (under the transversely isotropic/orthotropic assumption) of both regions as a function of the bone apparent density [8].

The different structures of cortical and trabecular bone result in different mechanical properties. Bone mechanical properties are highly variable according to species, age, anatomical site, liquid content, etc., Table (1).

The Cortical bone is an anisotropic material, meaning that its mechanical properties vary according to the direction of load. Cortical bone is often considered an orthotropic material. Orthotropic materials are a class of anisotropic materials characterized by three different Young’s modules E₁, E₂, E₃ according to the direction of load, three shear modules G₁₂, G₁₃, G₂₃ and three Poisson’s ratios ν₁₂, ν₁₃, ν₂₃. [9]

The mechanical characterization of trabecular bone is even more difficult. The mechanical properties of trabecular bone as a whole are due to the mechanical characteristics of single trabecular and to its highly porous structure.

The material properties of the model used in this work was based on results of W.R. Taylor, et al. and they are shown in Table (2). [10]

4. The Standardized Femur

The “Standardized Femur” model (large left femur, mod. 3310, Pacific Research Labs, Vashon Island, WA, USA) was used as reference geometry. It is a 3D solid model made available in public domain derived from a CT-scan dataset of a composite human femur, Fig. (1). The scientific
community found this model very useful. From 1998 many researchers downloaded the model from the International Society of Biomechanics Finite Element Mesh Repository. The model contains surfaces representing both cortical and cancellous bone [11]. The format of the standardized human femur file is IGES (CAD model) and this format is supported by ANSYS.

5. Finite Element Modeling of the Femur

The standardized femur file which was taken from the Pacific Research Labs as reference was found as two volumes. The out volume simulates the cortical bone while the inner volume simulates the trabecular bone. Many enhancements have been done on this model in order to be more suitable for this work practicability. The enhanced model contains 6 volumes. These 6 volumes contain 34 areas, 75 lines and 49 key points.

The element which is used in order to mesh the volumes is 'SOLID 187'. This element is defined by 10 nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element has plasticity, hyper elasticity, creep, stress stiffening, large deflection, and large strain capabilities. It also has mixed formulation capability for simulating deformations of nearly incompressible elastoplastic materials, and fully incompressible hyper elastic materials [12]. This element is used for 3-D model. The number of elements of this model after meshing process was 120150.

6. The Vibration Characteristics of the Standardized Femur

The natural frequencies of the standardized femur which are calculated by using ANSYS program are shown in Table (3)

Different values of bone density were used to calculate the first 11 natural frequencies of standardized femur at each density value, Table (4).

Table (4) explains how natural frequencies of the bone changed by changing bone's density , Fig. 2 (2) represents further explanation of the density effect on natural frequencies values for the second bending mode in Sagittal plane. Fig. (3) explains the shifts of the natural frequencies of the standardized femur for different densities' values

7. Comparison with experimental and numerical results

Comparison of the numerical calculated natural frequencies of the standardized femur of the present study with experimental natural frequencies measured by Couteau et al.(1998) [2] and Khalil et al.(1981) as cited in reference [2] is presented in Table (5). The percentage error between the results of the present study and Couteau et al. results for the first natural frequency is 1.1% and that for Khalil et al. is 21.9%. Fig. (4) compares the first five natural frequencies obtained in the present study with those of Couteau et al and Khalil et al.

The results of Couteau et al. were in agreement with the results found by Khalil et al. and The frequency values of Couteau et al. were 1.2 times higher than Khalil's values. Differences between Khalil's results and Couteau's results were certainly inherent to the different experimental conditions.[2]

The results of the present study are in agreement with the experimental results found by Couteau et al. and Khalil et al., differences can be traced from:

- First, the difference between the models used in each study, the present study used standardized femur while couteau et al and Khalil et al used fresh and cadaveric femur.

-Second, the difference in mechanical properties values used in each study, Table (6).

The natural frequencies of the present study's results compared with the first nine experimental natural frequencies obtained by Jaecques et al. 2004 .[14] as explained in Fig. (5).

The results of the present study are in agreement with the numerical results obtained by Jaecques et al.. Results of Jaecques et al, were obtained using a cadaveric femur, but the healthy conditions of the femur used are not described, mainly if it was a fresh or dried and rehydrated or other, and also the mechanical properties are not published.

Moreover, according to the Euler Bernoulli beam theory, the ratio of frequencies defined by F2/F1 is constant for both two planes (sagittal & frontal plane) and equal to 2.75 in free-free boundaries conditions [2]. Fig. (6) shows the values of (F2/F1) for different researches and the present study in comparison with the analytical value (2.75), the Fig. shows that all values are greater than the analytical value and the difference between them and the analytical value gives a conclusion that the results of the present study are in agreement with that of the other researches.

Fig. (7) shows the first four mode shape of cadaveric left femur that has been found by W.R. Taylor et al. [10]. It is clear that mode shapes are matching and containing the same scientific meaning of the first four mode shapes that have been found for the present study. Fig. (8) shows 2nd bending mode for both studies

8. Conclusions

According to the results of the present study, it can be concluded that:

1- It appeared that numerical results of the present study and those published (experimental and numerical studies) are in reasonable agreement.

2- Vibration analysis is a precise method in predicting the bone strength and health which depends highly on its shape and the distribution of its trabecular and cortical components. The shifts of the natural frequencies of the bone give high imagining about bone's state and health. The natural frequencies shift's shape of the bone fracture area after healing helps in knowing the degree of the succeeding in the healing at the fracture area by comparing it with the shift's natural frequencies diagram of the intact bone.

The shifts of the natural frequencies of the standardized femur for different value of density are due to the density of the bone increasing and the natural frequencies values will decrease and vice versa , this matter consider important and constitutive in diagnosing bone healthy.

3- Osteoporosis disease lead to changes in the density of the bone, it decreases the value of the density continuously and increasing the porosity inside the bone, the value of the bone's density changes in every point in the segments of the
bone and for this reason the average value of the bone's
density depends in diagnosing the osteoporosis, the bone's
density average can't be calculated easily. The shifts of the
natural frequencies of the bone would help in calculating
the average value of the bone's density and estimate the
health state of the bone.

9. References


[2] Beatrice Couteau, Marie-Christine Hobatho, Robert
Darmana, Jean-Claude Brignola, Jean-Yves Arlau "
Finite element modelling of the vibrational behaviour of
the human femur using CT-based individualized
geometrical and material properties " Journal of

[3] Dimarogonas, A. "Method And Apparatus For
Determining Bone Density And Diagnosing
osteoporosis" US patent No. 5836876, nov. 1998.

Naertc, J. Vander Slotena " The resonance frequencies and mode shapes of
dental implants: Rigid body behaviour versus bending
behaviour. A numerical approach " Journal of

[5] Yoon Hyuk Kim, Chang Hwan Byun and Taek Yul Oh "
EFFECT OF OSTEOPOROSIS ON NATURAL
FREQUENCIES IN MOUSE FEMUR " research,Trans

[6] Irina Ionescu, Ted Conway, Alexandra Schonning,
Mutlaq Almutairi, David W. Nicholson " Solid Modeling
And Static Finite Element Analysis Of The Human Tibia

[7] rhabia hürrem özdurak " Vibration Analysis In The
Diagnosis Of Bone Mineral Density In Healthy And
Osteopenia Radius Bone And Its Correlation To Muscle
Strength "Master’s thesis, Department of Physical
Education and Sports, Middle East Technical University,

[8] Ahmet C. Cilingir, Vahdet Ucara, Recep Kazana "
Three-Dimensional Anatomic Finite Element Modeling of
Hemi Arthroplasty of Human Hip Joint " Trends

[9] Biomechanics in dentistry from the site "
http://www.ansys.com/customure/
content/documentation/80/ansys/hlp-e-solid 187.html.

Klabundec, M.D. Warnera, M.C. Hobathod, L.
Rakotomananab, S.E. Clifta " Determination of
orthotropic bone elastic constants using FEA and modal
2002.

CONSTRUCTION AND TESTING OFA GRASPING
PLATE FOR HUMAN EXTRACAPSULAR FEMORAL
NECK FRACTURE FIXATION " PhD thesis, University
of Basrah, Mechanical engineering, October 2005.

[12] ANSYS Element Reference, from the site “
http://www.ansys.com/customure/
content/documentation/80/ansys/hlp-e-solid 187.html.

Simões " Experimental Modal Analysis of a Synthetic
Composite Femur ", Society for Experimental Mechanics,
2002.

Perre " Analysis of the fixation quality of cementless
hip prostheses using a vibrational technique "
K.U.Leuven, Department Mechanical Engineering,
Division of Biomechanics and Engineering


[17] Azra Alizad, Matthew Walsh & James F. Greenleaf "Vibrational Characteristics of Bone Fracture and Fracture
Repair: Application to Exercised Rat Femur” Journal of
300-308,2006.


Table (1) Ultimate strength and ultimate strain of cortical bone of the human femur as a function of age.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Property</th>
<th>Ultimate strength (MPa)</th>
<th>Ultimate Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-20</td>
<td>Tension</td>
<td>114</td>
<td>1.5</td>
</tr>
<tr>
<td>20-30</td>
<td>Compression</td>
<td>123</td>
<td>1.4</td>
</tr>
<tr>
<td>30-40</td>
<td>Bending</td>
<td>120</td>
<td>1.4</td>
</tr>
<tr>
<td>40-50</td>
<td>Torsion</td>
<td>112</td>
<td>1.3</td>
</tr>
<tr>
<td>50-60</td>
<td>Compression</td>
<td>93</td>
<td>1.3</td>
</tr>
<tr>
<td>60-70</td>
<td>Bending</td>
<td>86</td>
<td>1.3</td>
</tr>
<tr>
<td>70-80</td>
<td>Torsion</td>
<td>86</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>Tension</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td></td>
<td>Compression</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td></td>
<td>Bending</td>
<td>_</td>
<td>_</td>
</tr>
<tr>
<td></td>
<td>Torsion</td>
<td>_</td>
<td>_</td>
</tr>
</tbody>
</table>
Table (2) Mechanical properties of the Standardized femur used in the present study. [10]

<table>
<thead>
<tr>
<th></th>
<th>Young’s Modulus</th>
<th>Shear Modulus</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$</td>
<td>13.4 GPa</td>
<td>$G_x$</td>
<td>4.6 GPa</td>
</tr>
<tr>
<td>$E_y$</td>
<td>14.4 GPa</td>
<td>$G_y$</td>
<td>6.2 GPa</td>
</tr>
<tr>
<td>$E_z$</td>
<td>22.9 GPa</td>
<td>$G_z$</td>
<td>5.8 GPa</td>
</tr>
</tbody>
</table>

$\rho = 1940 \frac{Kg}{M^3}$

Table (3) Natural frequencies of standardized femur

<table>
<thead>
<tr>
<th>Mode shape</th>
<th>FREQ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st bending in sagittal plane</td>
<td>304.94</td>
</tr>
<tr>
<td>1st bending in frontal plane</td>
<td>319.79</td>
</tr>
<tr>
<td>Torsion</td>
<td>555.63</td>
</tr>
<tr>
<td>2nd bending in Sagittal plane</td>
<td>952.10</td>
</tr>
<tr>
<td>2nd bending in frontal plane</td>
<td>998.41</td>
</tr>
<tr>
<td>Mode 6</td>
<td>1779.1</td>
</tr>
<tr>
<td>Mode 7</td>
<td>1954.3</td>
</tr>
<tr>
<td>Mode 8</td>
<td>2585.1</td>
</tr>
<tr>
<td>Mode 9</td>
<td>2814.9</td>
</tr>
<tr>
<td>Mode 10</td>
<td>3060.9</td>
</tr>
<tr>
<td>Mode 11</td>
<td>3155</td>
</tr>
</tbody>
</table>

Table (4) Femur natural frequencies due to variable densities

<table>
<thead>
<tr>
<th>Set</th>
<th>$\rho =$1500</th>
<th>$\rho =$1600</th>
<th>$\rho =$1700</th>
<th>$\rho =$1800</th>
<th>$\rho =$1900</th>
<th>$\rho =$2000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The Natural Frequencies [Hz]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>346.80</td>
<td>335.78</td>
<td>325.76</td>
<td>316.58</td>
<td>308.14</td>
<td>300.33</td>
</tr>
<tr>
<td>2</td>
<td>363.68</td>
<td>352.13</td>
<td>341.62</td>
<td>331.99</td>
<td>323.14</td>
<td>314.95</td>
</tr>
<tr>
<td>3</td>
<td>631.89</td>
<td>611.82</td>
<td>593.55</td>
<td>576.83</td>
<td>561.45</td>
<td>547.23</td>
</tr>
<tr>
<td>4</td>
<td>1082.8</td>
<td>1048.4</td>
<td>1017.1</td>
<td>988.43</td>
<td>962.07</td>
<td>937.71</td>
</tr>
<tr>
<td>5</td>
<td>1135.4</td>
<td>1099.4</td>
<td>1066.6</td>
<td>1036.5</td>
<td>1008.9</td>
<td>983.32</td>
</tr>
<tr>
<td>6</td>
<td>2023.3</td>
<td>1959.1</td>
<td>1900.6</td>
<td>1847.0</td>
<td>1797.7</td>
<td>1752.2</td>
</tr>
<tr>
<td>7</td>
<td>2222.6</td>
<td>2152.0</td>
<td>2087.7</td>
<td>2028.9</td>
<td>1974.8</td>
<td>1924.8</td>
</tr>
<tr>
<td>8</td>
<td>2939.9</td>
<td>2846.5</td>
<td>2761.5</td>
<td>2683.7</td>
<td>2612.2</td>
<td>2546.0</td>
</tr>
<tr>
<td>9</td>
<td>3201.3</td>
<td>3099.6</td>
<td>3007.1</td>
<td>2922.3</td>
<td>2844.4</td>
<td>2772.4</td>
</tr>
<tr>
<td>10</td>
<td>3481.0</td>
<td>3370.4</td>
<td>3269.8</td>
<td>3177.7</td>
<td>3092.9</td>
<td>3014.6</td>
</tr>
</tbody>
</table>

Where:
Set 1: 1st bending in sagittal plane.
Set 2: 2nd bending in frontal plane.
Set 3: Torsion mode.
Set 4: 2nd bending in sagittal plane.
Set 5: 2nd bending in frontal plane.
Set 6: Mode 6.
Set 7: Mode 7.
Set 8: Mode 8.
Set 9: Mode 9.
Set10: Mode 10.
Table (5) Comparison of the present study’s results with Couteau et al. (1998) & Khalil et al. (1981) results

<table>
<thead>
<tr>
<th>$F_{num}$ (Hz) Present study</th>
<th>$F_{exp}$ (Hz) Couteau et al. (1998)</th>
<th>$F_{exp}$ (Hz) Khalil et al. (1981)</th>
<th>Mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>304.94</td>
<td>301.6</td>
<td>250</td>
<td>1st bending mode in Sagittal plane</td>
</tr>
<tr>
<td>319.79</td>
<td>353.3</td>
<td>315</td>
<td>2nd bending mode in frontal plane</td>
</tr>
<tr>
<td>555.63</td>
<td>612.0</td>
<td>563</td>
<td>Torsion mode</td>
</tr>
<tr>
<td>952.10</td>
<td>886.6</td>
<td>825</td>
<td>2nd bending mode in sagittal plane</td>
</tr>
<tr>
<td>998.41</td>
<td>931.6</td>
<td>879</td>
<td>2nd bending mode in frontal plane</td>
</tr>
</tbody>
</table>

$F_{num}$: Numerical natural frequency.  $F_{exp}$ : Experimental natural frequency.

Table (6) The mechanical properties values for current study and Couteau et al study

<table>
<thead>
<tr>
<th>E &amp; G in GPa</th>
<th>Standardized femur</th>
<th>Cadaveric femur</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$, $E_y$, $E_z$</td>
<td>$E_x$, $E_y$, $E_z$</td>
<td></td>
</tr>
<tr>
<td>13.4, 14.4, 22.9</td>
<td>11.6, 12.2, 19.9</td>
<td></td>
</tr>
<tr>
<td>$G_x$, $G_y$, $G_z$</td>
<td>$G_x$, $G_y$, $G_z$</td>
<td></td>
</tr>
<tr>
<td>4.6, 6.2, 5.8</td>
<td>4, 5.4, 5</td>
<td></td>
</tr>
<tr>
<td>$V_x$, $V_y$, $V_z$</td>
<td>$V_x$, $V_y$, $V_z$</td>
<td></td>
</tr>
<tr>
<td>0.42, 0.23, 0.23</td>
<td>0.42, 0.23, 0.23</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1 The standardized human femur
Fig. 2 The changing in natural frequencies due to changing in densities.

Fig. 3 Shifts of the natural frequencies of the standardized femur in sagittal plane due to changing of the density.
Fig. (5-4) shows the first four mode shape of cadaveric lef.

![Graph showing comparison of natural frequencies](image1)

**Fig. 4** Comparison of the first five natural frequencies obtained in the present study results with Couteau et al and Khalil et al. results

![Graph showing natural frequencies](image2)

**Fig. 5** Compares the natural frequencies obtained in the present study with S.V.N Jaecques et al. & J.F Dias et al.
The present study

Fig. 6 the values of \( F_2/F_1 \) for different researches comparison with the analytical value (2.75).

Fig. 7 Experimental mode shapes for the first 4 modes of vibration[10].
Fig. 8 Comparison between mode four for the present study with W.R. Taylor et al.

Fig. 9 shows how the bone density of the total hip decreases with age. The units are standardized bone density in (mg/cm²). The lines show the average values, and for each age, race and gender a range of values occurs in the ordinary population.[17].