New modified tanh-function method for nonlinear evolution equations

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Abstract

The aim of this paper is to calculate the Traveling waves solutions by using a new technique which is called modified tanh-function method, which are successfully performed to get analytical solutions for Korteweg-deVries (KdV)–Burgers' equation and (2+1)-dimensional Calogero–Bogoyavlenskii–Schiff (CBS) equation. As a result, when the equation parameters are taken as special values, some new solitary wave solution are obtained. Moreover we find in this work that the modified tanh-function method give some new results which are easier and faster to compute by the help of a symbolic computation system. The results obtained were compared with standard tanh-function method.

Keywords: Tanh-function method, KdV–Burgers' equation, Calogero–Bogoyavlenskii–Schiff (CBS) equation.

1-Introduction

For the time being, nonlinear partial differential equations (PDEs) have been the subject of all-embracing studies in various branches of nonlinear sciences, because most of the phenomena that arise in mathematical physics and engineering fields can be described by nonlinear (PDEs). NLPDEs are frequently used to describe many problems, such as, solid state physics, fluid mechanics, plasma physics, chemical kinematics, chemistry, nonlinear optics, biology and many others. In particular, a traveling wave solutions special case of analytical solutions for nonlinear (PDEs), which can provide physical information about the problem and help us to understand the mechanism that governs these physical problem. And thus lead to more applications.

Many powerful methods to construct exact solutions of nonlinear partial differential equations have been established and developed such as the tanh-coth method [1-2], sine-cosine method [3], homogeneous balance method [4-5], exp-function method [6], first-integral method [7], Jacobi elliptic function method [8], and (G'/G)-Expansion method [9-11].

In this work, we will employ the modified tanh-function method to find the exact traveling wave solutions for the Burgers–KdV equation [12]

$$u_t + \epsilon u u_x - v u_{xx} + \lambda u_{xxx} = 0, \quad (1)$$

where ϵ, v and λ are arbitrary real constants with $\epsilon v \lambda \neq 0$. This equation is the simplest form of the wave equation in which the nonlinearity (uu_x), the dispersion (u_{xxx}) and the dissipation (u_{xx}) all occur. It arise as a model for the propagation of waves on an elastic tube [13], the flow of liquids containing gas bubbles and weakly nonlinear plasma waves with certain dissipative effects. Eq. (1) does not have the Painleve property, which was first showed by Gibbon et al. [14]. It can be collection between of the Burger's equation $(\epsilon \neq 0, v \neq 0, \lambda = 0)$ and the KdV equation $(\epsilon \neq 0, v = 0, \lambda \neq 0)$.

We start with the (2 +1)-dimensional Calogero–Bogoyavlenskii–Schiff (CBS) equation in the form [15]

$$u_{xt} + u_{xxxz} + 4u_x u_{xz} + 2u_{xx} u_z = 0. \quad (2)$$

The CBS equation was first investigated by Bogoyavlenskii and Schiff by many different ways. Bogoyavlenskii used the modified Lax formalism, whereas Schiff derived the same equation by reducing the self-dual Yang–Mills equation [16].

Our paper is organized as follows: in Section 2, we present the description of modified tanh-function method, and Section 3, we apply this method to the nonlinear Korteweg-de Vries (KdV)–Burgers' equation and (2 +1)-dimensional Calogero–Bogoyavlenskii–Schiff (CBS) equation pointed out above, conclusions are given in Section 4.

2-The Modified Tanh-function Method

In this section, we describe the modified tanh-function method for finding traveling wave solutions of nonlinear partial differential equations (PDEs). Suppose that a nonlinear partial differential equations in two independent variables, x and t are given by

$$P(u, u_x, u_t, u_{xt}, u_{xx}, u_{tt}, \dots) = 0. \quad (3)$$

Where $u = u(x,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ and it's various partial derivatives, in which highest order derivatives and nonlinear terms are involved. To determine the modified tanh-function method, we take the following seven steps.

Step 1. To find the traveling wave solutions of (3), we introduce the wave variable.

$$u(x,t) = u(\zeta), \qquad \zeta = (x - ct) \quad (4)$$

where the constant c is generally termed the wave velocity. Substituting (4) into (3), we obtain the following ordinary differential equations (ODE) in ζ (which illustrates a principal advantage of a traveling wave solution; i.e., a PDE is reduced to an ODE):

$$P(u, cu', cu'', c^2 u'', u'', \dots) = 0 \quad (5)$$

Step 2. It is necessary to integrate (5) as many times as possible and set the constants of integration to be zero for simplicity.

Step 3. We suppose that Eq. (5) has the following formal solution:

$$u(\zeta) = S(Y) = a_0 + \sum_{i=1}^{m} a_i y^i + b_i y^{i-1} \sqrt{\sigma\left(1 + \frac{y^2}{\mu}\right)}. \quad (6)$$

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Where m is a positive integer, and a_0, a_i, b_i, σ and μ are constants, while Y is given by

$Y = \tanh(\zeta)$ (7)

Then, the independent variable ζ in Eq. (7) leads to the following derivatives:

$$\frac{d}{d\zeta} = (1-y^2)\frac{d}{dy}, \quad (8)$$

$$\frac{d^2}{d\zeta^2} = (1-y^2)\left\{(1-y^2)\frac{d^2}{dy^2} - 2y\frac{d}{dy}\right\},$$

$$\frac{d^3}{d\zeta^3} = 2(1-y^2)(3y^2-1)\frac{d}{dy} - 6y(1-y^2)^2\frac{d^2}{dy^2} + (1-y^2)^3\frac{d^3}{dy^3},$$

and so on.

Step 4. The positive integer m can be accomplished by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (5) as follows:

Now, if we define the degree of $u(\zeta)$ as $D[u(\zeta)] = m$, then the degree of other expressions is defined by

$$D\left[\frac{d^q u}{d\zeta^q}\right] = m+q,$$
$$D\left[u^r\left(\frac{d^q u}{d\zeta^q}\right)^s\right] = mr + s(q+m). \quad (9)$$

therefore, we can get the value of m in (6)

Step 5. Substituting (6) along with (8) into (5) and we obtain polynomials in y^i and $y^i\sqrt{\sigma(1+\frac{y^2}{\mu})}$, then we collect each coefficient of the resulted polynomials to zero, yields a set of algebraic equations for a_0, a_i, b_i, c, σ and μ

Step 6. Solving these algebraic equations by Maple or Mathematica, we get the values of a_0, a_i, b_i, c, σ and μ.

Step 7. Substituting these values into (6) and (4), we can obtain the exact traveling wave solutions of Eq. (3).

3-Applications of Modified Tanh-function Method

3.1. Burgers–KdV equation

In order to solve (1) by the modified tanh-function method, we use the wave transformation $u(x,t) = u(\zeta)$, with wave variable $= x + ct$. We change the Eq. (1) to the following NLODEs:

$$cu' + \frac{\epsilon}{2}(u^2)' - vu'' + \lambda u''' = 0 \quad (10)$$

Integrating (10) once with respect to ζ and setting the constant of integration to zero, we obtain

$$cu + \frac{\epsilon}{2}u^2 - vu' + \lambda u'' = 0. \quad (11)$$

Balancing the order of u^2 with the order of u'' in (11), we find M = 2. So the solution takes the following form

$$u = a_0 + a_1 y + a_2 y^2 + b_1\sqrt{\sigma(1+\frac{y^2}{\mu})} + b_2 y\sqrt{\sigma(1+\frac{y^2}{\mu})} \quad (12)$$

where a_0, a_1, a_2, b_1, b_2, σ and μ are unknown constants to be determined later. Substituting (12) into (11), with computerized symbolic computation, equating to zero the coefficients of all power y^i, yields a set of over-determined algebraic equations for a_0, a_i, b_i, c, σ and μ. Solving the algebraic equations by Maple or Mathematica, we can obtain the following results:

Case 1:

$$a_0 = \frac{2v}{\epsilon}, a_1 = \frac{-2v}{\epsilon}, a_2 = 0, b_1 = b_1, b_2 = b_2, c = -2v, \sigma = 0, \lambda = 0$$

Case 2:

$$a_0 = \frac{-2v}{\epsilon}, a_1 = \frac{-2v}{\epsilon}, a_2 = 0, b_1 = b_1, b_2 = b_2, c = 2v, \sigma = 0, \lambda = 0$$

Case 3:

$$a_0 = \frac{6v}{5\epsilon}, a_1 = \frac{-12v}{5\epsilon}, a_2 = \frac{6v}{5\epsilon}, b_1 = b_1, b_2 = b_2, c = \frac{-12v}{5}, \sigma = 0, \lambda = \frac{-v}{10}$$

Case 4:

$$a_0 = \frac{-18v}{5\epsilon}, a_1 = \frac{-12v}{5\epsilon}, a_2 = \frac{6v}{5\epsilon}, b_1 = b_1, b_2 = b_2, c = \frac{12v}{5}, \sigma = 0, \lambda = \frac{-v}{10}$$

Case 5:

$$a_0 = \frac{-6v}{5\epsilon}, a_1 = \frac{-12v}{5\epsilon}, a_2 = \frac{-6v}{5\epsilon}, b_1 = b_1, b_2 = b_2, c = \frac{12v}{5}, \sigma = 0, \lambda = \frac{v}{10}$$

Case 6:

$$a_0 = \frac{18v}{5\epsilon}, a_1 = \frac{-12v}{5\epsilon}, a_2 = \frac{-6v}{5\epsilon}, b_1 = b_1, b_2 = b_2, c = \frac{-12v}{5}, \sigma = 0, \lambda = \frac{v}{10}$$

Case 7:

$$a_0 = \frac{2v}{\epsilon}, a_1 = \frac{-2v}{\epsilon}, a_2 = 0, b_1 = 0, b_2 = 0, c = 2v, \sigma = \sigma, \lambda = 0$$

Case 8:

$$a_0 = \frac{-2v}{\epsilon}, a_1 = \frac{-2v}{\epsilon}, a_2 = 0, b_1 = 0, b_2 = 0, c = 2v, \sigma = \sigma, \lambda = 0$$

Case 9:

$$a_0 = \frac{6v}{5\epsilon}, a_1 = \frac{-12v}{5\epsilon}, a_2 = \frac{6v}{5\epsilon}, b_1 = 0, b_2 = 0, c = \frac{-12v}{5}, \sigma = \sigma, \lambda = \frac{-v}{10}$$

Case 10:

$$a_0 = \frac{-18v}{5\epsilon}, a_1 = \frac{-12v}{5\epsilon}, a_2 = \frac{6v}{5\epsilon}, b_1 = 0, b_2 = 0, c = \frac{12v}{5}, \sigma = \sigma, \lambda = \frac{-v}{10}$$

Case 11:

$$a_0 = \frac{-6v}{5\epsilon}, a_1 = \frac{-12v}{5\epsilon}, a_2 = \frac{-6v}{5\epsilon}, b_1 = 0, b_2 = 0, c = \frac{12v}{5}, \sigma = \sigma, \lambda = \frac{v}{10}$$

Case 12:

$$a_0 = \frac{18v}{5\epsilon}, a_1 = \frac{-12v}{5\epsilon}, a_2 = \frac{-6v}{5\epsilon}, b_1 = 0, b_2 = 0, c = \frac{-12v}{5}, \sigma = \sigma, \lambda = \frac{v}{10}$$

In view of this, we obtain the following solitons and kink solutions:

$u_1(x,t) = \frac{-2v}{\epsilon}[1 + \tanh(x + 2vt)]$,

$u_2(x,t) = \frac{2v}{\epsilon}[1 + \tanh(x + 2vt)]$,

$u_3(x,t) = \frac{6v}{5\epsilon}[1 - 2\tanh(x - \frac{12}{5}vt) + \tanh^2(x - \frac{12}{5}vt)]$,

$u_4(x,t) = \frac{6v}{5\epsilon}[-3 - 2\tanh(x + \frac{12}{5}vt) + \tanh^2(x + \frac{12}{5}vt)]$,

$u_5(x,t) = \frac{-6v}{5\epsilon}[1 + 2\tanh(x + \frac{12}{5}vt) + \tanh^2(x + \frac{12}{5}vt)]$,

$u_6(x,t) = \frac{6v}{5\epsilon}[3 - 2\tanh\left(x - \frac{12}{5}vt\right) - \tanh^2\left(x - \frac{12}{5}vt\right)]$,

If we set $v = 2$ and $\epsilon = 5$ in (u_6), we obtain the solitary wave solution

$u = \frac{12}{25}[3 - 2\tanh(x - \frac{24}{5}t) - \tanh^2(x - \frac{24}{5}t)]$ (13)

This is exactly the same solution obtained by Soliman as given in [12], when we assume $v = 2, \epsilon = -5$ and $\mu = -1/5$.

$$u = \frac{v^2}{25\epsilon\mu}[9 - 6\tanh(x - \frac{6v^2}{25\mu}t) - 3\tanh^2(x - \frac{6v^2}{25\mu}t)] \quad (14)$$

The solitary wave and behaviour of the solutions $u_2(x,t)$ and $u_3(x,t)$ are shown in Figures 1 and 2 respectively for some fixed values of the $v = 2$ and $\epsilon = 5$.

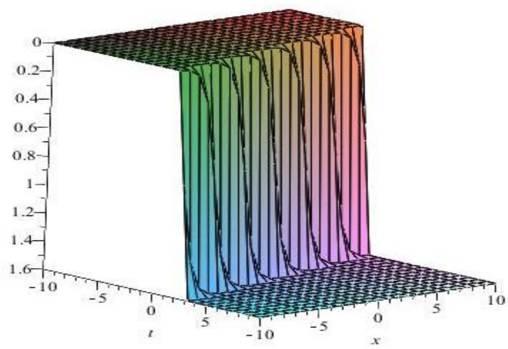

Figure 1 The kink solution of $u_2(x,t)$ for $v = 2$ and $\epsilon = 5$.

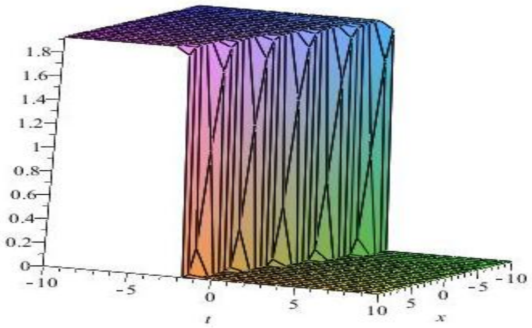

Figure 2 The kink solution of $u_3(x,t)$ for $v = 2$ and $\epsilon = 5$.

3.2. The (2+1) - dimensional Calogero–Bogoyavlenskii – Schiff equation

Using the traveling wave variable $u(x,z,t) = u(\zeta)$, with wave variable $\zeta = x + z - Vt$ in Eq.(2) that converts the given PDE into an ODE and on integrating once, therefore integrating with respect ζonce yields:

$$-Vu' + 3(u')^2 + u''' = 0. \quad (15)$$

Balancing u''' with $(u')^2$ in (15), we obtain $m = 1$. Therefore, the solution (6) can be written as

$$u = a_0 + a_1 y + b_1\sqrt{\sigma(1 + \frac{y^2}{\mu})}. \quad (16)$$

Where a_0,a_1,b_1,σ and μ constants to be determined later. Substituting (16) into (15), collecting the coefficients of Y^i we obtain the following system of algebraic equations for a_0,a_1,b_1,σ and μ .

$y^0 = 3b^2{}_1\sigma + 3a^2{}_1\mu - \sigma a_1\mu = 0,$

$y^1 = 6\mu b_1 + 6\mu a_1 b_1 = 0,$

$y^2 = 3\mu a^2{}_1 + 6b^2{}_1\sigma\mu + 9a^2{}_1\mu^2 + v\mu a_1 - 3b^2{}_1\sigma - 18\mu^2 a_1 + 2a_1\mu = 0,$

$y^3 = v\mu b_1 + 2\mu b_1 - 6\mu a_1 b_1 - 15b_1\mu^2 + 12\mu^2 a_1 b_1 = 0,$

$y^4 = 3v\mu^2 a_1 + 3b^2{}_1\sigma\mu^2 - 6b^2{}_1\sigma\mu - 18\mu^3 a_1 + 6\mu^2 a_1 - 9a^2{}_1\mu^2 + 9a^2{}_1\mu^3 = 0,$

$y^5 = 4b_1\mu^2 + 2vb_1\mu^2 - 12\mu^2 a_1 b_1 - 12\mu^3 b_1 + 6\mu^3 b_1 a_1 = 0,$

$y^6 = 3v\mu^3 a_1 + 6\mu^3 a_1 - 3b^2{}_1\sigma\mu^2 + 3a^2{}_1\mu^4 - 9a^2{}_1\mu^3 - 6\mu^4 a_1 = 0,$

$y^7 = v\mu^3 b_1 - 6\mu^3 b_1 a_1 + 8\mu^3 b_1 + 3\mu^2 b_1 = 0,$

$y^8 = 3a^2{}_1\mu^4 + 2\mu^4 a_1 + v\mu^4 a_1 = 0.$

On solving the above set of algebraic equations by Maple, we have the following sets of solutions

Case 1:

$a_0 = a_0, a_1 = 2, b_1 = 0, \mu = \mu, v = 4,$

Case 2:

$a_0 = a_0, a_1 = 1, b_1 = \pm\sqrt{-\frac{1}{\sigma}}, \mu = -1, v = 1,$

Case 3:

$a_0 = a_0, a_1 = a_1, b_1 = 0, \mu = 0, v = 0,$

In view of this, we obtain the following kink solutions:

$u_1(x,t) = a_0 + 2\tanh(x + z - 4t),$

$u_2(x,t) = a_0 + \tanh(x + z - t) + \sqrt{\frac{-1}{\sigma}}\sqrt{\sigma(1 - y^2)}$,

$u_3(x,t) = a_0 + \tanh(x + z - t) - \sqrt{\frac{-1}{\sigma}}\sqrt{\sigma(1 - y^2)}$

This solution is the exact same solution obtained by [15],when $A = -1$.

Some of our obtained traveling wave solutions are represented in the Figures 3, for some fixed values of the $a_0 = 2, t = 0.5$

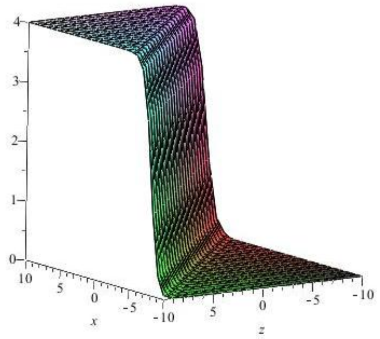

Figure 3 The kink solution of u_3(x, t) for $a_0 = 2, t = 0.5$

4. Conclusion

A modified tanh-function method is suggested and applied to the Korteweg-de Vries (KdV) – Burgers' equation and (2+ 1)-dimensional Calogero–Bogoyavlenskii–Schiff (CBS) equation. The results obtained by the suggested method have been compared with those obtained by the standard tanh-function method. We see that all the results obtained by the standard tanh-function are found by the modified tanh-function and in addition some new solutions are attained. this means the standard tanh method does not have this capability. Also, the solutions contain free parameters. These solutions will be very useful in various physical situations. Finally, numerical simulations are given to complete the study.

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طريقة الظل الزائدي المعدلة للمعادلات التفاضلية الغير خطية

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الملخص

الهدف من دراسة هذا البحث حساب الحلول الانتقال الموجة باستخدام تقنية جديدة التي تسمى طريقة الظل الزائدي المعدلة ، التي يتم تنفيذها بنجاح للحصول على الحلول التحليلية لمعادلة kdv-Burgers ومعادلة CBS بالبعد (2+1). كذلك النتيجة ، عندما معلمات المعادلة تاخذ قيما خاصة نحصل على بعض الحلول الجديدة للموجة الانفرادية. لقد وجدنا في هذه الدراسة بان طريقة الظل الزائدي المعدلة تعطي بعض النتائج الجديدة والتي تكون اسرع واسهل للحساب بمساعدة نظام الحوسبة الرمزي. ولقد تم مقارنة النتائج مع طريقة الظل الزائدي القياسية.

الكلمات المفتاحية: دالة الظل الزائدي ، الموجات المتنقلة ، معادلة KdV – Burgers ، معادلة CBS