Determination local geoid Heights Using RTK-DGPS/Leveling and transformation methods

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Abstract
Geodesy is concerned with the relative positioning of points and the gravity field of the earth. For this task, a well-defined coordinate system is needed on which measurements are normally tied to a set of reference points called a geodetic datum (geoid or ellipsoid). The Global Positioning System GPS gives accurately the three-dimensional position of a point (latitude, longitude, and ellipsoidal height) and can measure under all weather conditions. The coordinates of the GPS reference to the World Geodetic System1984 (WGS 84), a global ellipsoid having its origin as the mass center of the earth, and height, referenced to the surface of the ellipsoid. In this research, using RTK-DGPS technique Data collection for study local and leveling, and Earth Gravitational Models (EGM2008, EGM96) for determined to geoid undulation utilized 2D polynomial models, and then Using a surface interpolation (kriging) approach, the coordinate and the computed geoidal heights of some well selected points.

Keywords: Geoid height, Real Time Kinematic, Differential Global Position System, World Geodetic, System1984

تحديد الارتفاعات الجيوديسية المحلية باستخدام (الرصد المتحرك اللحظي - جهاز التموضع التفاضلي العالمي) للموانئ و طريقة التحويل

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الخلاصة

علم الجيوديسيا يختص بتحديد المواقع بالنسبة للفضاء، وقد تمت لليست النتائج في مجال الجيوديسية للأراض. هذه المهمة تنطوي على تحديد النقطة على سطح الأرض بواسطة قياسات ترتيب عادة ما تكون مركبة من نقاط المرجعية (الجيود أو الإهليلجي) نظام تحديد المواقع العالمي يعطي اتجاهات لمسار وأبعاد نقطة (خطوط الطرز) المرجعية، والارتفاع الإهليلجي. يمكن قياس هذه الطرق بواسطة نمادج تحديد المواقع العالمي المرجع ل념ام الجيوديسية العالمي 1984، وجود الإهليلجي العالمي أصبحًا عن مركز كتلة الأرض و مرجع الارتفاع إلى سطح الإهليلجي. في هذه البحث استخدمت تقنية الجي بي اس التقاطع - الرصد المتحرك اللحظي لجمع البيانات لتحديد ميزان النمادج المتعددة للمغزاء الإهليلجي الإرصاد EGM2008, EGM96 لتحديد أطراف الارتفاعات باستخدام نمادج متعددة النمادج التفاعلية الإرصاد وبعدها تم استخدام طريقة الكريجين (kriging) لاستخدام السطوح لأطراف الارتفاعات وتمويح الجيوديسية المحورية لبعض النقاط المتفرقة.

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Introduction

One of the basic goals of geodesy is the determination of the geoid which is the equipotential surface of the earth gravity field and which coincides on the average with the mean sea level [1]. The geoid surface is more irregular than the ellipsoid of revolution often used to approximate the shape of the physical Earth, but considerably smoother than the Earth’s physical surface. The transformation of ellipsoidal heights to orthometric heights therefore requires that the geoid height refer to the same reference ellipsoid. In the case of GPS-derived ellipsoidal heights the geocentric WGS84 ellipsoid are used by using the equation (1)[2].

\[ H = h - N \]  

Orthometric heights (H) are defined as the geometric distance, measured along the plumb line, from the geoid to a point of interest above. Ellipsoid heights (h) are defined as the geometric distance, measured along a normal straight line to a reference ellipsoid, from the reference ellipsoid to a point of interest above. The geoid height (or geoid undulation, N) is defined as the geometric distance, measured along a normal straight line to a reference ellipsoid, from the reference ellipsoid to a point on the geoid above. Equations (1) has been used for the determination of orthometric heights from ellipsoidal heights and a geoid model, this is called GPS/leveling [3] The relationships between orthometric, ellipsoidal and geoid heights are shown in Figure-1.

![Figure 1 - Illustration of the relationships between orthometric, ellipsoidal and geoid heights][4]

Global Geopotential Models (GGM)

Global Geopotential models (GGMs) describe the Earth’s gravitational potential in terms of an infinite series of spherical harmonics outside the Earth attracting masses. They are determined by a combination of satellite and terrestrial observations and used as reference fields in the determination of local and regional geoids. The geopotential is usually given as a truncated set of harmonic coefficients, obtained when solving a Laplace equation in spherical coordinates described [5].

\[ V(r, \theta, \lambda) = \frac{GM}{r} \left\{ 1 + \sum_{n=1}^{N_{\text{max}}} \sum_{m=0}^{l} \left( \frac{a}{r} \right)^{l} \left[ \bar{C}_{nm} \cos m \lambda + \bar{S}_{nm} \sin m \lambda \right] P_{nm}(\sin \theta) \right\} \]  

Where:

- \( GM \) Earth’s gravity constant
- r magnitude of radius vector
- n, m degree and order of spherical harmonics
- \( P_{nm} \) Legrendre functions
- \( \bar{C}_{nm}, \bar{S}_{nm} \) coefficients of spherical harmonics
- \( \theta \) Latitude
- \( \lambda \) Longitude

The disturbing potential \( T \) at a point \( V (r, \theta, \lambda) \) is the differences between the actual gravity potential of the Earth and the normal potential associated with the a rotating equipotential ellipsoid at \( V \). Based on equation (2) the spherical harmonic representation of \( T \) is:

\[ T(r, \theta, \lambda) = \frac{GM}{r} \sum_{n=1}^{N_{\text{max}}} \sum_{m=0}^{l} \left( \frac{a}{r} \right)^{l} \left[ \bar{C}_{nm} \cos m \lambda + \bar{S}_{nm} \sin m \lambda \right] P_{nm}(\sin \theta) \]  

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Equation (3) have been expanded for several numerous processes to get the element of the Earth’s gravity field such as gravity anomalies (Δg) and geoid height (N). The relationship between the coefficient of spherical harmonic with gravity anomalies (ΔgGM) and geoidal height (NGM) is given by the following formula, respectively:[6]

$$\Delta g_{GM} = \frac{GM}{r^2} \sum_{n=2}^{N_{\text{max}}} \sum_{m=0}^{l} \left( \frac{\alpha}{r} \right)^l \left[ \bar{C}_{nm} \cos m \lambda + \bar{S}_{nm} \sin m \lambda \right] \overline{P}_{nm}(\sin \theta) \quad (4)$$

$$N_{GM} = \frac{GM}{r^2} \sum_{n=2}^{N_{\text{max}}} \sum_{m=0}^{l} \left( \frac{\alpha}{r} \right)^l \left[ \bar{C}_{nm} \cos m \lambda + \bar{S}_{nm} \sin m \lambda \right] \overline{P}_{nm}(\sin \theta) \quad (5)$$

The development of accurate potential coefficient models is dependent on accurate analyses of the perturbations of the orbits of artificial satellites (e.g. GPS ) and from the combination of such information with surface gravity data, and relatively recently with satellite altimeter data [6].

In this study, used two models ( EGM96, EGM2008). Earth Gravitational Model 1996 EGM96 is a geopotential model of the Earth consisting of spherical harmonic coefficients complete to degree and order 360, This model was developed by Goddard Space Flight Center, The National Imagery and Mapping Agency (NIMA) and the Ohio State University (OSU). The model gives the geoidal separation from WGS84 ellipsoid and utilized worldwide for converting ellipsoidal elevation values to Orthometric heights. [7]. Earth Gravitational Model 2008 (EGM2008) is a spherical harmonic model of the Earth’s gravitational potential developed by the Department of Defense. However, National Geospatial-Intelligence Agency (NGA) is developing a comprehensive Earth Gravitational Model Implementation Plan for use within the National System for Geospatial-Intelligence. The Implementation Plan will include specific data, format, file and algorithm guidance for system developers on EGM2008 and future geoid releases.[8] Together with other satellite-only and combined GGMs that have been available after the GRACE and CHAMP missions, the EGM08 model represents the state-of-art in global gravity field mapping and it contributes significantly to the continuing efforts of the geodetic community for a highly accurate reference model of Earth’s gravity field.[8] EGM2008 is complete to degree and order 2159, and contains additional coefficients up to degree 2190 and order 2159. Over areas covered with high quality gravity data, the discrepancies between EGM2008 geoid undulations and independent GPS/ Leveling values are on the order of 5 to 10 cm.[8]

**Real Time Kinematic DGPS**

RTK surveying is a carrier phase based relative positioning technique that, like the previous methods, employs two or more receivers simultaneously tracking the same satellites . This method is suitable when: (1) the survey involves a large number of unknown points located in the vicinity (i.e., within up to about 10.15 km) of a known point; (2) the coordinates of the unknown points are required in real time; and (3) the line of sight, the propagation path, is relatively unobstructed [10]. The RTK approach is a differential positioning technique that uses known coordinates of a reference station occupied by one receiver to determine coordinates of unknown points visited by a rover receiver [11].Because of its ease of use as well as its capability to determine the coordinates in real time, this method is the preferred method by many users. The typical nominal accuracy for these dual-frequency systems is 1 centimeter ± 2 parts-per-million (ppm) horizontally and 2 centimeters ± 2 ppm vertically.

**Modeling local GPS/levelling geoid technique**

The GPS/levelling technique simply involve the use of ellipsoidal heights derived from the Global Positioning System and orthometric heights obtained via the levelling process, to determine the geoid height and subsequently the geoid model . In modelling local GPS/levelling geoid with geometric approach, a geoid reference benchmarks network having coverage of entire area is constituted. One of the limitations of local geoid determination is datum inconsistency problem. But, in this study, this problem is not going to be considered because the focus is on testing surface fitting algorithms as a geometrical approach for modeling a local geoid using GPS and Leveling data. [12]. The geoid reference benchmarks are generally selected from the common points of order GPS benchmarks and the 1st , the 2nd and 3rd orders levelling network points, With existing n reference benchmarks having GPS ellipsoidal and levelling heights (and hence with known geoid heights: $N_{\text{GPS/lev.}} = h_{\text{GPS}} - H_{\text{levelling}}$)
in a local area, the general equation of polynomial interpolation to estimate GPS/levelling geoid heights at unknown points in the area can be given as (equation 6):

\[ N(x, y) = \sum_{m=0}^{L} \sum_{n=0}^{M} a_{mn} x^m y^n \]

(6)

where \( x \) and \( y \) represent the position coordinates, \( a_{mn} \) symbolize the polynomial coefficients, and \( L \) is the degree of the polynomial. The position coordinates can be constituted in various ways, and in this study they are obtained from the ellipsoidal geographical coordinates as following:

\[ x = (\phi - \phi_0) \quad \text{and} \quad y = (\lambda - \lambda_0) \]

where \( \phi_0 \) and \( \lambda_0 \) are the arithmetic averages of the latitudes and longitudes of the data set. For modeling the geoid of this area, a 1st, 2nd and 3rd degrees polynomials were used as a trend functions (shows equations 7, 8 and 9).

\[ N(x, y) = a_0 + a_1 x + a_2 y \]

(7)

\[ N(x, y) = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2 \]

(8)

\[ N(x, y) = a_0 + a_1 x + a_2 y + a_3 xy + a_4 x^2 + a_5 y^2 + a_6 xy^2 + a_7 x^2 y + a_8 x^3 + a_9 y^3 \]

(9)

The \( (a_0, a_1, \ldots, a_9) \) polynomial coefficients, determined according to Least Squares Adjustment (LSA) method.

The Study Area

Baghdad University Compass region (The Baghdad government), which covers about (2.903225 Square Kilometers). The survey for this area were accomplished using Real Time Kinematic - Differential Global Position System RTK-DGPS, type Topcon Hiper-II. The study region located in the middle of the Iraqi country, Latitude (33º 16' 32.1") to (33º 16' 2.9") N, Longitude (44º 22' 10.1") to (44º 23' 18.5") E. A total of 34 points (bench marks) were observed using both GPS and leveling . One of the best available data is QuickBird satellite image of 0.6 spatial resolutions, shown Figure-2.

![Figure 2 - Satellite image QuickBird, 0.6m sensors show study area (Baghdad University)](image)

Data Acquisition

The Easting and Northing of 34 points located on the studied area have been navigated using GPS survey, with the Universal Transverse Mercator UTM projection and world geodetic system 1984 (WGS-84). A total of points (bench marks) were observed using both GPS and leveling. Figure-2 shows the distribution of these GPS/leveling. The survey for this study were accomplished using Differential Global Position System (DGPS), type Topcon Hiper-II.

Results and Discussion

These methods is based on the direct determination of the orthometric (H) and ellipsoidal (h) heights, for the purpose of calculating the undulation height values N of the geoid of these points. And also determine the geoid height using of the Earth Gravitational Model 2008 (EGM2008) and EGM96 using matlab. The Table-1 Illustrate the results for all modeling mathematical using in this research.
For all measuring modeling described in this paper, the average, min., max., and root mean square error were calculated. The results are shown in Table-2.

Table 2 - Statistical analysis for all modeling

<table>
<thead>
<tr>
<th>Geoid model</th>
<th>Min. (m)</th>
<th>Max. (m)</th>
<th>Average (m)</th>
<th>RMSE (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N observed</td>
<td>-1.59</td>
<td>-1.41</td>
<td>-1.4982</td>
<td>0.0682</td>
</tr>
<tr>
<td>N (1st order)</td>
<td>-1.5033</td>
<td>-1.4946</td>
<td>-1.4982</td>
<td>0.0028</td>
</tr>
<tr>
<td>N (2nd order)</td>
<td>-1.5326</td>
<td>-1.4564</td>
<td>-1.4982</td>
<td>0.0287</td>
</tr>
<tr>
<td>N (3rd order)</td>
<td>-1.543</td>
<td>-1.455</td>
<td>-1.4982</td>
<td>0.0085</td>
</tr>
<tr>
<td>N (EGM 2008)</td>
<td>-1.6244</td>
<td>-1.5905</td>
<td>-1.6081</td>
<td>0.0024</td>
</tr>
<tr>
<td>N (EGM96)</td>
<td>-2.2542</td>
<td>-2.2193</td>
<td>-2.2351</td>
<td>0.0047</td>
</tr>
</tbody>
</table>
Figure 3 - Modelling Surface and Digital Terrain Model using 1st order

Figure 4 - Modelling Surface and Digital Terrain Model using 2nd order

Figure 5 - Modelling Surface and Digital Terrain Model using 3rd order
Conclusions:

Despite the fact that the GPS heighting is fairly muddled methodology since it is gathers a few strategies for situating and gravity field determination GPS, leveling and geoid modeling Geoid modeling utilizing geometrical interpolation procedure has been discussed; the likelihood of utilizing a lower request polynomial for modeling geoid. This is using high degree polynomial as depending on well distributed and sufficient number of GPS/Leveling points that are called as benchmarks and by the way expressing the surface of geoid as an analytical surface is one of the ways of modeling. This polynomial that means to make it more accurate with some techniques is possible. Applying weighted corrections from the reference points to the interpolation points or according to the least squares collocation method can be considered in to these techniques.

Reference


