Focusing of Charged Particles Beam by Double Quadrupole Triplet Electrostatic Lenses

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Abstract
A study was presented to understand the behavior of charged particles beam through consideration systems of double Quadrupole Triplet Electrostatic Lenses reaching to the optimum design. In this work, tracing the path of charged particles beam has been done within the free field space and double quadrupole electrostatic lens, by using matrices to described particle trajectories throughout the system, matrix representation deals with ion beam as bunched and representing phase ellipse for both horizontal and vertical planes. The present work investigate the effect of changing the main parameters of double Quadrupole Triplet Electrostatic Lenses such as [extraction region shape, length of quadrupole, the distance between the quadrupole lenses, voltage applying on lenses and second field free region length].

A computer program written in (MATLAB) language to trace the path of charged particles beam with a system of double Quadrupole Triplet Electrostatic Lenses has been built in the present work. The results indicate good focusing properties.

Introduction
Study the focusing of charged particle beam by the electrostatic quadrupole lenses is very important because when the ion beam passed through the lens deflects the ion beam toward lens axis, then the lens focuses beam or compresses them to smallest possible radius. That means there is spatial distribution of beam intensity in the transport plane, where the lens can make modified copy of beam distribution in another plane along the direction of propagation.

According to the importance of ion beam transport systems, many researches are interested and conducted for developing this field of physics. In this section we introduced some of these researches which are the closet to our research field. The ion optical properties reviewed in [1], [2] for electric and magnetic element used for beam transport. Matrix methods and ellipse representations used to describe the charged particles motion in a transverse fields devices for quadrupole lenses in all types, deflectors, Wien velocity filter, etc. and in longitudinal fields such as accelerators x. The investigation of the main parameters effects [3],[4] in designing the medium
range ion implanter, also built a computer program to trace the path of charged particles beam within ion implanter system, the later contain c-magnet mass analyzer, acceleration tube, triplet quadrupole lenses and elements treated with matrices representation. The studied beam properties, paraxial path equation, paraxial approximation, thin lens approximation, quadrupole-muotolets and strong focusing by studying Liouville's theorem and emittance.

The fundamentals of charge particle motion in ion optical system, paraxial trajectory equation, [5],[6] has been studied electrostatic charge particle lens, focusing properties of electrostatic lens, magnetic lens approximation and describing the charged particles motion in different type of ion optical system.

A design and evaluation of an electrostatic quadrupole triplet lens constructed to focus ion beams of up to 200keV in energy done by [7], and the investigation of the magnetic lens and electrostatic lens has been done in [8], using matrix description and how focusing ion beam. Also they were studied optical properties of electrostatic lens. In [9], a design of the focusing lens system has been done by using SIMION computer program which is used to trace and simulate the ion beam during its transport. Recently an investigation tool has presented to deals with electrostatic lenses [10] and [11].

Matrix Representation

Matrix formalism is widely used to calculate trajectories for individual particle or for a virtual particle representing the central path of a whole beam [5], in another word the matrix description is a mathematical method to organize information about the transverse motions of particles concerning the main beam axis. Matrices are particularly helpful when dealing with systems with a large number of different element, the effects on particles of a single element and combinations of elements are described by the familiar rules of matrix algebra [12]. The total matrix of beam transports system which is obtained by multiplying separated matrices describes the individual elements of the system. Also the information of the transverse motion of particles relative to that of central trajectory, at any location in the beam line, may be determined from its initial value by means of a transfer matrix. That is the transfer matrix contained complete information on the physics properties of the system elements. The present study includes two types of matrices; first-order transfer, which can be classified as linear matrix and phase space matrix.

Linear Ray Matrix and Linear Phase Space Matrix

The ion beam charge consists of many charged particle with a certain momentum (p), at the certain position and direction, this particle can be characterized by six coordinates in Cartesian coordinates (x, y, z, px, py, pz) [13], that is by six row column matrix so for each particle in beam.

\[
X = \begin{bmatrix}
  x \\
  y \\
  z \\
  p_x \\
  p_y \\
  p_z \\
\end{bmatrix}
\]

\[..........................(1)\]
This is propagated along the z-axis, there are only four coordinates \((x,y,p_x,p_y)\)[12]. That main dealing with an orthogonal coordinates system. Then each particle in the beam line is described by four component vectors as shown Figure (1).

\[
X = \begin{bmatrix}
x \\
x' \\
y \\
y'
\end{bmatrix}
\]

...................(2)

Where: \(x\): the horizontal displacement of ray with respect to the central trajectory. \\
\(x'\): the angle of the ray makes in the horizontal plane with respect to the central trajectory. \\
\(y\): the vertical displacement of ray with respect to the central trajectory. \\
\(y'\): the vertical angle of the ray makes with central trajectory.

\[
X_i(out) = \sum_{j=1}^{4} R_{ij} X_j(in)
\]

...............(3)

Where:

\(R_{ij}\): The \(4 \times 4\) linear matrix, in the case of mid plane symmetry which leads to no coupling in \((x)\) and \((y)[12,14]\). This makes \(R\)-matrix simple to the following expression:

\[
R = \begin{bmatrix}
R_{11} & R_{12} & 0 & 0 \\
R_{21} & R_{22} & 0 & 0 \\
0 & 0 & R_{33} & R_{34} \\
0 & 0 & R_{43} & R_{44}
\end{bmatrix}
\]

...............(4)
That mean motion of particle in x plan does not depend on motion in y plan, so $R$-matrix may be reduced to two $2 \times 2$ sub-matrixes, the first matrix denoted by $R_x$, represents the horizontal plane, and the second matrix denoted by $R_y$ represents the vertical plane:

$$
R_x = \begin{bmatrix}
R_{11} & R_{12} \\
R_{21} & R_{22}
\end{bmatrix},
R_y = \begin{bmatrix}
R_{33} & R_{34} \\
R_{43} & R_{44}
\end{bmatrix}.
$$

Generally, $R$-matrix elements are the first order coefficients of the Taylor series expansion of the output coordinates in term of the input coordinate [15]. According to Liouville's theorem the determinant of the transfer matrix is united, when there is no charge in the particles energy. Also the product of several matrices, each of unit determinant, is united too [16].

A more convenient method is to represent a bundle of ray by six-dimensional phase space, ellipsoid, that contains all the points representing all the particle in the beam, that is, particles in the beam are assumed to occupy the volume enclosed by the ellipsoid, each point representing a possible ray[14]. The projection of the ellipsoid in any two dimensions is called phase ellipse. The shape and orientation of this phase space ellipse change as the beam moves along the beam line, but the energy-normalized area remains constant. This area is called the beam emittance, which is found to be equal to the determinant of phase space matrix. The general algebraic equation of an ellipse centered on the origin is:

$$
A x^2 + 2Bxx' + Cx'^2 = D 
$$

This equation of ellipse rotates with an angle, with x-axis while, the slope of beam ellipse depends on the magnitude of (B) which is constant in equation (6) In the case of (B=0) a upright phase space ellipse(waist)is obtained, which always defines a minimum beam size or so-called beam waist, the existence of the later depends on the nature of the beam phase ellipse [18]. In the present study the beam extracted from source is considered to be as upright ellipse with semi axes representing the maximum displacement and maximum divergence, which aligned parallel to the coordinate.

$$
X^T M_m X = D 
$$

Where:

$M_m$: is a positive definite symmetric matrix,

$X^T$: is the transpose of $X$.

The $M_m$ and $X$ matrices defined as:

$$
M_m = \begin{bmatrix}
A & B \\
B & C
\end{bmatrix},
X = \begin{bmatrix}
x \\
x'
\end{bmatrix}
$$

Based on Liouville's theorem the determinate of $M_m$-matrix is equal to unit, that means (D =1).

So equation becomes:

$$
X^T M_m X = 1 
$$
By considering \((\sigma)\) as the inverse of \(M_m\)–matrix

\[
\sigma = M_m^{-1} = \begin{bmatrix}
\sigma_{11} & \sigma_{12} \\
\sigma_{12} & \sigma_{22}
\end{bmatrix}
\]  

......................(9)

Where \((\sigma)\) is the phase space matrix.

Then the equation defining the ellipse may be written as:

\[
X^T \sigma^{-1} X = 1 \]  

...............(10)

Or equivalently:

\[
\sigma_{22} x^2 - 2\sigma_{12} xx' + \sigma_{11} x'^2 = \text{det} \sigma \]  

...............(11)

A typical output beams ellipse, characterized by \(\sigma_{11}, \sigma_{22}\) and \(\sigma_{12}\) is illustrated in Figure (2).

![Two-dimensional phase space ellipse based on the \(\sigma\) matrix][30]

Figure (2): Two-dimensional phase space ellipse based on the \(\sigma\) matrix[30].

The beam envelope, which represent the maximum spatial extent of ellipse \((x_{\max} = \sqrt{\sigma_{11}})\) also the maximum angular divergence of the beam within the phase ellipse is \((x_{\max}' = \sqrt{\sigma_{22}})\) the parameter \((\sigma_{12})\) defines the orientation of the ellipse relative to the \(x\) and \(x'\) axes[19].

In the present study \(\sigma\)-matrix has been used with transport notation. The particle of phase space ellipse has a normal Gaussian distribution [20]. Under the linear transformation, the normal distribution of particles in the ellipse is transformed into another normal distribution. To find the relation between \(\sigma\)-matrices in two positions of the beam line, i.e. \((\sigma(\text{out}))\) and \((\sigma(\text{in}))\), the equation of ellipse can be used at input location [21]:

By \(\sigma(\text{out}) = R\sigma(\text{in})R^T\)  

...............(12)

This equation represents the relation between \(\sigma\)-matrix in two locations of the beam line.
Drift Space Region

Drift space regions, sometime called free regions, exist at every beam transport line; these regions locate between any two elements along beam transport line. In the regions there is no force effect on charged particle, in other words, these regions are extended to effect the previous. Figure (3) indicates the particle path in the drift space regions where the particle position and divergence \((x_o, x_o')\) are transferred through drift space length \((S)\) and causing new position and divergence \((x_1, x_1')\). Sometimes the beam envelope forming a waist which occur when the beam transfer from converge to diverge, also the beam envelope depends on the length of the drift space as illustrated in figure (3)[5].

![Figure (3): Particle path in the drift space region.](image)

Since: \(\tan x_o' = x_o'\) (paraxial ray)

\[
x_1 = x_o + x_o' S
\]

...............(13)

Where:

\(x_1' = x_o'\)

From equations the drift space preserves the angle but changes the position by \((x_o' S)\).

In the matrix form [22]:

\[
\begin{bmatrix}
x_1 \\
x_1'
\end{bmatrix} = \begin{bmatrix}
1 & S \\
0 & 1
\end{bmatrix} \begin{bmatrix}
x_o \\
x_o'
\end{bmatrix}
\]

...............(14)

And

\[
\begin{bmatrix}
y_1 \\
y_1'
\end{bmatrix} = \begin{bmatrix}
1 & S \\
0 & 1
\end{bmatrix} \begin{bmatrix}
y_o \\
y_o'
\end{bmatrix}
\]

...............(15)
From the above one can note that the main factor affect at the drift space region is the length of this region \((S)\). The main characteristic of drift space is:
1) Limited by two elements of the system.
2) No force effect on the particle motion (each particle moves under its drift only).
3) The trajectories of individual particle become straight line.
4) The main parameter in drift space region is the length of this region is \((S)\) has space.

**Parameters and results**

Study has been done for the focusing properties of ion beam the when beam pass through double quadrupole triple lens. The calculation of the phase space ellipse and beam envelope of output beam for first order has been done, and the effect of changing the main parameters as (extraction region shape, length of quadrupole, the distance between lenses, voltage applying on lenses). The effect of these parameters is calculated by applying a computer program written in (MATLAB) language.

Computer program in (MATLAB) language were written to calculate the main parameters effect on the beam transport along the system consist from channel of triplet quadrupole lenses. The flow chart of this program indicate in figure (4). Input date using in the program used in the present study are: The dimension of extraction region\([x_0=10\, \text{mm}, \, y_0=10\, \text{mm}, \, x'=0.0006 \, \text{mrad}, \, y'=0.0006 \, \text{mrad}]\), the distance from ion source \((S_o)=80\, \text{mm}\), the length of first free region \((S_1)=200\, \text{mm}\), pole voltage \((V_a)=750\, \text{V}\), quadrupole lens length \((L_q)=100\, \text{mm}\),distance between lens \((L_d)=20\, \text{mm}\).
Figure (4): Flow chart of main program

Start

Initial conditions
$S_o, \; x_o, x'_o, y_o, y'_o$

Extraction region matrix
$\alpha_{x1} = x_o^2, \; \alpha_{x2} = x'_o^2,$
$\alpha_{y1} = y_o^2, \; \alpha_{y2} = y'_o^2$

Plot ellipse $(x_{max}, x'_{max})$
$(y_{max}, y'_{max})$

1$^{st}$ free region, equations (15,16)

Plot ellipse $(x_{max}, x'_{max})$
$(y_{max}, y'_{max})$

Input no. of lens

Triplet quadrupole lenses
$\alpha_{x1}, \; \alpha_{x2}, \; \alpha_{y1}, \; \alpha_{y2}, \; \sigma_{x1}, \; \sigma_{x2}, \; \sigma_{y1}, \; \sigma_{y2}$

Plot ellipse $(x_{max}, x'_{max})$
$(y_{max}, y'_{max})$

2$^{nd}$ free region, equations (15,16)

Plot ellipse $(x_{max}, x'_{max})$
$(y_{max}, y'_{max})$

End
Double Quadrupole Triplet Lens.

Double quadrupole triplet lens system consists of two triplet quadrupole lens separated by drift space region. Here, we try to study the behavior of charged particle beam passing through this system by studying the main parameters of each one of quadrupole triplet lens in addition to the effect of free drift space between them as in Figure (5).

![Diagram of Double Quadrupole Triplet Lens](image)

Figure (5): The double quadrupole triplet lens.

**Lens Length ($L_q$)**

Now take some different designs for double quadrupole triplet lens system by changing the lens length ($L_q$) with values ($L_q=100, 150\text{mm}$) and fixed all other factors value ($L_d=50\text{mm}, S_2=75\text{mm}, V_a=750\text{V}$). Figure (6) represents beam envelope along the system of horizontal plane the behavior of ion beam has diverge action in the first free region while converge action appear when beams enter the lenses system just at value ($L_q=150\text{mm}$), the lens action convert to diverge at the end of the system. That means the lens length for the system consists of two quadrupole triplet lens change its focusing properties by changing the lens length.
Figure (6): Beam envelope versus distance for different values of $L_q$ for horizontal plane for double quadrupole triplet lens.

Figure (7) indicates beam envelope for vertical plane when passing through double quadrupole triplet lens, the behavior of beam has diverge action in the first lens and converge action in second lens and this converge increasing by increasing the length of lens, from the same figure one could note that the focus of ion beam is best in ($L_q=150\text{mm}$) this result agrees with [3,16].

Figure (7): Beam envelope versus distance for different values of $L_q$ for vertical plane for double quadrupole triplet lens.

From the two last figures one could conclude that the behavior of beam passing through system of two quadrupole triplet lens do not have the same focusing properties for the horizontal and vertical plane.
Distance between Lenses ($L_d$)

To give a clear perspective of distance between lenses effect on the charged particle beam passing through the double quadrupole triplet lens, ($L_d$) will be changed with values (20,50mm) and fixed all factors that effect on ion beam behavior ($L_q=100$mm, $S_2=75$mm, $V_a=750$V). Figure (8) indicates the beam envelope of charged particle beam in horizontal plane. From this figure one may note there is no variation on behavior for ion beam in two different value of ($L_d=20,50$mm), there is no change in the beam envelope at the first lens. When the beam enters the second lens there is converge action appearing.

![Figure (8): Beam envelope versus distance for different values of $L_d$ for horizontal plane for double quadrupole triplet lens.](image)

Figure (9) shows nearly the same behavior of beam envelope which appears in vertical plane as in horizontal plane, that mean the double quadrupole triplet lens has the same focus properties for both horizontal and vertical plane.

![Figure (9): Beam envelope versus distance for different values of $L_d$ for horizontal plane for double quadrupole triplet lens.](image)
Second Free Region ($S_2$)

Here, the effect of changing the length of second free region ($S_2=50, 75, 100$ mm) with fixed all other factors ($L_q=100$ mm, $L_d=50$ mm, $V_a=750$ V) will be studied.

Figure (10) shows beam envelope of charged particle beam in horizontal plane, the effect of second free region is convergence for all beams with the same behavior at the different value of ($S_2=75, 50, 100$ mm) this converge increases by increasing ($S_2$). The effect of the second free region located between the double quadrupole triplet lenses as converging action refers to the action of the double quadrupole triplet as convergence lens.

![Figure (10): Beam envelope versus distance for different values of $S_2$ for horizontal plane for double quadrupole triplet lens.](image1)

Figure (11) shows the beam envelope of changed particle beam passing through a double quadruple triplet lens. For vertical plane, the beam envelope has nearly the same results as obtained for horizontal plane.

![Figure (11): Beam envelope versus distance for different values of $S_2$ for vertical plane for double quadrupole triplet lens.](image2)
Lens Voltage ($V_a$)

The voltage applied between pole of lenses effect on the charged particle beam to be parallel to the axis of lens that is, converge and forced this desirable action. Here, we take different values ($V_a=750,1250V$) with fixing other parameters that effect in double quadrupole triplet lens design ($L_q=100\text{mm}$, $L_d=50\text{mm}$, $S_2=75\text{mm}$).

Figure (12) illustrates the beam envelope of charged particle beam in horizontal plane. There is converge action at the end of this system, that means the beam exit from first lens has good focus action when entering the second lens containing with converge action, that means decreasing the value voltage leads to increasing focus beam, this converge seems nearly as straight line at voltage (1250V).

![Figure (12): Beam envelope versus distance for different values of $V_a$ for horizontal plane for double quadrupole triplet lens.](image1)

![Figure (13): Beam envelope versus distance for different values of $V_a$ for vertical plane for double quadrupole triplet lens.](image2)
Figure (13) shows beam envelope of charged particle beam in vertical plane. The behavior of beam envelope is the same for the different value of voltage as converge, also this converge increases by increasing the voltage. Figure (14) shows the rotation and the shape of phase space ellipse of ion beam passing through double quadrupole triplet lens. When comparing this system with that for single triplet lens, we find that the converge action increases by adding the second quadruple triplet lens along the system, the extraction region is up right phase space ellipse, in the first free region is already be diverge in the phase space ellipse. When the ion beam passing through the system, the phase space ellipse is changed so that there is converge action appears in both horizontal and vertical plane.

![Horizontal plane](image)

![Vertical plane](image)

Figure (14): The phase space ellipse in double quadrupole triplet lens for (a) Horizontal plane and (b) Vertical plane.
Conclusions

From the results of this research, it can be concluded the following:
1) The ion beam in field free regions depends strongly on the length of these regions, in addition to the focal length of the pervious element.
2) For double triplet quadrupole lens:
   a. Increasing of \(L_q, V_a\) there is focusing action in vertical plane, but focusing action in horizontal decreasing by increasing \(L_q, V_a\).
   b. The focus action increasing by increasing \((L_d, S_2)\) in both plane horizontal and vertical plane.
   c. From result we find the migration ratio for horizontal plan nearly equal %5 while it equal %2 for vertical plane, that means the focusing properties for vertical plane is better than that horizontal plane.

References