Teaching Average to Secondary School Students

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Abstract

This research is mainly concerned with novel strategies to teach average. It looks into the relevance understanding of some properties of the average (the arithmetic mean) to students' general education in order to access its importance and place in curriculum design and to assess the effects of different teaching strategies, materials and media of presentation on student's understanding of that concept.

Teaching materials were designed and put into practice with a group of students.
One of the two groups followed the teaching materials that was taught by us, using innovatory and various other teaching strategies. The other group was taught it by their usual teacher using standard teaching methods. We compared the learning outcome of both methods, and evaluated teaching methods and materials.

Keywords*- statistics, average (the arithmetic mean), curriculum, teaching strategies.*

Introduction

The teaching of statistics is currently increasing substantially in many countries, due to its widely recognised place in the general education of citizens.
The mean (average) is a key concept in the understanding of the 'real world' and further statistical concepts. Research is vital to inform effective teaching to overcome difficulties.

Often, the word average is used when it is clear from the context that it is the arithmetic mean which is being referred to. Through this research, the use of the word average where it is quoted refers to the arithmetic mean.

Six periods of 75 minutes were allocated to teach the materials, which consisted of lesson one to lesson five and the final one for an assessment. Teaching materials were designed and 'trailed' with a group of year eight, from a comprehensive school (11-18) in the UK. For a complete description of each lesson (see appendix 1).

One of the two parallel top groups trailed the teaching materials. This set was taught by us, using innovatory and various other teaching strategies. The other parallel group was taught it by their usual teacher using standard teaching methods.
We compared the learning outcome of both methods, and evaluated teaching methods and materials. We shall call the first set 'group 1' and the second set 'group2'.
To determine the development of children's understanding of some properties of the arithmetic mean and to assess the effects of different teaching methods on children's understanding of that concept, nine properties have been assessed in a random order.

Emphasis was placed on the use of real data which is relevant and of interest to the pupils. Various methods of assessment were used to compare the pupils' understanding of the average (arithmetic mean) with that of a parallel group who have been taught using current teaching materials. We also discussed ways in which the teaching materials and their implications could be improved and whether they have been effective.

The main purposes of this research is to determine the development of student's understanding of some properties of the arithmetic mean and to assess the effects of different teaching methods, materials and media of presentation on student's understanding of that concept. The following issues will be investigated:

1. How pupils develop mathematical definitions of the mean. "A well developed notion of representativeness should include an understanding of the mean and how it works"

2. The connections that pupils make – or fail to make – between the arithmetic Mean which is used in real life and that which they are learning about in mathematics class.

3. To look at the way that appropriate teaching can ensure that concept is understand.

4. Knowing something about difficult of the properties would help to establish the best time to introduce the properties of the mean into the classroom.

The Average (Arithmetic Mean)

Education today prepares student to live in a world in which it will be vital that they can collect, present and introduce information in many different forms. Sometimes this information will be deliberately designed to mislead. The arithmetic mean is usually taught at secondary school as well as being used in subjects such as geography and Sciences. Although the simplicity of the technical side of its computation makes the arithmetic mean appear to be so straightforward and simple, researches indicate a lack of understanding of the mean, even after many years of formal schooling [20], [10], and [19]. These studies have been carried out in experimental situations and are not concerned with normal classroom practice. The arithmetic mean usually referred to as the average is possibly the most commonly encountered statistics in everyday contexts. The media and press regularly refer to average temperature, income and spending, and statistics arguments are used in advertising many products.

Learning about average mean) is probably the first time a pupil encounters a number that expresses a relationship among particular numbers. Further, the average (arithmetic mean) is a fundamental concept in the understanding of many other aspects of statistics course.

Although the simplicity of the technical side of the computation makes the arithmetic mean appear to be so straightforward and simple this is not the case."But the university students have difficulties solving weighted average problems and that when they solve these problems incorrectly; they do so as if they were solving simple average problems.

The first aim is to analyse the concept of the arithmetic mean into its properties. Goodchild (1988) identified three types of meaning for the mean, i.e. the mean as [8]:

"1. A representative number
2. A measure of location
3. An expected value"

Nine fundamental properties were selected for investigation: These properties tap three aspects of the concept:

1. Statistical, the properties 1,3,4, and 8 are basic to arithmetic average as a mathematical function.
2. Abstract, the properties 2 and 5. These properties allow non observed values.

It is representative of a group of individual values. This is the essence of the properties 6, 7, and 9

For each one of these properties we used different tasks, varying material used in the teaching and medium of presentation, investigating, concrete and numerical. The properties were tested by tasks that were presented in form of tests, each property was tested twice, using text and numerical form.

Teaching Strategies

Statistics has received, to date, less attention than other mathematical topics. Most research has been carried out in experimental situations and very little has been concerned with normal classroom practice, and much has been with very young or 18+ college students rather than with the 11-16 age range.

Statistics is different from mathematics, but it is usually taught as part of mathematics by teachers who often have no tainting in statistics and who teach mathematics in very traditional ways. Statistic is therefore often taught in the same way as mathematics, typically with an introduction to new work, example to provide drill, and, practice and perhaps at the end, a few more examples that involve applications of the work.

Research indicates that, many students' and teachers' understanding of the mean is as an incompletely developed algorithm which only deals with simple arithmetic mean problems, and that any understanding is only superficial. Batanero et al (1994) find that "knowledge of computational rule not only does not imply any deep understanding of the underlying concept, but may actually inhibit the acquisition of a more complete conceptual knowledge.[2]" They summarise, learning a formula is a poor substitute for gaining an understanding of the basic underlying concept.

Although the teaching of statistics is often justified be referring to the need to prepare pupils for the demand of an 'information society', teachers all too often use in instruction only tasks requiring pupils to compute averages. This may not be sufficient to develop pupils' statistical literacy skills and to make sure they can reasonably interpret statistical arguments encountered in media, or on the job.

Several teaching strategies were used to teach mean and its properties. The following are by Garfield, J. et al. (1988), for overcoming difficulties in learning statistics:

1. Introduce the mean through activities and simulations, not abstractions [7].

2. Try to stimulate the students the feeling that mathematics relates usefully to reality and is not just symbols, rules and conventions.

3. Use visual illustration and emphasise exploratory data methods.

4. Point out to pupils common misuses of statistics (say, in news and advertisements).

5. Use strategies to improve pupils' rational concepts before approaching proportional reasoning.
6. Use computer in different ways to aid in the teaching of the properties of the mean."

Other strategies were used, and some of them recommended by Goodchild (1988) [8]. Pupils have plenty of experience of working with real data which they have generated themselves from random processes. Pupils have been given the opportunity to consider and discuss, in the classroom, what is meant by the word 'average' when it occurs in a variety of contexts.

We encouraged them to look out for its use in newspaper articles, TV news, advertisements, supermarket and other places. Pupils were asked to generate a set of data, then work in a group of four to describe and interpret the data.

Liz Gibson et al (2006) [11] Discussed teaching statistics using a problem solving approach at undergraduate level. While Gal (1995) [5] argues that if students are only asked to calculate the average (mean) of a small set of numbers little is revealed about students' understanding of the average as a tool for solving data-based problems. " He stated that students should ask to compute and estimate averages from several different contexts (e.g. a series of numbers, in a table, scatter plot or histogram, or grouped figures).

We used small groups, so that able pupils could join groups of more able pupils. We used different groups for different activities, so that pupils of all abilities could learn to live together and value each other's specific contribution and achievements, and also to give the opportunity to each student in the class to meet and work with all the student in his class. The grouping system of ability groups is flexible and can be used to cater for a top set group of varying ability. Differentiation can occur in groups, so that more able pupils are less likely to be labelled out as being 'different'. Differentiation by outcome was suitable for discussion activities, as the outcome was left to the pupils, with clear planning by the teacher.

Assessment

"Teacher assessment lies at the heart of the learning progress in that new learning must match to what a pupils already knows and can do" (Dearing, 1993) [4].

Assessment is very important to identify the learning needs of pupils and provide a reliable means of evaluating a pupil's performance against the wider standard of attainment, and to monitor pupils' learning and long term trends of understanding.

"If there are no examinations, standards will fall; pupils will have nothing to aim for and no incentive to work hard". (Hargreaves, 1982) [9]. Thienhuong Hoang (2007) [16] examined several instructional practices and studied the relationships between classroom teaching practices and mathematics achievement .Computing the mean does nothing to test the students' understanding of the functional .characteristics of the mean as representative statistics or model of the given data, and so it is a bad task. It can also be criticised because it is based on a small sample of meaningless .

figures. The size of the data set in a written examination question is often constrained by examination conditions and the availability of relevant technical resources.

For this reason pupils' knowledge and understanding were assessed by more than one method

1. Through their contribution to class discussion and written comments.
2. By the formal marking of class/ homework.
3. From the grades they achieved in the formal test.
4. From project performance.

It is hoped that this wide spread of assessments styles will enable all pupils to have an opportunity of reaching their full potential, bearing in mind their previous knowledge and their ability.

Results

Table 1 shows percentages of student achieving correct answer for the same properties for group 1 and 2

Table (1) Percentage of students achieving correct answers for each property by group

Property number	Types of meaning for the mean	Percentage of correct answer	
		Group1	Group2
1	Statistical	98%	88%
2	Abstract	85%	95%
3	Statistical	45%	28%
4	Statistical	100%	85%
5	Abstract	60%	25%
6	Representative	63%	70%
7	Representative	35%	18%
8	Statistical	8%	3%
9	Representative	55%	50%

Properties 1, 3, 4 and 8 as 'mathematical function' deal with the idea of the data distribution and its relationship to the mean, which is similar to Goodchild's research and indicates that the mean is a measure of location. We found that very few students offered correct judgement for the task measuring property 8, nearly 88% judged correctly on the task measuring property 1 and 100% for property 4, and 45% for property 3.

Alternatively, properties 6, 7 and 9 address a representative interpretive of the mean.

Approximately 63%, 35% and 55% and correctly solved the task measuring properties 6, 7 and 9 respectively.

Goodchild's research indicates that the mean as-location was a commonly understood interpretation of the mean, while a mean as a representative measure of interpretation was proved more difficult for person to grasp. While properties 3 and 9 perhaps to be almost twice as difficult to understand as did properties 5 and 6. 100% of the pupils in group 1 and 85% in group 2 fully understood the way in which the data on either side of the mean must balance.

Properties 1 and 4 were relatively easy to understand, while most pupils found properties 6, 7 and 9 relatively difficult. As Pollatsek et al. (1981) [16], and Mevarech (1983) argue, most pupils find it relatively easy to understand the mean as computational construct (property 1 and 4), and relatively difficult to understand the mean as representative value [13]. All properties addressed the mean as a mathematical function were relatively easily understood by pupils except property 8, which addresses the mean as expected values.

It is interesting to note that property 8 (the mean equal to the expected value) was one of the more difficult properties for the student to understand. 15% of group 1 and 5% of group 2 correctly solved the tasks exhibiting this property.

In general the statistical aspect of the mean was easily understood by the majority of subjects, while the abstract and representative aspects prove more difficult to master, and, overall, group 1 understood the properties better than group 2.

It is possible that the judgements on the properties were at two levels of difficulty, firstly because the question measuring them were formulated at two levels of complexity. Secondly, the differences in the difficulty of the questions presented in numerical format were significantly easier to solve than were question presented in context,It is not straight forward to do valid research on the research on teaching methods and materials, because it can be difficult to control other factors that might affect the outcomes.

We compared one year sets. The pupils in both classes have the same prior experience of mean and followed the same syllabuses and teaching in other subjects. As recommended by Hawkins et al. (1992) "A better design would be one with two classes of a similar mix of students taught in the same academic year where one method is used with one class and a different one with the other class [10]".

The differences between our innovatory and conventional teaching methods were very clear, as pupils in group 1 achieved significantly higher scores at the end of the course than those receiving conventional teaching (group 2). However, a similar controlled study comparing teaching methods was undertaken at the University of Essex with students on a first year introductory probability and statistics course (Hawkins et al. (1992, based on Harding et al. 1982). Four matched groups were given a course comprising either lecture only, or lecture followed by tape-slides, or tape-slides followed by lectures or tape-slides only. No differences were found between the groups in examination results, attitudes and so on but, costed over a five year period, the lectures alone were found to be slightly cheaper than the other methods [10]. Thienhuong Hoang (2007) identified significant relationships between classroom teaching practices and mathematics achievement[21].

Care has to be taken in any interpretation as it is difficult to isolate the factors that caused the different outcomes. We may have been more committed to our pupils' success than the teacher using conventional teaching discuss their lessons with one another, the method.To isolate the effect of contamination because the pupils might two classes are taught by two different teachers. This is supported by Hawkins et al. (1992) [10].

Mokros et al. (1995) from his work with student as well as adults concludes that the "arithmetic mean is a mathematical object of unappreciated complexity (belied by the 'simple' algorithm for finding it) and that it should only be introduced relatively late in the middle grades, well after students have developed a strong foundation of the idea of representativeness.[14]"

Finally, with the increasing use of technology in teaching statistics, research is needed on the appropriate ways to use computers, calculators, multimedia, and Internet resources, to determine their effect on pupils learning and to suggest the ways in which instruction should be changed to best utilise technology. Information related to the learning processes is essential to curricular design.

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Appendix 1

Lesson 1: The Development of Statistical literacy The overall objectives of this lesson to focus on how students think about the mean as a particular mathematical definition and relationship, and hence to highlight ambiguity inherent in the usage of the term 'average' and the range of possible misinterpretation.

The lesson started with the television advertisement for Peugeot cars. This advertisement include statements as do their newspaper advertisement **" The average person doesn't notice car ads" and "There is no such thing as an average person" .**

The teacher can use any clips from the media and these should be adapted to students' background and area of interest. Students in group of four need to discuss the meaning of average in the advertisement and within the context of four sentences in which the world average is embedded, then compare their interpretation.

Students then needed to generate data set by playing 'telepathy games', which is guessing any number between 1 and 10 then calculate and compare the mean using a scientific calculator with and without using the average button. Finally , students asked to plot the data and the mean on the line number, then describe and interpret the mean with respect to the data if possible.

Lesson 2: Using Formula on the Excel Spreadsheet

The main purpose of this lesson to create data set from average (arithmetic mean) using Excel to manipulate the data. The formulae are used to illustrate the first and the fourth properties of the mean.

Students need to demonstrate what they know about averages by constructing several data sets that could reflect a particular average using Formula and graphical display on Excel. The graphics used are either bar of histogram to shows the distribution of the data and indicate the position of the mean for the data.

Lesson 3: Relevance of Arithmetic Mean to Real Life

The aim of the presentation was to highlight the importance use of the mean in real life. A discussion took place about Rothampsted Station , and the work of a statistician. A lively discussion followed about the importance role of the mean to summarise the data. The discussion was extended to talk about research in agricultural, crops, pests, soil insects and the environment.

Lesson 4: Enrichment and Consolidation to Previous Lessons

The idea behind this lesson is for the first half an hour, to review of all the properties that have done in the last three lessons, and to introduce property 3 and 5(see table 1).

Although most students know the procedure for finding the mean of a set of values, the mathematical relationship itself remain opaque. Teacher need to place emphasis on the relationship of the mean by refereeing to the data set.

For the last half an hour, the teacher may introduce property 8 through investigation, using matchboxes or ears cotton swabs as it is safer to use with young students.

The teacher provided students with matchboxes or cotton swabs upon which was printed ‘average content 35 matches or swabs’. The students were asked to empty the contents of 28

boxes and to count the number of matches in each box, the result was: 35, 40, 38,..., 38. They were asked what they expected the average in the 29^{th} box to be?

The activity can promote discussion about whether or not the average contents of a match box is actually what you expect to find in the box, hence average (arithmetic mean) is equal to the expected value.

Lesson 5: Frequency Table, Calculated and estimated Mean of Grouped Data

This lesson addresses how students construct and interpret representativeness, when asked to describe a real data set (property 7 and 9 in Table 1).

Students may calculate or estimate the mean from grouped data, frequency table and chart, then discuss the nature of the data from which an average was generated. Students can compare two data sets, of different sizes using the range as a dispersion measure.

The teacher may encourage students to hypothesise about the range of the data from which a reported average was derived, and more able students may perhaps be encouraged to comments on average, and consider whether the reported average represents the data, or is a biased due to outliers.

Lesson 6: Assessment

ملخص البحث

نزيرة خليل داخل

– قسم الرياضيات – كلية الرياضيات وعلوم الحاسوب –

جامعة الكوفة

يهتم هذا البحث بدراسة أساليب حديثة في تدريس الأحصاء . حيث يبحث في أهمية فهم بعض صفات الوسط الحسابي في تعليم الطلاب بصورة عامة ، وتقييم تاثير بعض أساليب التدريس المختافة على فهم الطلاب الى هذا الفكرة.
منهج دراسي صمم وطبق على مجموعة من الطلاب ، قمنا بتدريس المجموعة الأولى باستعمال طرق حديثة وبعض الطرق الأخرى. أما المجموعة الثانية فقد درست من قبل مدرسهم باستعمال الطريقة المعتادة في التدريس. تمت المقارنة بين نتائج الطريقتين وكذلك تقييم أساليب التدريس المستعملة والمنهج الدراسي.

مفتاح الكلمات– الأحصاء، المتوسط الحسابي، المنهج، أساليب تدريس