Studying the Electron Energy Distribution Function (EEDF) and Electron Transport Coefficients in SF$_6$ – He Gas Mixtures by Solving the Boltzmann Equation

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Received 22/5/2016  
Accepted 5/10/2016

Abstract:  
The Boltzmann equation has been solved using (EEDF) package for a pure sulfur hexafluoride (SF$_6$) gas and its mixtures with buffer Helium (He) gas to study the electron energy distribution function EEDF and then the corresponding transport coefficients for various ratios of SF$_6$ and the mixtures. The calculations are graphically represented and discussed for the sake of comparison between the various mixtures. It is found that the various SF$_6$ – He content mixtures have a considerable effect on EEDF and the transport coefficients of the mixtures.

Key words: (EEDF) Program, EEDF, Boltzmann Equation, SF$_6$ – He Mixture, Transport Coefficients.

Introduction:  
Because of its importance for computing reaction rates, the EEDF has a fundamental role in plasma modeling [1,2]. The EEDF assumes that elastic collisions are dominating; therefore the effect of inelastic collisions (ionization or excitation) on the distribution function is insignificant [3]. To describe the EEDF several functions are mentioned such as Maxwellian, Druyvesteyn, generalized Maxwellian – Druyvesteyn, and solution of Boltzmann equation [4,5]. There are many computational resources and numerical techniques used to find the transport properties [6]. The latter can be derived from EEDF, thus the choice of EEDF would affect the results of plasma modeling. The (EEDF) software package gives results of the kinetic and transport coefficients of plasma in the mixture of gases by numerically solving the Boltzmann equation of EEDF in plasma under an electric field with the two-term approximation [7]. It is known that SF$_6$ gas has been vastly applied as a medium for insulation in high – voltage equipment [8]. At atmospheric pressure, SF$_6$ has premium properties; the breakdown strength of the gas is higher than air. Due to these properties, it is of great use in the field of the electrical industry. The SF$_6$ mixtures with some inert gases are very significant in
practical applications like electrical discharge [9]. Therefore, there is a need to have accurate data about the electrical discharge for electrical engineering applications. In this paper we present calculated data for a mixture of a pure SF$_6$ and its mixtures with He under a steady state electric field by solving the Boltzmann equation with two-term approximation utilizing (EEDF) software package. We study the effects of various percentages of mixtures on the EEDF and electron transport coefficients.

Method of Numerical Solution in the Code

In this section, the following summaries are extracted from the program manual [7]. By solving Boltzmann equation one can obtain the growth of the distribution function in a six – dimensional phase space. The Boltzmann equation involved here is [7] 

$$\frac{d}{dt} f_0(u) = I_E(u) + I_{el}(u) + I_{in}(u) + I_{ee}(u)$$  

(1)

where $f_0(u)$ is the isotropic part of the distribution function $u$ is electron energy, the term $I_E(u)$ expresses heating of the electrons in the electric field and the terms $I_{el}(u)$, $I_{in}(u)$ and $I_{ee}(u)$ describe elastic, inelastic and electron – electron collisions, respectively. The $dn_e/dt$ is a term for the conservation of the electron density given by 

$$dn_e/dt = n_e (\bar{v}_i - \nu_{att} - \nu_{rec})$$  

(2)

where $\bar{v}_i$, $\nu_{att}$, $\nu_{rec}$ are the frequencies of ionization, attachment and recombination, respectively. They are expressed in terms of suitable integrals of $f_0(u)$. The EEDF code solves the Boltzmann equation numerically using an iterative method. To calculate $(dn_e/dt)^n$ value, the code solves Boltzmann equation with equation (2) which is then substituted in the equation (1). As for any code that uses iteration procedure, the number of iterations is restricted by the value $M_{max}$. Then the iteration process is ended at some criterion and function $f_0^n$ is considered in looking for a solution. When the distribution function is set, some characteristics of the plasma are calculated. The following equations are used in the program and in the calculations of this research

The mean electron energy [7] 

$$\bar{u} = \int_0^\infty u^{3/2} f_0(u) du$$  

(3)

The electron mobility [7] 

$$\mu_e = -\frac{1}{3} \int_0^\infty \frac{u^{3/2} \nu_{f_0}}{v_{m(u)}} \frac{df_0}{du}$$  

(4)

where $v_{m}$ represents the electron momentum – transfer collision frequency. The electron diffusion coefficient is [7] 

$$D_e = \frac{1}{3} \int_0^\infty \frac{u^{3/2}}{v_{m(u)}} f_0 du$$  

(5)

So, the drift velocity is 

$$W_e = -\frac{E}{3} \int_0^\infty \frac{u^{3/2}}{v_{m(u)}} \frac{df_0}{du}$$  

(6)

and the characteristics energy is [7] 

$$\mu_{ch} = e \frac{D_e}{\mu_e}$$  

(7)

Results and Discussion:

Figure (1a) shows the mean electron energy versus SF$_6$ content in SF$_6$ – He mixtures at $E/N = 200$ Td, where $E$ is the electric field and $N$ is the density of the gas mixture. Clearly, the mean electron energy of SF$_6$ – He and mixtures sharply decreases with increasing SF$_6$ content. Such behavior can be attributed to the variation of EEDF for both mixtures due to the difference of the ionization potential of both gases [10]. The influence of the electron distribution function EEDF versus the mean electron energy for different ratios of electric field to the SF$_6$ density $(E/N)$ is shown in Figure (1b). It is obvious that the EEDF is strongly affected by changing the parameter $E/N$. The higher the values of $E/N$ at lower mean electron energy, is the lower the EEDF near the origin of energy domain. On the other hand, as the values of mean electron energy

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increases the EEDF develops as the $E/N$ increases. The higher values of $E/N$ lead to the extension of the EEDF curves towards higher energy tail. This behavior can be attributed to the electric field that heats up the electrons and thus the energy of cold electrons increases [11]. Subsequently, the mean electron energy depends on the ratio of $E/N$ as well as on the electron transport coefficients.

Regarding to entire mixtures of $\text{SF}_6$, it is clear that the EEDF of the electron hardly differs near energy space, while it significantly shifts to the right as the $\text{SF}_6$ content is decreased at higher mean electron energy as shown in Figures (2). The higher concentration of $\text{SF}_6$ in He gas increases the EEDF at the proximity of the origin of energy domain. This leads to decrease the mean electron energy as shown in Figure (1a). All EEDF curves approximately have common range of mean electron energy ($12 - 14 \text{ eV}$) where they meet. As the mean electron energy increases, the EEDFs tend to distribute depending on the concentration of both He and $\text{SF}_6$ mixtures. Obviously, the EEDF versus mean electron energy of pure $\text{SF}_6$ gas has lower profile for fixing others parameters. As the concentration of He increases in the mixtures composition, the EEDFs are raised. In other words, the properties of both gases in the mixtures come to affect the EEDF profiles. Thus, higher EEDF profile in Figure (2) is dominated by the presence of higher percentage concentration of He gas in the mixture. The behavior of EEDF versus mean electron energy of $\text{SF}_6$ – He mixtures are quite analogy to that of $\text{SF}_6$ – $\text{CHF}_3$ and $\text{SF}_6$ – $\text{CF}_4$ mixtures gas [12].

Figure (3a) shows the EEDF versus the mean electron energy for $\text{SF}_6$ (75%) + He (25%) mixture at various values of $E/N$. Obviously, the effect of increasing $E/N$ leads to shifting the EEDF to the right towards higher mean electron energy. In other words, for constant mean electron energy, the higher reduced electric field tends to rise up the profile of EEDFs. This can be attributed to the gain of more energy by the electrons between collisions [13]. The effect of $E/N$ is just the energy that is imported to electrons. At the last stage of the EEDF the curve tail falls rapidly due to reduce of the mean electron energy by various inelastic collisions [14]. Further, the EEDF does not now differ at near energy space. Figure (3b) shows again the EEDF versus the mean electron energy but now for $\text{SF}_6$ (25%) – He (75%) mixture at various values of $E/N$. The influence of increasing $E/N$ is clear in both Figures (3a and 3b) for the pure $\text{SF}_6$ and its mixture with He. By comparing the two parts of Figure (3a – 3b) one finds that the influence of decreasing the $\text{SF}_6$ is to raise and shift the EEDF profile towards increasing the mean electron energy. The tail of the EEDF for lower concentration of $\text{SF}_6$ in the $\text{SF}_6$ (25%) – He (75%) mixture takes place at 35.6 eV of mean electron energy (Fig. 3b), while the higher percentage concentration of $\text{SF}_6$ in the mixture leads to the curve tail to be located at 24.2 eV of the mean electron energy (Fig. 3a). The entire curves for all percentage concentrations of the $\text{SF}_6$ - He mixtures are located between the extreme EEDF of pure $\text{SF}_6$ and the extreme EEDF of pure He on the other side. In other words, different percentages of buffer gas lead to different behaviors of EEDF versus mean electron energy.

Having known the effect of increasing the mean electron energy on EEDF, the effect of reduced field strength $E/N$ on mean electron energy for different concentrations of He in $\text{SF}_6$ are plotted in Figure (4a). It is obvious that as $E/N$ increases the mean electron energy is nonlinearly increased. This is in good agreement with recognized known fact about the relation between
them [14]. The curves are raised as the percentage concentration of buffer gas increases.

The characteristic energy for the mentioned mixtures versus $E/N$ is shown in Figure (4b). The characteristic energy increases gradually as $E/N$ increases in nonlinear mode. Increasing the percentage concentration of He leads to a jump in the profile up, however, lower concentration of SF$_6$ gives distinguished curve at higher $E/N$ totally differs from higher concentration.

Figure (5a) shows the dependence of electron diffusion coefficient on $E/N$ for various percentage concentrations of SF$_6$ and buffer He. For pure SF$_6$, as $E/N$ increases the electron diffusion coefficient slowly increases until the dependence becomes linear. Mixing He with SF$_6$ yields upper curve depending on the concentration of both gases. Lower percentage of SF$_6$ gives upper profile of the relation. Clearly, as the percentage concentration of SF$_6$ decreases at higher $E/N$, the gap between adjacent curves is increased.

When electric field is applied $E$, the electrons move anti-parallel while positive ions move parallel to the direction of $E$ [15]. This gives a drift velocity. Figure (5b) expresses the relation between drift velocity and $E/N$ for pure SF$_6$ and some concentrations with He. The drift velocity versus $E/N$ is gradually increased until the dependence becomes linear for pure SF$_6$ and some higher concentrations of it with He, although it seems that at lower percentage of SF$_6$ in He at higher $E/N$, the curves bend down. Such a behavior is in a good agreement with the calculated electron drift velocity as function of $E/N$ by Mote Carlo method for Nitrogen [16]. Further, for comparison with other work, the drift velocity versus $E/N$ of SF$_6$ – He mixtures have similar variations as for SF$_6$ – N$_2$ and SF$_6$ – Xe mixtures [8].

Conclusions:

By solving the steady state Boltzmann equation, the electron energy distribution function and transport coefficients, are calculated for SF$_6$ – He for different mixtures. Results show that the mean electron energy of pure SF$_6$ and mixtures with He decrease exponentially with increasing SF$_6$ content. Furthermore, there is an intense dependence of EEDF, noticed as $E/N$ varies for various concentrations of SF$_6$ content with He. As $E/N$ increases, the EEDF tends to shift towards higher mean electron energy. The effect of decreasing SF$_6$ concentration in the mixture is to raise the EEDF profile versus $E/N$. Also, a nonlinearly increased dependence is observed between the mean electron energy with $E/N$. Similar behavior is noticed for the characteristic energy versus $E/N$ for the various mixtures. Although there is a linear relationship between electron diffusion coefficient and $E/N$ for various percentage concentrations of SF$_6$ and buffer He, it is not linear for lower values of $E/N$. In addition to that, the lower concentration of SF$_6$ in the gas mixture leads to bend down of the curve of drift velocity versus $E/N$. 
Fig. (1a): Mean Electron Energy versus SF₆ Content for SF₆-He Mixture. (1b): EEDF Versus Mean Electron Energy for Various reduced Electric Fields in SF₆, Pressure is 760 torr, the Electron Concentration is $1 \times 10^{16}$ cm$^{-3}$ as used in the calculations.

Fig. 2: Electron Energy Distribution Functions of SF₆-He Mixtures, in the Calculations the Electron Concentration is $1 \times 10^{16}$ cm$^{-3}$, the Pressure is 760 torr, and $E/N = 200$ Td ($1 \text{Td} = 10^{-21}$ Vm$^2$).

Fig. (3a): Electron Energy Distribution Functions Versus Mean Electron Energy of SF₆ (75%) –He (25%) Mixture for Different $E/N$. Figure (3b): The Electron Energy Distribution Function Versus Mean Electron Energy of SF₆ (25%) – He (75%) Mixture, the Electron Concentration is $1 \times 10^{16}$ cm$^{-3}$, and the Pressure is 760 torr, as Used in the Calculations.
Fig. (4a): Mean Electron Energy Versus Reduced Field of SF₆ and its Mixture with He. Figure (4b): The Characteristic Energy Versus Reduced Field of SF₆ and its Mixture with He, the Electron Concentration is $1 \times 10^{16} \text{cm}^{-3}$, and the Pressure is 760 torr, as Used in the calculations.

Fig. (5a): Electron Diffusion Coefficient Versus Reduced Field of SF₆ and its Mixture with He. Figure (5b): The Drift Velocity Versus Reduced Field of SF₆ and its Mixture with He, the Electron Concentration is $1 \times 10^{16} \text{cm}^{-3}$, and the Pressure is 760 torr, as Used in the Calculations.

References:


software package for calculations of the electron energy distribution functions in gas mixtures.  


دراسة دالة توزيع طاقة الإلكترونات ومعاملات النقل (الانتقال الإلكترونية) في سادس فلوريد الكبريت المخلوط بالهيليوم عن طريق حل معادلة بولتزمان

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الخلاصة:

تم حل معادلة بولتزمان باستخدام برنامج (EEDF) لغاز سادس فلوريد الكبريت الم نقى (SF₆) وخلط من غاز الهيليوم لدراسة دالة توزيع طاقة الإلكترونات (الانتقال) المقابلة EEDF ومن ثم معاملات النقل (الانتقال) للغازين SF₆–He. تم تمثيل الحسابات بيانيا ومناقشتها من أجل تحليلها بين مختلف مخاليط الغازين SF₆–He وحسب النسبة المئوية لكلهما. بينت النتائج أن مختلف مخاليط محتوى غاز SF₆ لها تأثير كبير على دالة توزيع طاقة الإلكترونات والانتقال المخلوط EEDF ومعاملات النقل للمرحلة منSF₆–He وخلط غاز SF₆–He. 

الكلمات المفتاحية: برنامج (EEDF)، دالة توزيع طاقة الإلكترونات EEDF ، معادلة بولتزمان، حليلومSF₆–He ، سادس فلوريد الكبريت، هيليوم، معاملات النقل (الانتقال).