3a\ quasi $M$-$\theta$-ii-continuous functions in bi-Supra topological spaces

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Abstract.

In this paper, we introduce a new definition in bi-supra topological space, called $M$-$\theta$-ii-open and via this definition, we introduce a new types of functions called quasi $M$-$\theta$-ii-continuous functions which unifies some weak forms of quasi $\theta$-ii-continuous functions in bi-supra topological spaces and investigate their properties.

Key Words and Phrases   bi-supra topological space , $M$-$\theta$-ii-open set, $M$-$\theta$-ii-closed set, quasi $M$-$\theta$-ii-continuous.
في الفضاءات ثنائية التبولوجية -ii M- θ حول الدوال شبه الضعيفة من النوع-ii M- θ

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1. Introduction

In 1987, Noiri and Popa introduced M-open set and M-closed set and investigated a new class of functions called quasi $\theta$-continuous functions[8], Maghrabi and Juhani introduced M-continuous function, pre-M-open function and pre-M-closed function [3]. In this paper by using M-$\theta$-ii-open sets in [12] denoted another sets is M-$\theta$-i-open in bi-supra topological spaces every M-$\theta$-ii-open (resp. M-$\theta$-ii-closed) set is M-$\theta$-i-open (resp. M-$\theta$-i-open) sets but the converse is not true[10],

the quasi M-$\theta$-ii-continuity is introduced and studied in bi-supra topological spaces

Let $X$ be non-empty space, let $\mathcal{S}_o(X)$ be the set of all semi open subset of the space $X$, (for short $\mathcal{S}T$) and let $\mathcal{P}_o(X)$ be the set of all pre open subset of $X$ (for short $\mathcal{P}T$). Then, we say that $(X,\mathcal{S}T,\mathcal{P}T)$ is a bi-supra topological space, [5] Moreover, basic properties of quasi M-$\theta$-ii-continuous functions are investigated, also, relationships between quasi M-$\theta$-ii-continuous functions and graphs are investigated.

2. Preliminaries.

Throughout this paper $(X,\mathcal{T}_X)$ and $(Y,\mathcal{T}_Y)$ (Simply, X and Y) represent topological spaces on which no separation axioms are assumed, unless otherwise mentioned. The closure, the interior and the complement a subset of $A$ is denoted by $\text{cl}(A)$, $\text{int}(A)$ and $(A^c \text{ or } X\setminus A)$ respectively. A subset $A$ of a space $X$ is said to be regular open[7] if it is the interior of its closure, i.e., $(A = \text{int}(\text{cl}(A)))$. The complement of a regular-open set is referred to as a regular-closed set. A union of regular-open sets is called $\delta$-open [7] The complement of a $\delta$-open set is a $\delta$-closed set. A subset $A$ of a space $(X,\mathcal{T}_X)$ is a $\theta$-open set[9] if there exists an open set $U$ containing $x$ such that $U \subseteq \text{cl}(U) \subseteq A$. The set of all $\theta$-interior points of $A$ is said to be the $\theta$-interior set and denoted by $\theta$-int $(A)$. A subset $A$ of $X$ is $\theta$-open if $A = \theta$-int $(A)$. The family of all $\theta$-open sets of a space $X$ is a topology on $\mathcal{T}_X$. The union of all $\theta$-open (resp. $\delta$-open) sets contained

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in A is called the \(0\)-interior (resp. \(\delta\)-interior) of A and it is denoted by \(\theta\)-int(A) (resp. \(\delta\)-int(A)). The intersection of all \(0\)-closed (resp. \(\delta\)-closed) sets containing A is called the \(0\)-closure (resp. \(\delta\)-closure [4]) of A and it is denoted by \(0\)-cl(A) (resp. \(\delta\)-cl(A)).

We recall the following definitions and results, which are useful in the sequel

**Definition 2.1**[8] Let \((X, \mathcal{T}_X)\) be a topological space. Then a subset A of X is said to be:

(i) an \(M\)-open set, if \(A \subseteq \text{cl}(\text{int}_\theta(A)) \cup \text{int}(\text{cl}_\delta(A))\),

(ii) an \(M\)-closed set if \(\text{int}(\text{cl}_\theta(A)) \cap \text{cl}(\text{int}_\delta(A)) \subseteq A\).

**Definition 2.2**[8] Let \((X, \mathcal{T}_X)\) be a topological space and \(A \subseteq X\). Then:

(i) the \(M\)-interior of A is the union of all \(M\)-open sets contained in A and is denoted by \(M\)-int(A),

(ii) the \(M\)-closure of A is the intersection of all \(M\)-closed sets containing A and is denoted by \(M\)-cl(A).

**Definition 2.3**[3] A function \(f: (X, \mathcal{T}_X) \rightarrow (Y, \mathcal{T}_Y)\) is said to be:

(i) \(M\)-continuous [1] if \(f^{-1}(U)\) is \(M\)-open in X, for each U is \(\mathcal{T}_Y\)

(ii) pre-\(M\)-open [2] if, \(f(U)\) is \(M\)-open in Y, for each U is \(M\)-open in X.

(iii) pre-\(M\)-closed [2] if, \(f(U)\) is \(M\)-closed in Y, for each U is \(M\)-closed in X.

**Definition 2.4**[11] A function \(f: (X, \mathcal{ST}_X, \mathcal{PT}_X) \rightarrow (Y, \mathcal{ST}_Y, \mathcal{PT}_Y)\) is called a quasi-\(\theta\)-continuous function if \(f^{-1}(V)\) is a \(\theta\)-open set in X for every \(\theta\)-open set V of Y.

3. A new type of bi-supra topological space

**Definitions 3.1** Let \((X, \mathcal{ST}_X, \mathcal{PT}_X)\) be a bi-supra topological space, and let G be a subset of X. Then G is said to be an \(ii\)-open set if \(G=(A \cup B) \cup \emptyset\) where \(A \in \mathcal{ST}, B \in \mathcal{PT}\) such that \(A \notin \mathcal{PT}, A \cap B \neq \emptyset\). The Complement of \(ii\)-open set is called \(ii\)-closed set.
Definitions 3.2 A subset $A$ of a space $(X, \mathcal{S}_x, \mathcal{T}_x)$ is called $\theta$-$ii$-open set if there exists an $ii$-open set $U$ containing $x$ such that $U \subseteq ii$-cl$(U) \subseteq A$.

Definitions 3.3 The set of all $\theta$-$ii$-interior points of $A$ is said to be the $\theta$-$ii$-interior set and denoted by $\theta$-$ii$-int$(A)$, so a subset $A$ of $X$ is $\theta$-$ii$-open if and only if $A = \theta$-$ii$-int$(A)$.

Definitions 3.4 A union of regular-$ii$-open sets is called $\delta$-$ii$-open. The complement of a $\delta$-$ii$-open set is a $\delta$-$ii$-closed set.

Definitions 3.5 A subset $A$ of a bi-supra topological space $(X, \mathcal{S}_x, \mathcal{T}_x)$ is called an $M$-$ii$-open set if $A \subseteq ii$-cl$(\theta$-$ii$-int$(A))$ $\cup$ $ii$-int$(\delta$-$ii$-cl$(A))$. The union of all $M$-$ii$-open (resp. $\delta$-$ii$-open) sets contained in $A$ is called the $M$-$ii$-interior (resp. $\delta$-$ii$-interior) of $A$ and it is denoted by $M$-$ii$-int$(A)$ (resp. $\delta$-$ii$-int$(A)$). The intersection of all $M$-$ii$-closed (resp. $\delta$-$ii$-closed) sets containing $A$ is called the $M$-$ii$-closure (resp. $\delta$-$ii$-closure) of $A$ and it is denoted by $M$-$ii$-cl$(A)$ (resp. $\delta$-$ii$-cl$(A)$).

Definition 3.6 A subset $A$ of a space $(X, \mathcal{S}_x, \mathcal{T}_x)$ is an $M$-$\theta$-$ii$-open set if and only if for each $x \in A$ there exists an $M$-$ii$-open set in $X$ such that $M$-$ii$-cl$(G) \subseteq A$.

Remark 3.7 Every $M$-$\theta$-$ii$-open (resp. $M$-$\theta$-$ii$-closed) set is an $M$-$ii$-open (resp. $M$-$ii$-closed) set, and every $\theta$-$ii$-open (resp. $\theta$-$ii$-closed) set is $M$-$ii$-open (resp. $M$-$ii$-closed) but the converse is not true as in that example.

The implication between some types of sets are given by the following diagram

\[
\text{M-$\theta$-$ii$-open} \Rightarrow \text{M-$ii$-open} \Rightarrow \text{0-$ii$-open}
\]

Example 3.8 Let $X = \{a, b, c\}$ and $\mathcal{T}_x = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, X\}$. Then

- $\emptyset, \{b, c\}, \{a,c\}, \{c\}, X$, $\mathcal{S}_x = \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, X = \mathcal{T}_x^c$
- $\mathcal{P}_x = \emptyset, \{a\}, \{b\}, \{a, b\}, X$

$ii$-open in $X = \emptyset, \{a, c\}, \{b, c\}, X$, $ii$-closed in $X = \emptyset, \{b\}, \{a\}, X$

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\[= \{ \emptyset, X \}, \text{ in } X \text{ 0-ii- open}\]

\[= \{ \emptyset, X \}, \text{ in } X \text{ 0-ii-closed}\]

Regular –ii-open in \(X= \{ \emptyset, X \},\)

\[\delta-\text{ii-open in } X = \{ \emptyset, X \},\]

\[\delta-\text{ii-closed in } X = \{ \emptyset, X \},\]

\[\text{M-ii-open in } X = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X \},\]

\[\text{M-ii-closed in } X = \{ \emptyset, \{b, c\}, \{a, b\}, \{b, c\}, \{a\}, X \},\]

\[\text{M-0-ii-open in } X = \{ \emptyset, \{a, b\}, \{a, c\}, \{b, c\}, X \}, \text{ so the sets } \{a\}, \{b\}, \{c\} \text{ are M-ii-open sets but not M-0-ii-open sets in } X \text{ and not 0-ii-open in } X .\]

**Definition 3.9** A subset \(A\) in a bi-supra topological space \((X, ST_X, PT_X)\) is called M-0-ii-closure and denoted by \(M-0-\text{ii-cl}(A)\) and is defined to be the set of all points \(x\) of \(X\) such that for each an M-0-ii-open set in \(X\), \(M-0-\text{ii-cl}(G) \cap A \neq \emptyset.\)

**Definition 3.10** A subset \(A\) in bi-supra topological space \((X, ST_X, PT_X)\) is said to be M-0-ii-closed if \(M-0-\text{ii-cl}(A) = A\). The complement of a M-0-ii-closed set is an M-0-ii-open set.

**Lemma 3.11** For a subset \(A\) of a topological space \((X, T_X)\) (resp.bi-supra topological space \((X, ST_X, PT_X)\) the following statements are hold:

(i) If \(A \subseteq F_i, F_i \text{ is an M-closed }[2]\) (resp. M-ii-closed )set in \(X\), then \(A \subseteq M-\text{cl}(A) \subseteq F_i, (\text{resp. } M-\text{ii-cl}(A) \subseteq F_i).\)

(ii) If \(G_i \subseteq A, G_i \text{ is an M-open}[3] \text{ (resp. M-ii-open) set in } X\), then \(G_i \subseteq M-\text{int}(A) \subseteq A \text{ (resp. } G_i \subseteq M-\text{ii-int}(A) \subseteq A).\)

(iii) \(A\) is M-ii-closed in \(U\) if \(M-\text{ii-d}(A) \subseteq A\)
(iv) $M\text{-}\text{ii-cl}(A) = A \cup M\text{-}\text{ii-d}(A)$.

The set of $\theta$-boundary (resp. $\theta\text{-}\text{ii}$-boundary, $M\text{-}\text{ii}$-boundary, $M\text{-}\text{ii}$-border) of $A$ is denoted by $\theta\text{-}F_r (A)$ (resp. $\theta\text{-}\text{ii}\text{-}F_r (A), M\text{-}\text{ii}\text{-}F_r (A), M\text{-}\text{ii}\text{-}b(A)$).

**Proposition 3.12** Let $A$ be a subset of $(X,\mathcal{T}_X)$ a topological space (resp. $(X,\mathcal{S}\mathcal{T}_X,\mathcal{P}\mathcal{T}_X)$ a bi-supra topological space). Then, the following statements are hold:

(i) $\theta\text{-}F_r(A) = \theta\text{-}\text{cl}(A) \setminus \theta\text{-}\text{int}(A)$ (resp. $\theta\text{-}\text{ii}\text{-}F_r(A) = \theta\text{-}\text{ii}\text{-}\text{cl}(A) \setminus (\theta\text{-}\text{ii}\text{-}\text{int}(A)$)

(ii) $M\text{-}F_r(A) = M\text{-}\text{cl}(A) \setminus M\text{-}\text{int}(A)$ [4] (resp. $M\text{-}\text{ii}\text{-}F_r(A) = M\text{-}\text{ii}\text{-}\text{cl}(A) \setminus (M\text{-}\text{ii}\text{-}\text{int}(A)$)

(iii) $M\text{-}b(A) = A \setminus M\text{-}\text{int}(A)$ [2] (resp. $M\text{-}\text{ii}\text{-}b(A) = A \setminus M\text{-}\text{ii}\text{-}\text{int}(A)$).

We recall the following definitions and results, which are useful in the sequel:

**Definition 3.13** A function $f: (X,\mathcal{S}\mathcal{T}_X,\mathcal{P}\mathcal{T}_X) \rightarrow (Y,\mathcal{S}\mathcal{T}_Y,\mathcal{P}\mathcal{T}_Y)$ is called $M\text{-}\text{ii}$-continuous if $f^{-1}(V)$ is $M\text{-}\text{ii}$-open in $X$ for every $ii$-open set $V$ in $Y$.

**Example 3.14** Let $X = \{a, b, c\}$ and $\mathcal{T}_X = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then

$\{\emptyset, \{b, c\}, \{a, c\}, \{c\}, X\} \in \mathcal{S}\mathcal{T}_X = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\} = \mathcal{T}_X^c$

$\mathcal{P}\mathcal{T}_X = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$

$ii$-open in $X = \{\emptyset, \{a, c\}, \{b, c\}, X\}$, $ii$-closed in $X = \{\emptyset, \{b\}, \{a\}, X\}$

$M\text{-}\text{ii}$-open in $X= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$M\text{-}\text{ii}$-closed in $X = \{\emptyset, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\}, \{a\}, X\}$

New, let $Y=\{a, b, c\}$ and , $\mathcal{T}_X = \{\emptyset, \{a\}, \{c\}, \{a, c\}, Y\}$, $\mathcal{T}_Y = \{Y, \{b, c\}, \{a, b\}, b, \emptyset\}$. Then
\( ST_y = \{ \emptyset, \{ a \}, \{ c \}, \{ a, c \}, \{ a, b \}, \{ b, c \}, \{ Y \} \), \( PT_y = \{ \emptyset, \{ a \}, \{ c \}, \{ a, c \}, Y \} \)

\( ii \)-open in \( Y = \{ \emptyset, \{ a \}, \{ b \}, \{ c \}, \{ a, b \}, \{ b, c \}, \{ Y \} \), \( ii \)-closed in \( Y = \{ \emptyset, \{ c \}, \{ a \}, \{ Y \} \} \).

If \( f: (X, ST_x, PT_x) \rightarrow (Y, ST_y, PT_y) \) is defined by \( f(a)=b, f(b)=a, \) and \( f(c)=c \) then \( f \) is \( M-ii \)-continuous.

**Theorem 3.15** Let \( (X, ST_x, PT_x) \) and \( (Y, ST_y, PT_y) \) be two bi-supra topological spaces and \( f:(X, ST_x, PT_x) \rightarrow (Y, ST_y, PT_y) \) be a function. Then the following statements are equivalent:

(i) \( f \) is \( M-ii \)-continuous

(ii) \( M-ii-\text{cl}(f^{-1}(B)) \subseteq f^{-1}(ii-\text{cl}(B)) \) for each \( B \subseteq Y \)

(iii) \( f(M-ii-\text{cl}(A)) \subseteq ii-\text{cl}(f(A)) \) for each \( A \subseteq X \)

(iv) \( f^{-1}(ii-\text{int}(B)) \subseteq M-ii-\text{int}(f^{-1}(ii-b(B))) \) for each \( B = Y \)

**Proof:** (i) \( \rightarrow \) (ii) Since \( B \subseteq ii-\text{cl}(B) \subseteq Y \) which and is an \( ii \)-closed set, then by hypothesis, \( f^{-1}(ii-\text{cl}(B)) \) is \( M-ii \)-closed in \( X \). Hence, by Lemma 3.11, \( M-ii-\text{cl}(f^{-1}(B)) \subseteq f^{-1}(ii-\text{cl}(B)) \) for each \( B \subseteq Y \).

(ii) \( \rightarrow \) (iii) Let \( A \subseteq X \). Then \( f(A) \subseteq Y \), hence by hypothesis, \( M-ii-\text{cl}(A) \subseteq M-ii-\text{cl}(f^{-1}(f(A))) \subseteq f^{-1}(ii-\text{cl}(f(A))). \) Therefore, \( f(M-ii-\text{cl}(A)) \subseteq f(f^{-1}(ii-\text{cl}(A))) \subseteq ii-\text{cl}(f(A)) \).

(iii) \( \rightarrow \) (i) Let \( V \subseteq Y \) be an \( ii \)-closed set. Then, \( f^{-1}(V) \subseteq X \). Hence, by (iii), \( f(M-ii-\text{cl}(f^{-1}(V))) \subseteq ii-\text{cl}(f(f^{-1}(V))) \subseteq ii-\text{cl}(V) = V \). Thus \( M-ii-\text{cl}(f^{-1}(V)) \subseteq f^{-1}(V) \) and hence \( f^{-1}(V) \) is \( M-ii \)-closed in \( X \). Hence, \( f \) is \( M-ii \)-continuous.

(iv) \( \rightarrow \) (i) Let \( U = Y \) be an \( ii \)-open set. Then by assumption, \( f^{-1}(U) = f^{-1}(ii-\text{int}(U)) \subseteq M-ii-\text{int}(f^{-1}(U)) \). Hence, \( f^{-1}(U) \) is \( M-ii \)-open in \( X \). Therefore, \( f \) is \( M-ii \)-continuous.
Corollary 3.16 Let \((X,\mathcal{ST}_x,\mathcal{PT}_x)\) and \((Y,\mathcal{ST}_y,\mathcal{PT}_y)\) be two bi-supra topological spaces and \(f:(X,\mathcal{ST}_x,\mathcal{PT}_x)\rightarrow(Y,\mathcal{ST}_y,\mathcal{PT}_y)\) be a function

(i) If \(f\) is M-\(ii\)-continuous, then \(M-\text{ii-b}(f^{-1}(B)) \subseteq f^{-1}(\text{ii-b}(B))\) for each \(B \subseteq Y\)

(ii) If \(f\) is M-\(ii\)-continuous, then \(M-\text{ii-Fr}(f^{-1}(B)) \subseteq f^{-1}(\text{ii-Fr}(B))\) for each \(B \subseteq Y\)

Theorem 3.16 Let \((X,\mathcal{ST}_x,\mathcal{PT}_x)\) and \((Y,\mathcal{ST}_y,\mathcal{PT}_y)\) be two bi-supra topological spaces and \(f:(X,\mathcal{ST}_x,\mathcal{PT}_x)\rightarrow(Y,\mathcal{ST}_y,\mathcal{PT}_y)\) be a function. Then the following statement are equivalent:

(i) \(f\) is M-\(ii\)-continuous

(ii) \(f(M-\text{ii-d}(A)) \subseteq \text{ii-cl}(f(A))\) for each \(A \subseteq X\), where \(M-\text{ii-d}(A)\) is (The set of all M-\(ii\)-limit points of \(A\) is called M-\(ii\)-derived set of \(A\) and denoted by \(M-\text{ii-d}(A)\)).

Proof: (i) \(\rightarrow\) (ii) since \(f\) is M-\(ii\)-continuous then by theorem (3.15(iii)) \(f(M-\text{ii-cl}(A)) \subseteq \text{ii-cl}(f(A))\) for each \(A \subseteq X\), so \(f(M-\text{ii-d}(A)) \subseteq f(M-\text{ii-cl}(A)) \subseteq \text{ii-cl}(f(A))\).

(ii) \(\rightarrow\) (i) Let \(U\) be an \(ii\)-closed subset of \(Y\). Then \(f^{-1}(U) \subseteq X\) hence by hypothesis \(f(M-\text{ii-cl}(f^{-1}(U))) \subseteq \text{ii-cl}(f(f^{-1}(U))) \subseteq \text{ii-cl}(U) = U\). Therefore by lemma (3.11(iii),(iv))

Thus \(M-\text{ii-cl}(f^{-1}(U)) = f^{-1}(U) \cup M-\text{ii-d}(f^{-1}(U)) \subseteq f^{-1}(U) \cup f^{-1}(U) = f^{-1}(U)\).

Hence \(f^{-1}(U) = M-\text{ii-cl}(f^{-1}(U))\) which is M-\(ii\)-closed set in \(X\) therefore \(f\) is M-\(ii\)-continuous.

Definition 3.17 A function \(f:\): \((X,\mathcal{ST}_x,\mathcal{PT}_x)\rightarrow(Y,\mathcal{ST}_y,\mathcal{PT}_y)\) is called an M-\(\theta\)-\(ii\)-continuous function, if for each \(x\) in \(X\) and each \(ii\)-open set \(V\) in \(Y\) containing \(f(x)\), there exists an M-\(\theta\)-\(ii\)-open set \(U\) in \(X\) containing \(x\) such that \(f(U) \subseteq V\).

Remark 3.18 Every M-\(\theta\)-\(ii\)-continuous function is M-\(ii\)-continuous but the converse is not true.

Proof: Directly from that definitions.
Let \( X = \{ a, b, c \} \)

\[ \mathcal{T}_x = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, X \} . \]

Then

\[ \{ \emptyset, \{b, c\}, \{a,c\}, \{c\}, X \} , \quad \mathcal{S}\mathcal{T}_x = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, X \} = \mathcal{T}_x^c \]

\[ \mathcal{P}\mathcal{T}_x = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, X \} \]

\( \text{M-}\theta\text{-ii-open in } X = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a,c\}, \{b, c\}, X \} \)

\( \text{M-}\theta\text{-ii-closed in } X = \{ \emptyset, \{b, c\}, \{a,c\}, \{c\}, \{a\}, X \} \)

And let \( Y = \{ a, b, c \} \)

\[ \mathcal{T}_y = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, Y \} , \quad \mathcal{T}_y^c = \{ Y, \{b,c\}, \{a,c\}, \{c\}, \emptyset \} \]

\[ \mathcal{S}\mathcal{T}_y = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, Y \} , \quad \mathcal{P}\mathcal{T}_y = \{ \emptyset, \{a\}, \{b\}, \{a,b\}, Y \} \]

\( \text{ii-open in } Y = \{ \emptyset, \{a,c\}, \{b,c\}, Y \} \)

\( \text{ii-closed in } Y = \{ \emptyset, \{a,c\}, Y \} \)

If \( f : (X,\mathcal{S}\mathcal{T}_x,\mathcal{P}\mathcal{T}_x) \rightarrow (Y,\mathcal{S}\mathcal{T}_y,\mathcal{P}\mathcal{T}_y) \) is defined by \( f(a)=a \) , \( f(b)=c \) , and \( f(c)=b \).

Then \( f \) is \( M-\theta\text{-ii-continuous} \).

4. qusai M-\( \theta\text{-ii-continuous} \) functions.

In this section, we introduces the concept of "qusai M-\( \theta\text{-ii-continuous} \) functions" and some examples with many properties of this concept.

**Definition 4.1** A function \( f: (X,\mathcal{S}\mathcal{T}_x,\mathcal{P}\mathcal{T}_x) \rightarrow (Y,\mathcal{S}\mathcal{T}_y,\mathcal{P}\mathcal{T}_y) \) is called quasi-\( \theta\text{-ii-continuous} \) function if \( f^{-1}(V) \) is \( \theta\text{-ii-open} \) set in \( X \) for every \( \theta\text{-ii-open} \) set \( V \) of \( Y \).

**Definition 4.2** A function \( f: (X,\mathcal{S}\mathcal{T}_x,\mathcal{P}\mathcal{T}_x) \rightarrow (Y,\mathcal{S}\mathcal{T}_y,\mathcal{P}\mathcal{T}_y) \) is called a quasi-M-\( \theta\text{-ii-continuous} \) function, if \( f^{-1}(V) \) is an M-\( \theta\text{-ii-open} \) set in \( X \) for every \( \theta\text{-ii-open} \) set \( V \) of \( Y \).
Example 4.3 See example (3.13) is holding definition

**Proposition 4.4** Every quasi-\( \theta - \text{ii} \)-continuous function is an \( \theta - \text{ii} \)-continuous function.

**Theorem 4.5** For a function \( (X, \mathcal{ST}_{X}, \mathcal{PT}_{X}) \rightarrow (Y, \mathcal{ST}_{Y}, \mathcal{PT}_{Y}) \), the following statements are equivalent:

(i) \( f \) is quasi \( \theta - \text{ii} \)-continuous,

(ii) For each \( x \in X \) and each \( \theta - \text{ii} \)-open \( V \) in \( Y \) contains \( f(x) \), there exists an \( \theta - \text{ii} \)-open set \( U \) in \( X \) contains \( x \) such that \( f(U) \subseteq V \),

(iii) \( f^{-1}(F) \) is \( \theta - \text{ii} \)-closed in \( X \), for every \( \theta - \text{ii} \)-closed \( F \) of \( Y \),

(iv) \( \theta - \text{ii-cl}(f^{-1}(B)) \subseteq f^{-1}(\theta - \text{ii-cl}(B)) \), for each \( B \subseteq Y \),

(v) \( f(\theta - \text{ii-cl}(A)) \subseteq \theta - \text{ii-cl}(f(A)) \), for each \( A \subseteq X \),

(vi) \( f^{-1}(\theta - \text{ii-int}(B)) \subseteq \theta - \text{ii-int}(f^{-1}(B)) \), for each \( B \subseteq Y \),

(vii) \( \theta - \text{ii-Fr}(f^{-1}(B)) \subseteq f^{-1}(\theta - \text{ii-Fr}(B)) \), for each \( B \subseteq Y \),

(viii) \( \theta - \text{ii-b}(f^{-1}(B)) \subseteq f^{-1}(\theta - \text{ii-b}(B)) \), for each \( B \subseteq Y \).

**Proof.** (i)→(ii). Let \( x \in X \) and \( V \subseteq Y \) be a \( \theta - \text{ii} \)-open set containing \( f(x) \). Then \( x \in f^{-1}(V) \). Hence by hypothesis, \( f^{-1}(V) \) is \( \theta - \text{ii} \)-open set of \( X \) containing \( x \). We put \( U = f^{-1}(V) \), then \( x \in U \) and \( f(U) \subseteq V \).

(ii)→(iii). Let \( F \subseteq Y \) be \( \theta - \text{ii} \)-closed. Then \( Y \setminus F \) is \( \theta - \text{ii} \)-open if \( x \in f^{-1}(Y \setminus F) \), then \( f(x) \) \( \in Y \setminus F \). Hence by hypothesis, there exists an \( \theta - \text{ii} \)-open set \( U \) containing \( x \) such that \( f(U) \subseteq Y \setminus F \), this implies that, \( x \in U \subseteq f^{-1}(Y \setminus F) \). Therefore, \( f^{-1}(Y \setminus F) = \cup x \in U \{ U : f^{-1}(Y \setminus F) \} \) which is \( \theta - \text{ii} \)-open in \( X \). Therefore, \( f^{-1}(F) \) is \( \theta - \text{ii} \)-closed.

(iii)→(i). Let \( V \subseteq Y \) be a \( \theta - \text{ii} \)-open set. Then \( Y \setminus V \) is \( \theta - \text{ii} \)-closed in \( Y \). By hypothesis, \( f^{-1}(Y \setminus V) = X \setminus f^{-1}(V) \) is \( \theta - \text{ii} \)-closed and hence \( f^{-1}(V) \) is \( \theta - \text{ii} \)-open. Therefore, \( f \) is quasi \( \theta - \text{ii} \)-continuous.
(i)→ (iv). If \( B \subseteq X \) then \( \theta\text{-ii-cl}(B) \) is \( \theta\text{-ii}-\)closed, then by hypothesis, \( f^{-1}(\theta\text{-ii-cl}(B)) \) is \( M\text{-} \theta\text{-ii}-\)closed in \( X \). Hence, by Lemma 3.11, \( M\text{-} \theta\text{-ii-cl}(f^{-1}(B)) \subseteq f^{-1}(\theta\text{-ii-cl}(B)) \) for each \( B \subseteq Y \).

(iv)→(v). Let \( A \subseteq X \). Then \( f(A) \subseteq Y \), hence by hypothesis, \( M\text{-} \theta\text{-ii-cl}(A) \subseteq M\text{-} \theta\text{-ii-cl}(f^{-1}(f(A))) \subseteq f^{-1}(\theta\text{-ii-cl}(f(A))) \). Therefore, \( f(M\text{-} \theta\text{-ii-cl}(A)) \subseteq f\; f^{-1}(\theta\text{-ii-cl}(f(A))) \subseteq \theta\text{-ii-cl}(f(A)) \).

(v)→(i). Let \( V \subseteq Y \) be a \( \theta\text{-ii}\text{-closed set}. Then, \( f^{-1}(V) \subseteq X \). Hence, by hypothesis, \( f(M\text{-} \theta\text{-ii-cl}(f^{-1}(V))) \subseteq \theta\text{-ii-cl}(f(f^{-1}(V))) \subseteq \theta\text{-ii-cl}(V) \). Thus \( M\text{-} \theta\text{-ii-cl}(f^{-1}(V)) \subseteq f^{-1}(V) \) and hence \( f^{-1}(V) \in M\text{-} \theta\text{-ii-\)closed in \( X \). Hence, \( f \) is quasi \( M\text{-} \theta\text{-ii-\)continuous,

(i)→(vi). \( \forall B \subseteq X \) then \( \theta\text{-ii-int}(B) \) is \( \theta\text{-ii-\)open, then by hypothesis, \( f^{-1}(\theta\text{-ii-int}(B)) \) is an \( M\text{-} \theta\text{-ii-\)open set in \( X \). Hence, by Lemma 3.11, \( f^{-1}(\theta\text{-ii-int}(B)) \subseteq M\text{-} \theta\text{-ii-int}(f^{-1}(B)) \), for each \( B \subseteq Y \).

(vi)→(i). Let \( V \subseteq Y \) be a \( \theta\text{-ii-\)open set. Then by assumption, \( f^{-1}(V) = f^{-1}(\theta\text{-ii-int}(V)) \subseteq M\text{-} \theta\text{-ii-int}(f^{-1}(V)) \). Hence, \( f^{-1}(V) \) is \( M\text{-} \theta\text{-ii-\)open in \( X \). Therefore, \( f \) is quasi \( M\text{-} \theta\text{-ii-\)continuous.

(vi)→(vii). Let \( V \subseteq Y \). Then by hypothesis, \( f^{-1}(\theta\text{-ii-int}(V)) \subseteq M\text{-} \theta\text{-ii-int}(f^{-1}(V)) \) and so \( f^{-1}(V) \setminus M\text{-} \theta\text{-ii-int}(f^{-1}(V)) \subseteq f^{-1}(V) \setminus f^{-1}(\theta\text{-ii-int}(V)) = f^{-1}(V \setminus \theta\text{-ii-int}(V)) \). By Proposition 3.12, \( M\text{-} \theta\text{-ii-Fr}(f^{-1}(V)) \subseteq f^{-1}(\theta\text{-ii-Fr}(V)) \).

Proposition 4.6 If \( f : (X, \mathcal{ST}_X, \mathcal{PT}_X) \rightarrow (Y, \mathcal{ST}_Y, \mathcal{PT}_Y) \) is quasi-M- \( \theta\text{-ii-\)continuous and \( g : (Y, \mathcal{ST}_Y, \mathcal{PT}_Y) \rightarrow (Z, \mathcal{ST}_Z, \mathcal{PT}_Z) \) is quasi- \( \theta\text{-ii-\)continuous then \( g \circ f \) is quasi-M- \( \theta\text{-ii-\)continuous .

Proof: let \( V \subseteq Z \) be a \( \theta\text{-ii-\)open set and \( g \) be a quasi- \( \theta\text{-ii-\)continuous function . Then \( g^{-1}(V) \) is \( 0\text{-ii-\)open in \( Y \) . But \( f \) is a quasi-M- \( \theta\text{-ii-\)continuous function , then \( (g \circ f)^{-1}(V) \) is an \( M\text{-} \theta\text{-ii-\)open set in \( X \), Hence \( g \circ f \) is an \( M\text{-} \theta\text{-ii-\)continuous function

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