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Calculation of the Longitudinal Electron Scattering Form Factors for the ^{19}F and ^{27}Al nuclei

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Abstract

For investigating the inelastic longitudinal electron scattering form factors F (q)'s. An expression for the transition charge density is studied where the deformation in nuclear collective modes is taken into consideration besides the shell model transition density. The inelastic longitudinal C2 and C4 form factors are calculated using this transition charge density for the ^{19}F and ^{27}Al nuclei. In this work, the core polarization transition density is evaluated by adopting the shape of Tassie model together with the derived form of the ground state two-body charge density distributions (2BCDD's). It is noticed that the core polarization effects which represent the collective modes are essential in obtaining a remarkable agreement between the calculated inelastic longitudinal F (q)'s and those of experimental data for all considered nuclei.

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Keywords: Electron Scattering, Form Factors.

حساب عوامل التشكل للاستطارة الالكترونية الطولية للنوى^{19}F و^{27}Al

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الخلاصة

تم دراسة عوامل التشكيل للاستطارة الطولية غير المرنة و المتضمنة انتقال كثافة الشحنة عندما ناخذ بنظر الاعتبار التشوه في الانماط التجميعية النووية الى جانب كثافة الانتقال لانموذج القشرة. حيث تم حساب عوامل التشكيل للاستطارة الطولية $C2$و$C4$ للنوى^{19}F و^{27}Al . ان تاثيرات استقطاب القلب لكثافة الانتقال حسبت بالاعتماد على شكل انموذج Tassie الى جانب الصيغة الرياضية المشتقة لتوزيعات كثافة الشحنة النووية ذو صيغة الجسيمين في الحالة الارضية (2BCDD's) . لقد وجد بان تأثير استقطاب القلب الذي يمثل نمط تجميعي يكون جوهريا للحصول على توافق جيد بين حسابات الاستطارة الطولية غير المرنة ($F(q)$'s) و القيم العملية لجميع النوى قيد الدراسة.

Introduction

Electron scattering from nuclei provides more accurate information about the nuclear structure for example size and charge distribution. It provides important knowledge about the electromagnetic currents inside the nuclei. Electron scattering have is provide a good test for such evaluation since it is sensitive to the spatial dependence on the charge and current densities [1- 3]. In electron scattering one can distinguish two types of scattering processes: first the nucleus is remained in its ground state before and after scattering process, this operation is called "elastic electron scattering". In the second type, the nucleus is left in different excited states after scattering event; this process is called "inelastic

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electron scattering" [4]. Radhi [5] described the inelastic electron scattering form factors for even-parity states of ^{27}Al. the higher configurations are taking into account through first-order perturbation theory, which allows particle–hole excitations from the 1*s*- and 1*p* shells core orbits to the higher 2*s*–1*d* and 1*f*–2*p* shells with $2\hbar\omega$ excitations. It is shown that the core polarization effects are essential in obtaining a reasonable description of the data with no adjustable parameters. Radhi [6] described the coulomb form factors of C4 transitions in even-even N = Z sd-shell outside the sd-shell model space, which are called core polarization effects. Also inelastic electron scattering to 2^+ and 4^+ states for the even-even N = Z sd-shell nuclei have been discussed by Radhi & Boucheback [7], considering the core-polarization effects.. Jassim et al. [8] studied the nuclear structure (energy levels, elastic and inelastic electron-nucleus scattering, and transition probability) of ^{23}Na, ^{25}Mg, ^{27}Al, and ^{41}Ca nuclei have been studied using shell-model. Radhi et al. [9] used the shell model and Hartree–Fock calculations to study the nuclear structure of positive and negative party states in ^{19}F nucleus.

The aim of the present work is to calculate the longitudinal C2 and C4 form factors for the inelastic electron scattering ^{19}F and ^{27}Al nuclei.

Theory

The many particle reduced matrix elements of the longitudinal operator, consists of two parts; one is for the model space and the other is for core polarization matrix element [10, 11]:

$$\left\langle f \left\| \hat{T}_J^L(\tau_Z,q) \right\| i \right\rangle = \left\langle f \left\| \overset{ms}{\hat{T}_J^L}(\tau_Z,q) \right\| i \right\rangle + \left\langle f \left\| \overset{cor}{\hat{T}_J^L}(\tau_Z,q) \right\| i \right\rangle \quad \text{.......(1)}$$

The model space matrix element has the form [11]:

$$\left\langle f \left\| \overset{ms}{\hat{T}_J^L}(\tau_Z,q) \right\| i \right\rangle = e_i \int_0^\infty d\mathrm{r}\, \mathrm{r}^2 j_J(q\mathrm{r}) \overset{ms}{\rho}_{J,\tau_Z}(i,f,\mathrm{r}) \quad \text{.......(2)}$$

where $\overset{ms}{\rho}_J(i,f,\mathrm{r})$ is the transition charge density of model space given by [12]:

$$\overset{ms}{\rho}_{J,\tau_Z}(i,f,\mathrm{r}) = \sum_{jj'(ms)} OBDM(i,f,J,j,j',\tau_z) \left\langle j \| \mathrm{Y}_J \| j' \right\rangle R_{nl}(\mathrm{r})\, R_{n'l'}(\mathrm{r}) \quad \text{.......(3)}$$

where *OBDM* is the One Body Density Matrix. The core- polarization matrix element is given by [12]:

$$\left\langle f \left\| \overset{cor}{\hat{T}_J^L}(\tau_Z,q) \right\| i \right\rangle = e_i \int_0^\infty d\mathrm{r}\, \mathrm{r}^2 j_J(q\mathrm{r}) \overset{core}{\rho}_J(i,f,\mathrm{r}) \quad \text{.......(4)}$$

where $\overset{core}{\rho}_{J\tau_Z}$ is the core- polarization transition density which depends on the model used for core polarization. To take the core- polarization effects into consideration, the model space transition density is added to the core-polarization transition density that describes the collective modes of nuclei. The total transition density becomes

$$\rho_{J\tau_Z}(i,f,\mathrm{r}) = \overset{ms}{\rho}_{J\tau_Z}(i,f,\mathrm{r}) + \overset{core}{\rho}_{J\tau_Z}(i,f,\mathrm{r}) \quad \text{.......(5)}$$

where $\overset{core}{\rho}_{J\tau_Z}$ is assumed to have the form of Tassie shape and given by [13].

$$\rho_{Jt_z}^{core}(i,f,\mathrm{r}) = \mathrm{N}\frac{1}{2}(1+\tau_z)\,\mathrm{r}^{J-1}\frac{d\rho_o(i,f,\mathrm{r})}{d\mathrm{r}} \quad \text{.......(6)}$$

where N is a proportionality constant It is determined by adjusting the reduced transition probability *B(CJ)* and given by [14].

$$\mathrm{N} = \frac{\int_0^\infty d\mathrm{r}\, \mathrm{r}^{J+2}\, \rho^{ms}_{Jt_z}(i, f, \mathrm{r}) - \sqrt{(2J_i+1)B(CJ)}}{(2J+1)\int_0^\infty d\mathrm{r}\, \mathrm{r}^{2J}\rho_o(i, f, \mathrm{r})} \quad \ldots\ldots(7)$$

Here, □□ (i, f, r) is the ground state two body charge density distribution derived as follow; we have produced an effective two body charge density operator by folding the two-body charge density operator with the two-body correlation functions $\tilde{f}_{ij}$ as [15]:

$$\hat{\rho}^{(2)}_{eff}(\vec{\mathrm{r}}) = \frac{\sqrt{2}}{2(A-1)} \sum_{i\neq j} \tilde{f}_{ij} \left\{ \delta\left[\sqrt{2}\,\vec{\mathrm{r}} - \vec{R}_{\mathrm{ij}} - \vec{\mathrm{r}}_{\mathrm{ij}}\right] + \delta\left[\sqrt{2}\,\vec{\mathrm{r}} - \vec{R}_{\mathrm{ij}} + \vec{\mathrm{r}}_{\mathrm{ij}}\right]\right\} \tilde{f}_{ij} \quad (8)$$

where $\vec{\mathrm{r}}_{\mathrm{ij}}$ and $\vec{R}_{\mathrm{ij}}$ are relative and center of mass coordinates and the form of $\tilde{f}_{ij}$ given by [16]:

$$\tilde{f}_{ij} = f(r_{ij})\,\Delta_1 + f(r_{ij})\left\{1 + \alpha(\mathrm{A})\, S_{ij}\right\}\Delta_2 \quad (9)$$

It is clear that Eq. (9) contains two types of correlations

1. The two-body short rang correlations (SRC's) presented in the first term of Eq. (9) and denoted by $f(r_{ij})$. Here Δ_1 is a projection operator onto the space of all two-body functions with the exception of $^3S_1$ and $^3D_1$ states. In fact, the SRC's are central functions of the separation between the pair of particles which reduce the two-body wave function at short distances, where the repulsive core forces the particles apart, and heal to unity at large distance where the interactions are extremely weak. A simple model form of two-body SRC's is given by [16]

$$f(r_{ij}) = \begin{cases} 0 & for\ r_{ij} \le r_c \\ 1 - \exp\left\{-\mu(r_{ij} - r_c)^2\right\} & for\ r_{ij} > r_c \end{cases} \quad (10)$$

where r_c (in fm) is the radius of a suitable hard core and μ =25 fm^{-2} [16] is a correlation parameter

2. The two-body tensor correlations (TC's) presented in the second term of Eq.(9) are induced by the strong tensor component in the nucleon-nucleon force and they are of longer range. Here Δ_2 is a projection operator onto the $^3S_1$ and $^3D_1$ states only. S_{ij} is the usual tensor operator, formed by the scalar product of a second-rank operator in intrinsic spin space and coordinate space and is defined by

$$S_{ij} = \frac{3}{r_{ij}^2}(\vec{\sigma}_i.\vec{r}_{ij})(\vec{\sigma}_j.\vec{r}_{ij}) - \vec{\sigma}_i.\vec{\sigma}_j \quad (11)$$

The parameter □□(A) is the strength of tensor correlations and it is non zero only in the $^3S_1 - {}^3D_1$ channels.

The ground state two body charge density distribution $\rho_{\mathrm{ch}}(\mathrm{r})$ is given by the expectation value of the effective two-body charge density operator of Eq.(8) and written as

$$\begin{aligned} \rho_{\mathrm{ch}}(\mathrm{r}) &\equiv \left\langle \Psi \,\middle|\, \hat{\rho}^{(2)}_{eff}(\vec{\mathrm{r}}) \,\middle|\, \Psi \right\rangle \\ &= \sum_{i<j} \left\langle i\ j \right| \hat{\rho}^{(2)}_{eff}(\vec{\mathrm{r}}) \left[\left| ij \right\rangle - \left| ji \right\rangle \right] \end{aligned} \quad (12)$$

where the two particle wave function is given by [17].

$$\left|ij\right\rangle = \sum_{JM_J}\sum_{TM_T} \left\langle j_i m_i\, j_j m_j \middle| JM_J \right\rangle \left\langle t_i m_{t_i}\, t_j m_{t_j} \middle| TM_T \right\rangle \left|(j_i j_j)JM_J\right\rangle \left|(t_i t_j)TM_T\right\rangle \tag{13}$$

where J and M_J adenote the total angular momentum and it's projection of a pair of particles formed by coupling j_i and j_j while T and M_T denote their total isospin and isospin projection formed by coupling t_i and t_j.

It is important to indicate that our effective two body charge density operator of Eq. (8) is constructed in terms of relative and center of mass coordinates thus the space –spin part $\left|(j_i j_j)\, JM_J\right\rangle$ of two-particle wave function constructed in *jj*-coupling scheme must be transformed in terms of relative and center of mass coordinates. This transformation can be achieved as follow:

1-Switching from *j–j* to λ-*S* coupling schemes as [18]:

$$\begin{aligned}\left|(j_i j_j)\, JM_J\right\rangle &\equiv \left|(\ell_i\ \tfrac{1}{2})\ j_i\ ,(\ell_j\ \tfrac{1}{2})\ j_j ; JM_J\right\rangle \\ &= \sum_{\lambda S} \hat{j}_i\ \hat{j}_j\ \hat{\lambda}\ \hat{S} \begin{Bmatrix} \ell_i & \ell_j & \lambda \\ \tfrac{1}{2} & \tfrac{1}{2} & S \\ j_i & j_j & J \end{Bmatrix} \left|(\ell_i\ \ell_j)\ \lambda\ (\tfrac{1}{2}\tfrac{1}{2})\ S\ ; JM_J\right\rangle\end{aligned} \tag{14}$$

where the notation $\hat{A}=(2A+1)^{1/2}$ and the bracket $\begin{Bmatrix} \cdots\cdots\cdots . \\ \cdots\cdots\cdots . \\ \cdots\cdots\cdots . \end{Bmatrix}$ is the 9-*j* symbol.

3. We next use the Brody – Moshinsky transformation brackets [18] to transform the spatial part of the two-body wave function $\left|(\ell_i \ell_j)\lambda\right\rangle$ in terms of relative and center of mass coordinates, i.e.

$$\left|(\ell_i \ell_j)\lambda\right\rangle \equiv \left|n_i\ \ell_i\ n_j\ \ell_j\ ;\lambda\right\rangle = \sum_{n\ell NL} \left\langle n\ell, N L;\lambda \middle| n_i\ \ell_i\ , n_j\ \ell_j\ ;\lambda\right\rangle \left|n\ell, N L;\lambda\right\rangle \tag{15}$$

where the coefficient $\left\langle n\ell, N L;\lambda \middle| n_i\ \ell_i\ , n_j\ \ell_j\ ;\lambda\right\rangle$ is an overlap integral and called a transformation bracket. For the purpose of extending the calculation to open shell nuclei we replace the factors $\hat{j}_i$ and $\hat{j}_j$ in Eq. (14) as

$$\begin{aligned}(2j_i+1) &\Rightarrow \eta_{n_i \ell_i j_i}\,(2j_i+1) \\ (2j_j+1) &\Rightarrow \eta_{n_j \ell_j j_j}\,(2j_j+1)\end{aligned} \tag{16}$$

where $\eta_{n_i \ell_i j_i}$ and $\eta_{n_j \ell_j j_j}$ are the occupation probabilities of the states $n_i\ \ell_i\ j_i$ and $n_j\ \ell_j\ j_j$, respectively. These parameters equal to (zero or one) for closed shell nuclei while for open shell nuclei they are larger than zero and less than one (i.e. $0<\eta_{n_i \ell_i j_i}<1$) and ($0<\eta_{n_j \ell_j j_j}<1$).

The longitudinal form factor is related to the charge density distribution through the matrix elements of multipole operators $\hat{T}_J^L(q)$ [12].

$$\left|F_J^L(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left|\left\langle f \left\| \hat{T}_J^L(q) \right\| i\right\rangle\right|^2 \left|F_{cm}(q)\right|^2 \left|F_{fs}(q)\right|^2 \tag{17}$$

where Z is the proton number in the nucleus and $F_{cm}(q)$ is the center of mass correction, which removes the spurious state arising from the motion of the center of mass when shell model wave function is used, and given by [14]:

$$F_{cm}(q) = e^{q^2 b^2 / 4A} \tag{18}$$

where A is the nuclear mass number and b is the harmonic oscillator size parameter. The function $F_{fs}(q)$ is the finite size correction, considered as a free nucleon form factor and assumed to be the same for protons and neutrons, and it takes the form [14]:

$$F_{fs}(q) = e^{-0.43 q^2 / 4} \tag{19}$$

Results, Discussion and Conclusions

The inelastic longitudinal $C2$ and $C4$form factors of ^{19}F and ^{27}Al nuclei are presented in Figures- (1, 2), respectively. The model space transition density is obtained by Eq. (3), where the *OBDM* elements of above nuclei are calculated by OXBASH code [19] using the USDB interaction [20].

The inelastic longitudinal C2 form factor of ^{19}F nucleus for the transitions from the ground state $\left(J_i^\pi\, T_i = 5^+/2\; 1/2\right)$ to the $\left(J_f^\pi\, T_f = 3^+/2\; 1/2\right)$ state at E_x = 1.554 *MeV* is presented in Fig.1-a, the dash curves represent the contribution of the model space where the configuration mixing is taken into account, the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects into consideration and the dotted symbols represent the experimental data [21]. This figure shows that the contribution of the model space (the dashed curve) cannot reproduce the experimental data since it underestimates the data for all values of momentum transfer. Considering the effect of core polarization together with the model space (the solid curve), leads to give an enhancement to the longitudinal C2 form factors and consequently makes the calculated results to be in a satisfactory description with those of the experimental data for all values of momentum transfer q .

The inelastic longitudinal C4 form factor of ^{19}F nucleus for the transitions from the ground state $\left(J_i^\pi\, T_i = 5^+/2\; 1/2\right)$ to the $\left(J_f^\pi\, T_f = 9^+/2\; 1/2\right)$ state at E_x = 2.78 *MeV* is presented in Figure-1(b)This figure show that the contribution of the model space cannot reproduce the experimental data since it underestimates the data for momentum transfer $q \le 1.5$ fm^{-1}. Considering the effect of core polarization together with the model space (the solid curves), leads to give an enhancement to the longitudinal C4 form factors and consequently makes the calculated results to be in a satisfactory description with those of the experimental data for all values of momentum transfer q.

Regarding to the ^{27}Al nucleus, The Calculation are presented for the transitions from the ground state $\left(J_i^\pi\, T_i = 5^+/2\; 1/2\right)$ to the $\left(J_f^\pi\, T_f = 1^+/2\; 1/2\right)$ state and $\left(J_f^\pi\, T_f = 7^+/2\; 1/2\right)$ at 0.844 MeV and 2.211 MeV, respectively. The sd-shell model calculation (dashed curve) of C2 form factor including CP effects in $1/2^+$ state at 0.844 MeV (solid curve) gives a good result in the first maximum up to q=1.9 fm^{-1} and the result is gradually shifted from the experimental data in which the curve overestimates this data, the deviation can be interpreted within the large bar errors as show in Fig.2-a

The experimental form factor for 2.211 MeV (7^+/2 1/2) in ^{27}Al doublet of the C2 and C4. The theoretical results of C2 (dashed curves), C4 (dotted curves) and the results of C2+C4 (solid curve) form factors for these states which shown in Fig.2-b. Considering the effect of core polarization together with the model space, leads to give an enhancement to the longitudinal C2+C4 form factors and consequently makes the calculated results to be in a satisfactory description with those of the experimental data for all values of momentum transfer q.

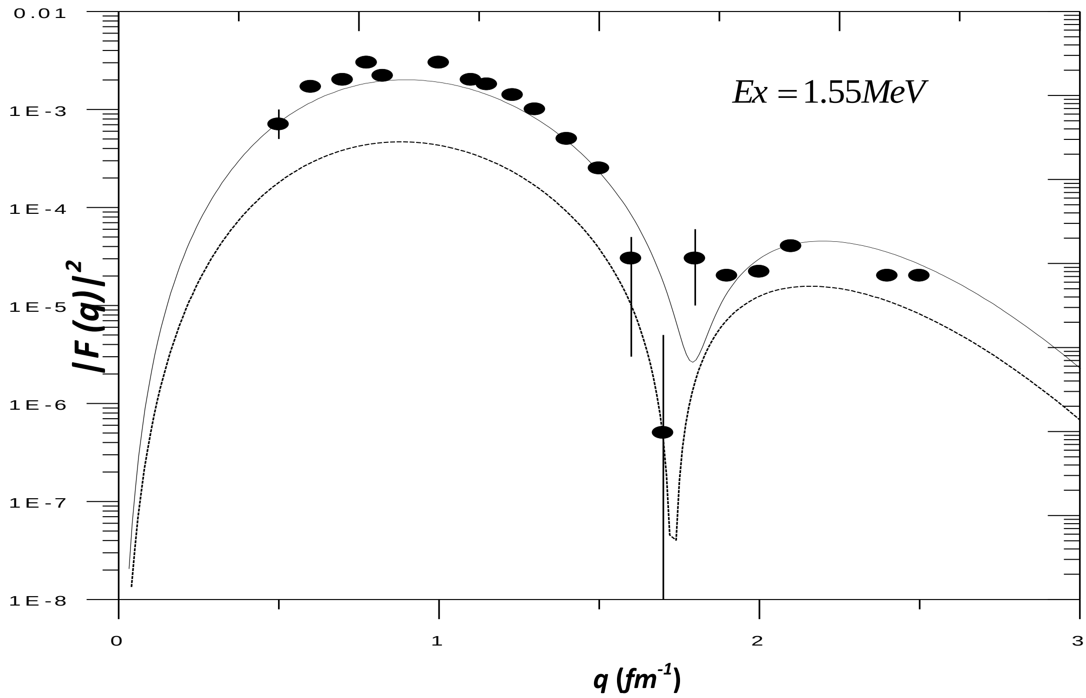

Figure 1(a)- The C2-form factors for the 3+/2-state, are compared with the experimental data [21].

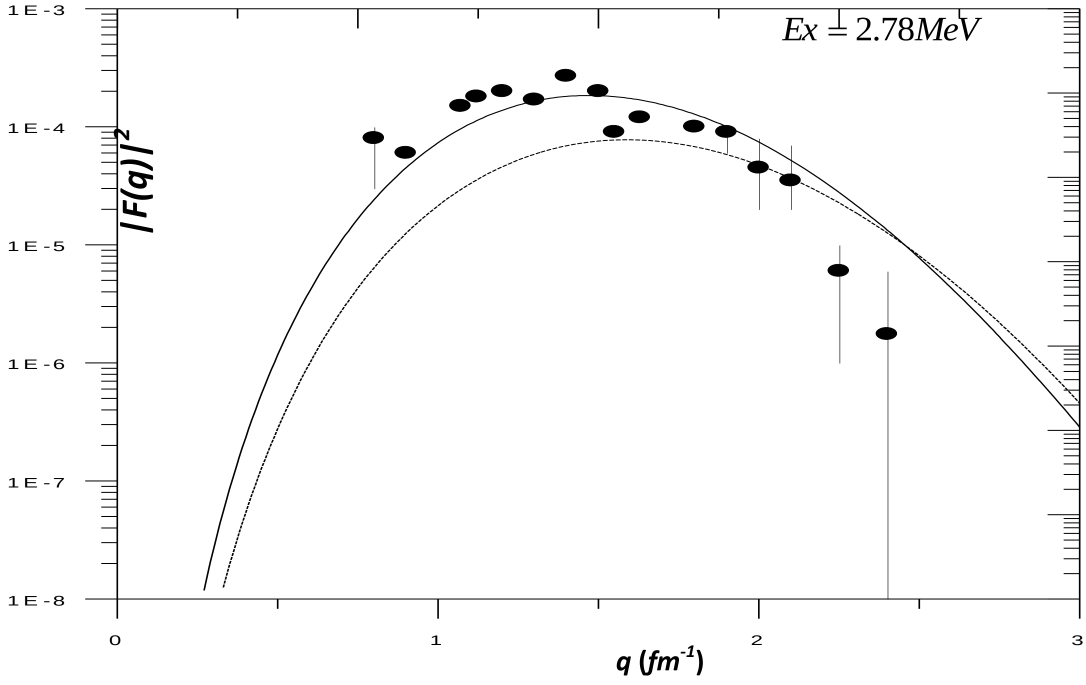

Figure 1(b)- The C4 form factors for the 9+/2-state, are compared with the experimental data [21].

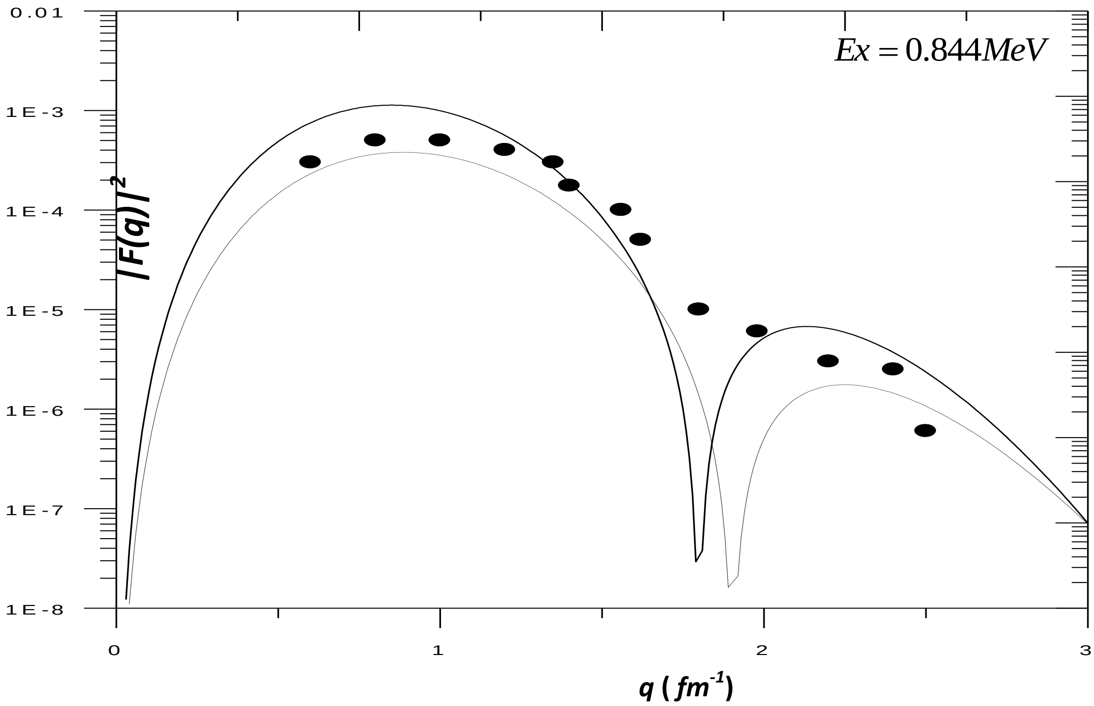

Figure 2(a)-The C2 form factors for the $1^{+}/2$-state, are compared with the experimental data [22].

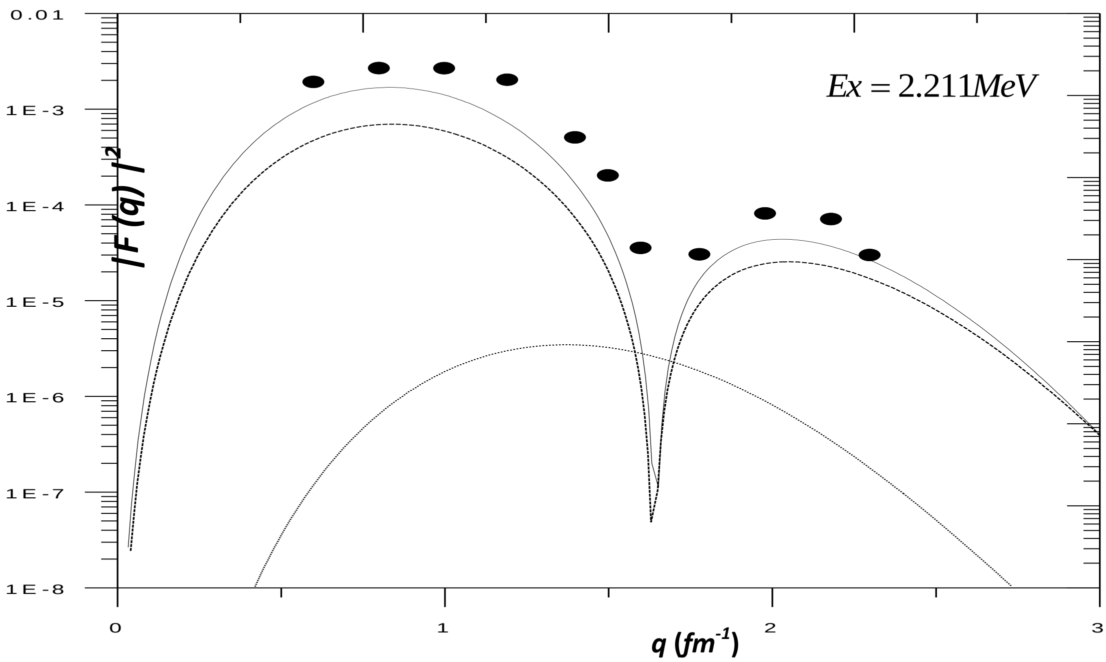

Figure 2(b)- The inelastic longitudinal (C2+C4) form factors for the $\left(7^{+}/2\ \ 1/2\right)$ are compared with the experimental data [23].

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