AIDING AND OPPOSING MIXED CONVECTION FROM AN ARRAY OF CIRCULAR CYLINDERS IN A SATURATED POROUS MEDIUM

Mohammed A. Ahmed
Mechanical Engineering Department, University of Anbar

ABSTRACT

Numerical solutions are presented for mixed convection from an array of circular cylinders embed in a saturated porous medium. The cylinders are at constant temperature(isothermal) and arranged in a staggered tube bank. Both aiding and opposing flow conditions are considered. Numerical calculations using finite difference method with body-fitted coordinates have covered a wide range of governing parameters(i.e.,$10 \leq Re \leq 100$, $0 \leq Gr \leq 400$ and $Pr = 0.7$). Results are presented for streamline, isotherms and the local and the average Nusselt number at different values of the governing parameters. The present results are compared with previous theoretical results and show good agreement.

Key words: Laminar flow, Mixed convection, Porous medium, Tube banks, Numerical study.

INTRODUCTION

Combined forced and natural convection heat transfer from an array of cylinders in a saturated porous medium continues to be an important problem, due to its fundamental nature as well as its many related engineering applications in compact heat exchangers, utilization of geothermal energy, solar power collectors and many more. For single cylinder problem, a good number of numerical studies have been carried out in the past. Cheng(1982) presented the problem of mixed convection over a horizontal, isothermal, cylinder embedded in a saturated porous medium using boundary layer approximation. Later, the problem was extended to include a non-isothermal cylinder by Minkowy etal.(1985) and constant flux cylinder by Huang etal.(1986). All three above studies are treated the case of aiding flow only. Badr and Pop(1988) considered the problem of aiding and opposing flows around a horizontal cylinder buried in a porous medium. Zhou and Lai(2002) considered the same problem were solved by Badr and Pop(1988) based on Darcy's law and using finite difference method with body-fitted coordinates. Mohammed(2007)studied the effect of flow direction on mixed convection from a horizontal cylinder in a saturated porous medium. The governing equations based on Darcy's law are expressed in a body- fitted coordinate system and solved numerically by explicit method. The direction of the flow varies between the vertically up ward(aiding flow) and vertically downward(opposing flow). Results are presented for Reynolds number(10-100), Grashof numbers(0-500) and Pr =0.7. the problem of forced convection heat transfer around single circular cylinder and an array of circular cylinders in the presence and no-presence of porous media were carried out by Mohammed and Ali(2004). Experimental results were very few and were reported only by Fand etal.(1987) for cross flow over a horizontal cylinder.

Numerical investigations of combined convection from the horizontal cylinder have been restricted to a single cylinder situations. The problem of mixed convection from array of cylinders has received relatively little attention. In the present work, the problem of aiding and opposing mixed heat transfer around array of cylinders in a saturated porous medium is solved numerically by finite difference method based on the Darcy law. The numerical results are compared with available numerical results in the literature for various ranges of Grashof numbers.

PROBLEM DESCRIPTION

In the present study, consider array of isothermal horizontal cylinders of diameter d and temperature T_w, embedded in a porous medium, with longitudinal pitch S_L and transfer pitch S_T, as shown in Fig.1. A uniform flow is vertically introduced to porous medium. When its direction is parallel to thermal buoyancy, it is called aiding flow; otherwise, its called opposing flow. To approximate the mixed heat convection flow problem, several assumptions have been made. The flow is steady, in compressible and two dimension. The properties of the porous medium and the fluid are isotropic and homogeneous.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The governing equations consist of a continuity equation, Navier-stokes equations, and an energy equation formulated in terms of velocity, pressure, and temperature. Under the Boussinesq approximation, these equations are[9].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u=\frac{-k}{m}\left(\frac{\partial p}{\partial x}\pm r.g\right) \tag{2}$$

$$v=\frac{-k}{m}\frac{\partial p}{\partial y} \tag{3}$$

$$s\,\frac{\partial T}{\partial t}+u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y}=a\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

In Eq.(2), the positive sign represents aiding flow and the negative sign is for opposing flows. After introducing the stream function, the above governing equations can be put into a dimensionless form as shown below.

$$\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}\pm\frac{Gr}{\mathrm{Re}}\frac{\partial q}{\partial Y}=0 \tag{5}$$

$$\frac{\partial q}{\partial t}+\frac{\partial\Psi}{\partial Y}\frac{\partial q}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial q}{\partial Y}=\frac{1}{\mathrm{Re.Pr}}\left(\frac{\partial^2 q}{\partial X^2}+\frac{\partial^2 q}{\partial Y^2}\right) \tag{6}$$

In terms of the body-fitted coordinates, the governing equations are given by

$$\frac{1}{J}\left[l'.\frac{\partial \Psi}{\partial z} + s'.\frac{\partial \Psi}{\partial h} + a'.\frac{\partial^2 \Psi}{\partial z^2} - 2b'.\frac{\partial^2 \Psi}{\partial z \partial h} + g'.\frac{\partial^2 \Psi}{\partial h^2} \right] = \pm \frac{Gr}{\mathrm{Re}}\left(-\frac{\partial q}{\partial z}.\frac{\partial X}{\partial h} + \frac{\partial q}{\partial h}.\frac{\partial X}{\partial z} \right) \tag{7}$$

$$\frac{\partial q}{\partial t} + \frac{1}{J}\left[-\frac{\partial \Psi}{\partial z}.\frac{\partial q}{\partial h} + \frac{\partial \Psi}{\partial h}.\frac{\partial q}{\partial z} \right] = \frac{1}{\mathrm{Re}.pr.J^2}\left[l'.\frac{\partial q}{\partial z} + s'.\frac{\partial q}{\partial h} + a'.\frac{\partial^2 q}{\partial z^2} - 2b'.\frac{\partial^2 q}{\partial z \partial h} + g'.\frac{\partial^2 q}{\partial h^2} \right] \tag{8}$$

The corresponding boundary conditions used for computations are given as follows:

At the upstream $\theta = 0, \quad \Psi = y$ (9a)

Along left side $\Psi = 1, \quad \frac{\partial q}{\partial Y} = 0$ (9b)

Along right side $\Psi = 0, \quad \frac{\partial q}{\partial Y} = 0$ (9c)

On the cylinders wall(right side) $\theta = 1, \quad \Psi = 0$ (9d)

On the cylinders wall(left side) $\theta = 1, \quad \Psi = 1$ (9e)

At the downstream $\frac{\partial \Psi}{\partial X} = 0, \frac{\partial q}{\partial X} = 0$ (9f)

In terms of new coordinates, the temperature gradient in the boundary conditions can be rewritten as

$$\frac{\partial q}{\partial Y} = \frac{1}{J\sqrt{g'}}\left(g'.\frac{\partial q}{\partial h} - b'.\frac{\partial q}{\partial z} \right) \tag{10a}$$

$$\frac{\partial q}{\partial X} = \frac{1}{J\sqrt{a'}}\left(a'.\frac{\partial q}{\partial z} - b'.\frac{\partial q}{\partial h} \right) \tag{10b}$$

Where J is the Jacobian of the transformation and a', b' and g' are the coefficients of coordinate transformation. They are given separately by

$$J = \frac{\partial X}{\partial z}\frac{\partial Y}{\partial h} - \frac{\partial Y}{\partial z}\frac{\partial X}{\partial h} \tag{11a}$$

$$b' = \frac{\partial X}{\partial z}\frac{\partial X}{\partial h} - \frac{\partial Y}{\partial z}\frac{\partial Y}{\partial h} \tag{11b}$$

$$a' = \frac{\partial X}{\partial h}\frac{\partial X}{\partial h} - \frac{\partial Y}{\partial h}\frac{\partial Y}{\partial h} \tag{11c}$$

$$g = \frac{\partial X}{\partial z}\frac{\partial X}{\partial z} - \frac{\partial Y}{\partial z}\frac{\partial Y}{\partial z} \tag{11d}$$

The geometrical relations used for this study $S_L/D = 2.3$ and $S_T/D = 1.3$. After a series of test runs, a mesh size(105x20) was chosen for all cases. The computation time step used in the present study was 10^{-3} .

CALCULATION THE LOCAL AND THE AVERAGE NUSSELT NUMBERS

From the balance of heat flux at the surfaces, the Nusselt number can be obtained as

$$Nu = -\frac{\partial q}{\partial n} \tag{12}$$

In terms the new coordinates, the Nusselt number in Eq.(12) can be rewritten as follows (Broughton etal., 1986)

$$Nu = \frac{1}{J\sqrt{g'}}\left(g'.\frac{\partial q}{\partial h} - b'.\frac{\partial q}{\partial x} \right) \tag{13}$$

Since the temperature along the wall cylinder is constant Eq.(13) becomes:

$$Nu = \frac{g'}{J\sqrt{g'}}\frac{\partial q}{\partial h} \tag{14}$$

The Nusselt number in Eq.(14) is the local Nusselt number. To find the average Nusselt, the local Nusselt number should be integrated by using numerical integration[1].

RESULTS AND DISCUSSION

Mixed convection heat transfer from an array of circular cylinders in a saturated porous medium is studied numerically for two cases when the forced flow is directed either vertically upward (aiding flows) or vertically downward (opposing flow). The validity of numerical model is tested on the problem of mixed convection around single cylinder in a saturated porous medium. Since there are no experimental data or numerical solutions available for the case of aiding and opposing mixed convection around array of circular cylinders in a saturated porous medium, our results against the numerical solution of Zhou and Lai(2002) are compared to validate the present model. Fig.2 shows that the average Nusselt numbers obtained in the present study are in good agreement with those obtained by Zhou and Lai (2002).

Fig.3 depicts the streamlines and isotherms for case of aiding flow for various Grashof numbers with fixed Re =100 and Pr =0.7. When thermal buoyancy is absent (i.e., Gr =0), the flow field is identical to that of a potential flow over an array of cylinders even the actual physical significance is different. With an increase in the Grashof number, the flow field is perturbed by thermal buoyancy and more fluid is accelerated through the central region. As a result, the thermal plume is extended further downstream.

Fig.4 gives the streamline and isotherm for case of opposing flow for different values of Grashof numbers at Re =100. it can be seen that the flow and temperature fields are very different from those of aiding flows. Also, it is found from Fig.4, a buoyancy-induced recirculating cell is present near the cylinders. Although the strength of the recirculating cell is considerably weaker than the primary flow, its presence can still be cleary observed. For a given Reynolds number, the strength of the recierculating cell increases with the Grashof number. Consequently, the buoyancy-influenced region extends both upstream and down stream.

Fig.5 shows the local Nusselt number distribution around cylinders surface at Re =100 and Gr = 0. it is clear from Fig.5. The maximum Nusselt number occurs near front stagnation point and the minimum Nusselt number occurs at the rear stagnation point (γ =180^o) for all cylinders. In aiding flows, see Fig.6, the local Nusselt number increases with increasing in Grashof number for all cylinders. The maximum Nusselt number remains near front stagnation point and the minimum Nusselt number remains at the rear stagnation point (γ =180^o) for all cylinders=180^o) for all cylinders. In opposing flows, see Fig.7, the minimum Nusselt number occurs at(γ =15^o) and the maximum Nusselt number occurs at (γ =180^o)for first cylinder. For second and third cylinder, the maximum Nusselt number occurs near front stagnation point and the minimum Nusselt number occurs at (γ =30^o) for second cylinder and at (γ =50^o) for third cylinder.

Fig.8 gives the average Nusselt number for each cylinder at Re =100. For aiding flows, the average Nusselt number increases with an increase in the mixed convection parameter Gr/Re for all cylinders. For opposing flows, the average Nusselt number for first cylinder decreases with an increase in the mixed convection parameter Gr/Re. After it reaches a minimum, which occurs at – 4< Gr/Re >-3, the average Nusselt number then starts to increase with mixed convection parameter. For second and third cylinders, the average Nusselt number increase with mixed convection parameter.

Fig.9 gives the total average Nusselt number for array of cylinders with different values of Reynolds numbers. From the curve we find that the average Nusselt number increase with increase in the mixed convection parameter for Re =10, 20 and 50. When Re =100, the average Nusselt number remain constant for opposing flows, which occurs at – 4< Gr/Re >-3, the average Nusselt number then starts to increase with mixed convection parameter.

CONCLUSIONS

Mixed convection from an array of circular cylinders embedded in a saturated porous medium has been studied numerically by finite difference method. The present results agree very well with the previous study(Zhou and Lai, 2002) . The results give detailed analysis of the local and the average Nusselt number with different Reynolds and Grashof numbers for aiding and opposing flows. From these results, the following conclusions can be drawn.

1. For aiding flows, the total heat transfer for array an of cylinders is found to increase with Grashof number at fixed Reynolds number. As Reynolds number increases the total heat transfer increases.
2. In opposing flows, the flow inertia of the primary flow tends to push the secondary flow downstream, and while the thermal buoyancy tends to lift the secondary cell upstream.

Consequently, the total heat transfer for an array of cylinders decreases when Grashof number increases.

REFERENCES

[1] AL-Khafaji, A., W. & Tooley, J., R., *Numerical Methods in Engineering Practice*, 1st Edition, CBS Publishing, Japan Ltd, 1986.

[2] Broughton, R., C. & Oliver, A., J., *A Numerical Model for Convection in Complex–Dimensional Geometries and it is Application to Buoyancy flow in Power Cable*, International Heat Transfer Conferences, vol. 2, pp. 447 - 451, 1986.

[3] Badr, H. M., and Pop, I., *Combined Convection from an Isothermal Horizontal Rod Buried in a Porous Medium,* Int. J. Heat and Mass Transfer, vol. 31, no. 12, pp. 2527 - 2541, 1988.

[4] Cheng, P., *Mixed Convection about a Horizontal Cylinder and Sphere in a fluid-Saturated Porous Medium,* Int. J. Heat and Mass Transfer, vol. 25, no. 8, pp. 1245 -1247, 1982.

[5] Fand, R. M., and Phan, R. T., *Combined Forced and Natural Convection Heat Transfer from a Horizontal Cylinder Embedded in a Porous Medium,* Int. J. Heat and Mass Transfer, vol. 30, no. 7, pp. 1351 - 1358, 1987.

[6] Huang, M. J., Yih, K. A., Chou, Y. L., and Chen, C. k., *Mixed Convection Flow over a Horizontal Cylinder or Sphere Embedded in a Saturated Porous Medium,* J. Heat Transfer, vol. 108, no. 2, pp.469 - 471, 1986.

[6] Minkowycz, M. J., Cheng, P., and Chang, C. H., *Mixed Convection about a non Isothermal Cylinder and Sphere in a Porous Medium, Numerical* Heat Transfer, vol. 8, no.3, pp.349 - 359, 1985.

[7] Mohammed, L., and Ali, N., *Fluid Flow and Heat Transfer around Circular Cylinders in the Presence and No- Presence of Porous Media,* J. Porous Media, vol. 7, no. 3, pp.239 - 247, 2004.

[8] Mohammed, A., *On The Effect Of Flow Direction on Mixed Convection from a Horizontal Cylinder in a Saturated Porous Meduim,* Anbar Journal of Engineering Sciences, vol. 1, no. 1, pp.77 – 87, 2007.

[9] Zhou, M. J., and Lai, F.C, *Aiding and Opposing Mixed Convection from a Cylinder in a Saturated Porous Medium*, J. Porous Media, vol. 5, no. 2, pp.103 - 111, 2002.

NOMENCLATURE

List of symbols

d	cylinder diameter,(m)
g	gravity acceleration,(m/s^2)
Gr	Grashof number, $kg\beta(T_w - T_\infty).d/\upsilon^2$
n	outer normal vector
Nu	local Nusselt number (h.d/k)
$\overline{Nu}$	average Nusselt number ($\overline{h}$.d/k)
h , $\overline{h}$	local and average heat transfer coefficients, $(w/m^2.k)$
J	Jacobain of transformation
K	porous medium permeability,(m^2)
k	thermal conductivity of porous medium, (w/m.k)
p	pressure,(pa)
Pr	Prandtl number, (υ/α)
Re	Reynolds number, $(u_\infty.d/\upsilon)$
S_L	longitudinal pitch(m)
S_T	transfer pitch(m)
t	time, (s)
T	temperature, (k)
u ,v	Darcy velocities components,(m/s)
u_∞	free stream velocity,(m/s)
x, y	cartesian coordinates,(m)
X, Y	dimensionless Cartesian coordinates, (X=x/d , Y=y/d)

Greek symbols

ζ , η	body-fitted coordinates
α	thermal diffusivity of porous medium, $k_m/(\rho\, cp)_f$ (m^2/s)
$\alpha', \beta', \lambda', \gamma'$	transforming coefficients
β	thermal expansion coefficient, (k^{-1})
ρ	density of fluid, (kg/m^3)
υ	kinematic viscosity, (m/s^2)
σ	ratio of thermal capacities between porous medium and fluid, $(\rho cp)_m/(\rho\, cp)_f$
τ	dimensionless time, $(t\, u_{\infty/}\sigma.d)$
γ	angular position
ψ	stream function
Ψ	dimensionless stream function, $(\psi/d.u_\infty)$
θ	dimensionless temperature, $(T - T_\infty)/(T_w - T_\infty)$

Subscripts

f	fluid
m	porous medium
w	wall
∞	free stream

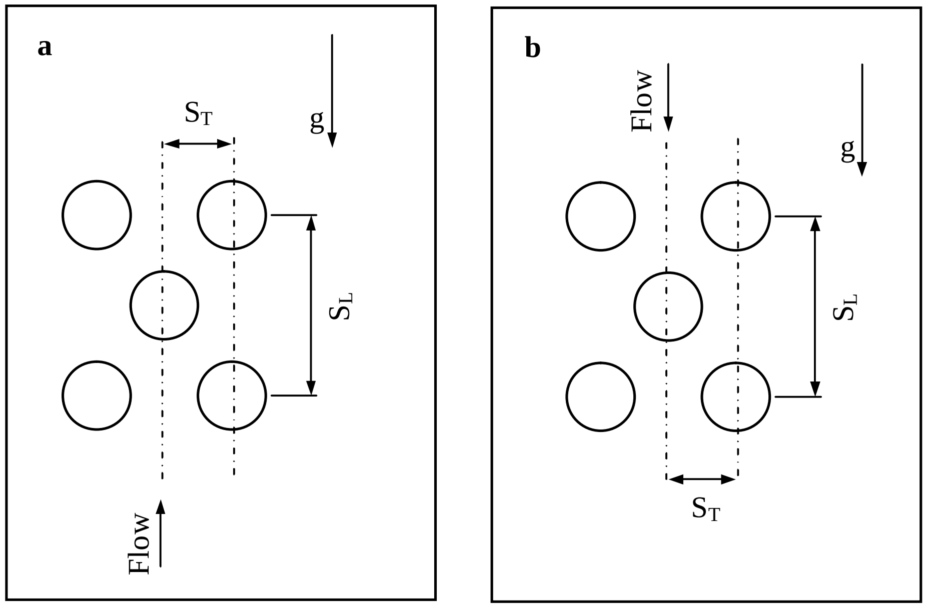

Figure (1): Mixed convection from an array of cylinders embedded in a saturated porous medium: (a) aiding flow, (b) opposing flow.

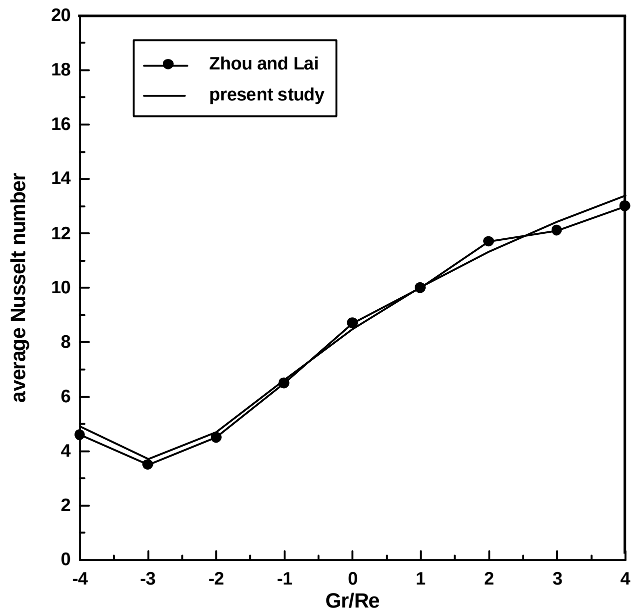

Figure (2): Comparison between present study and previous numerical result at Re =100.

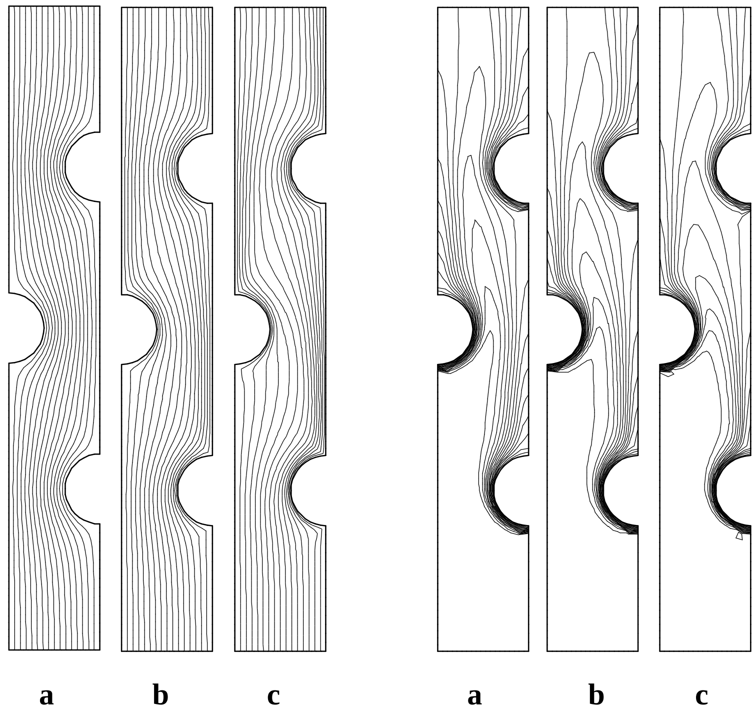

Figure (3): Streamline (left) and isotherms (right) contours for aiding flows at Re =100: (a) Gr = 0, (b) Gr =200, (c) Gr = 400.

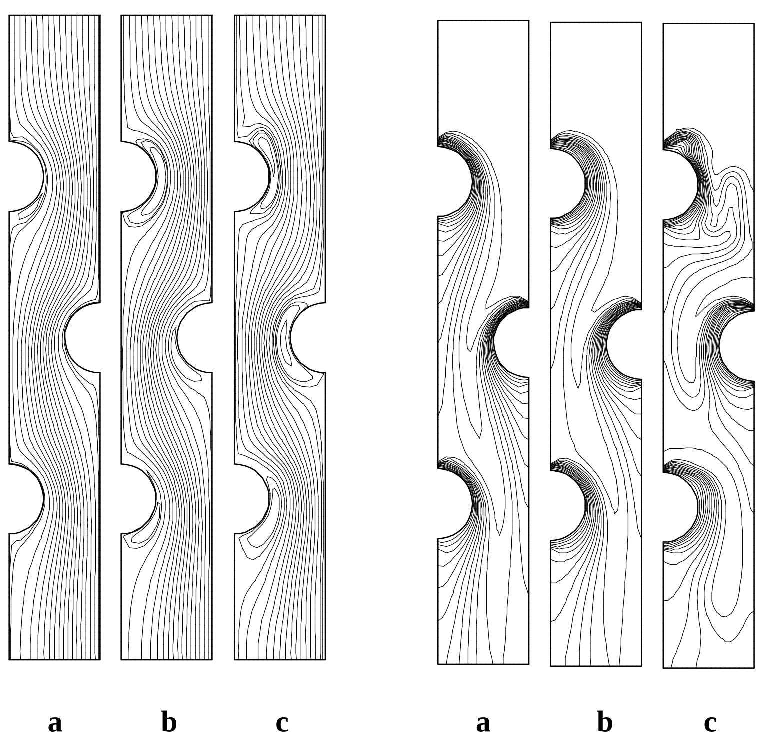

Figure (4): Streamline (left) and isotherms (right) contours for opposing flows at Re=100: (a) Gr =200, (b) Gr =300, (c) Gr =400.

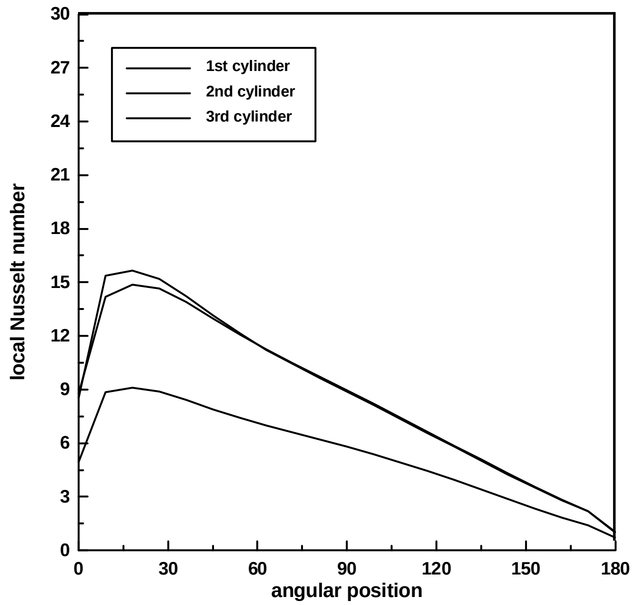

Figure (5): Local Nusselt number distribution around cylinders surfaces at Re =100 and Gr = 0.

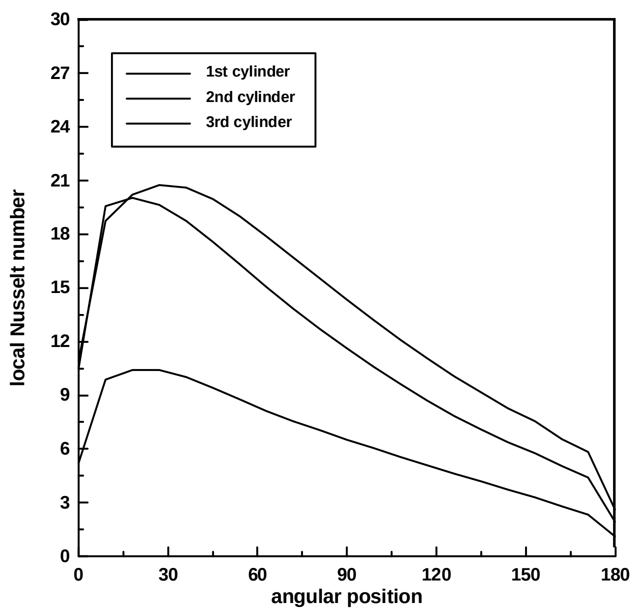

Figure (6): Local Nusselt number distribution around cylinders surfaces for aiding flows at Re =100 and Gr =400.

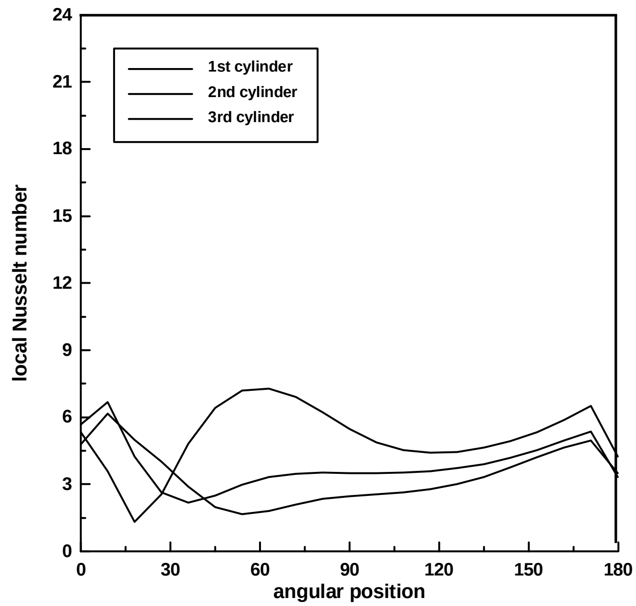

Figure (7): Local Nusselt number distribution around cylinders surfaces for opposing flows at Re =100 and Gr = 400.

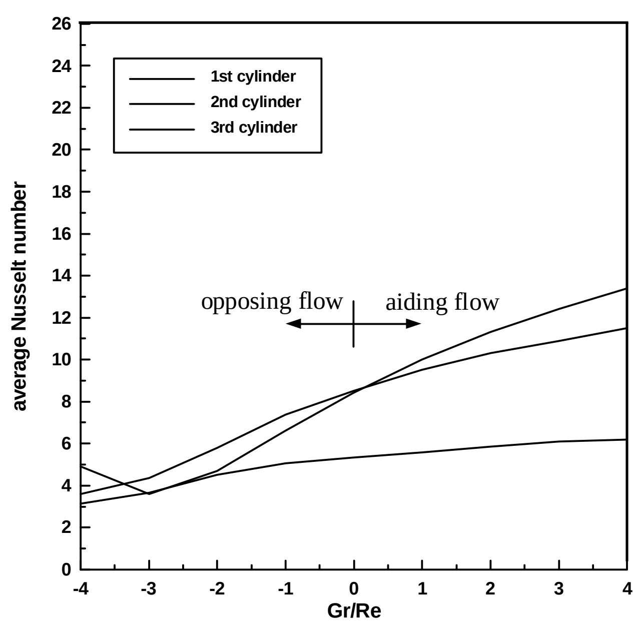

Figure (8): Variation of the average Nusselt number for three cylinders with Gr/Re at Re=100.

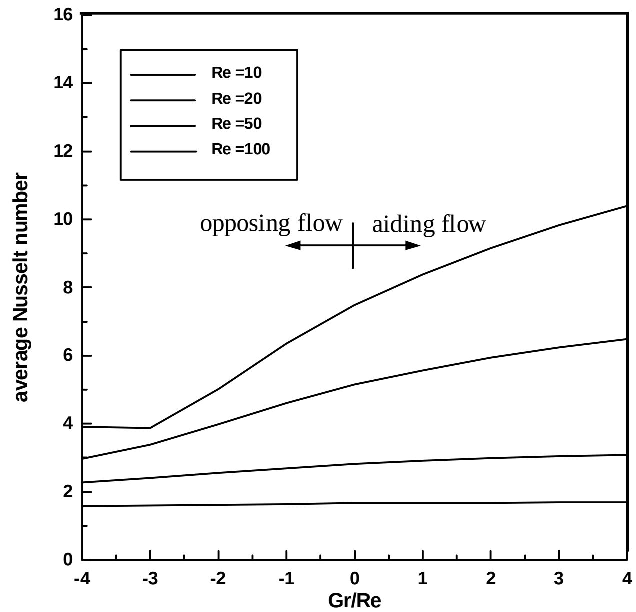

Figure (9): Variation of the total average Nusselt number for array of cylinders with Gr/Re for different Reynolds number.

الحمل المختلط المساعد والمعاكس من صف اسطوانات دائرية في وسـط مسامي مشبع

محمد عبد احمد
قسم الهندسة الميكانيكية، جامعة الانبار

الخلاصة

أجريت دراسة عددية للحمل المختلط من صف اسطوانات دائرية موضوعة في وسط مسامي مـشبع. الاسطوانات عند درجة حرارة ثابتة وحزمة الأنابيب مرتبة بشكل متخالف. تم دراسة كل مـن شـرط الجريان المساعد والمعاكس.استخدمت الحلول العددية طريقة الفـروق المحـددة مـع نظـام مطابقـة إحداثيات

الجسم وغطت مدى واسع من المتغيرات الحاكمة (100≤ عـدد رينولـدز ≤ 10 ، 400 ≤ عـدد كرشوف ≤ 0 و عدد براندتل= 0.7). مثلت النتائج لخطوط الانسياب، درجة الحرارة وعدد نسلت الموضعي و المعدل عند قيم مختلفة للمتغيرات الحاكمة. قورنت النتائج الحالية مع نتائج عددية متوفرة في البحوث السابقة و أوضحت توافق جيد.