On generalized Szasz-Bernstein –Type Operators

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Abstract

Resently Dr.R.P. Pathak and Shiv Kumar Sahoo in 2012 intrudes a new modified Szasz-Bernstein –type operators, in the present paper, we introduce generalize Szasz- Bernstein- type operators $\varphi B_n(f;x)$, we proved that the operators are converge to the function being approximation. In addition, we establish a Voronovaskaja- type asymptotic formula for this operators .

Keywords: Linear positive operators, Korovkin theorem, Voronovaskaja- type asymptotic formula.

الخلاصة

مؤخرا باثاك وكمرفي 2012 قدم مؤثرات جديدة من نوع زاز–برنستين المحسن ,في البحث الحالي نقدم تعميم هذا البحث نقدم تعميم مؤ,ثرات من نوع زاز برنستين $\varphi B_n(f;x)$, برهنا ان هذه المؤثرات تتقارب الى دالة التقريب بالإضافة الى ذلك نا قشنا صيغة فرونوفسكي لتلك المؤثرات.

الكلمات المفتاحية: مؤثرات خطية موجبة, مبرهنة كوروفكن, صيغة فرونوفسكي.

1. Introduction

In [Deo *et. al.*, 208 Introducing a new Bernstein type special operators $B_n(f;x)$

defined as : $B_n(f;x) = \sum_{k=0}^{\infty} p_{n,k}(x) f\left(\frac{k}{n}\right)$,

where, $p_{n,k}(x) = \left(1+\frac{1}{n}\right)^n \binom{n}{k} x^k \left(\frac{n}{n+1} - x\right)^{n-k}$, $0 \le x \le \frac{n}{n+1}$ (1.1)

Moreover, given the integral modification defined as:

$$L_x(f;x) = n\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{n} p_{n,k}(x) \int_0^{\frac{n}{n+1}} p_{n,k}(t) f(t)dt \qquad (1.2)$$

In (Mortici, 2009) defines a new class of linear positive operators which generalize the Szasz-Mirakjan operators for the analytic function $\varphi: \mathbb{R} \to [0,\infty)$ and

$$\varphi S_n: C^2([0,\infty)) \to C^\infty([0,\infty)) \quad , n \in \mathbb{N} = \{1,2,3, \dots\} \ as:$$

$$\varphi S_n(f;x) = \frac{1}{\varphi(nx)} \sum_{k=0}^{\infty} \frac{\varphi^{(k)}(0)}{k!} (nx)^k f\left(\frac{k}{n}\right) f \in C^2([0,\infty)). \quad (1.3)$$

Where $C^2([0,\infty)) = \left\{f \in C([0,\infty)) : \lim_{x\to\infty} \frac{f(x)}{1+x^2} \ exist\ and\ is\ finite\right\}$

If $\varphi(y) = e^y$, we have the classical Szasz-Mirakjan operators.

In (Pathak and Shiv, 2012) introduce the new modified operators as:

$$B_n(f;x) = n\left(1+\frac{1}{n}\right)^2 e^{-nx} \sum_{k=0}^{\infty} \frac{(nx)^k}{k!} \int_0^{\frac{n}{n+1}} p_{n,k}(t)\, f(t)dt. \quad (1.4)$$

Where, $x, t \in [0, \frac{n}{n+1})$.

In this paper, we investigate the new sequence of linear positive operators $\varphi B_n(f;x)$ define as:

$$\varphi B_n(f;x) = \frac{n}{\varphi(nx)}\left(n + \frac{1}{n}\right)^2 \sum_{k=0}^{\infty} \frac{\varphi^{(k)}(0)}{k!} (nx)^k \int_0^{\frac{n}{n+1}} p_{n,k}(t)\, f(t)dt. \quad (1.5)$$

We will prove that the operators are converging to the function being approximation. In addition, we discuss a Voronovaskaja- type asymptotic formula.

2. Preliminary Results

Lemma 1. [Morti2]

The φ Szasz-Mirakjan operators satisfies the following relation:

1) $\frac{1}{\varphi(nx)} \sum_{k=0}^{\infty} \frac{\varphi^{(1)}(nx)}{k!} (nx)^k k = nx \frac{\varphi^{(1)}(nx)}{\varphi(nx)}$ (2.1)

2) $\frac{1}{\varphi(nx)}\sum_{k=0}^{\infty}\frac{\varphi^{(1)}(nx)}{k!}(nx)^k k^2 =$

$$n^2x^2\frac{\varphi^{(2)}(nx)}{\varphi(nx)} + nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)} \qquad (2.2)$$

3) $\frac{1}{\varphi(nx)}\sum_{k=0}^{\infty}\frac{\varphi^{(1)}(nx)}{k!}(nx)^k k^3 =$

$$n^3x^3\frac{\varphi^{3}(nx)}{\varphi(nx)} + 3n^2x^2\frac{\varphi^{(2)}(nx)}{\varphi(nx)} + nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)} \qquad (2.3)$$

4) $\frac{1}{\varphi(nx)}\sum_{k=0}^{\infty}\frac{\varphi^{(1)}(nx)}{k!}(nx)^k k^4 = n^4x^4\frac{\varphi^{(4)}(nx)}{\varphi(nx)} + 6n^3x^3\frac{\varphi^{3}(nx)}{\varphi(nx)} + 7n^2x^2\frac{\varphi^{(2)}(nx)}{\varphi(nx)}$

$$+nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)} \qquad (2.4)$$

Lemma 2.

For any $x \in [0,\infty)$, $n \in \mathbb{N}$ and $e_i = x^i, i = 1,2,3,4$

the operators φB_n satisfies the following relations:

1) $\varphi B_n(e_0; x) =$

$$1 \qquad (2.5)$$

2) $\varphi B_n(e_1; x) =$

$$\frac{n}{(n+1)(n+2)}\left[nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)} + 1\right] \qquad (2.6)$$

3) $\varphi B_n(e_2; x) =$

$$\frac{n^2}{(n+1)^2(n+2)(n+3)}\left[n^2x^2\frac{\varphi^{(2)}(nx)}{\varphi(nx)} + 4nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)} + 2\right] \qquad (2.7)$$

4) $\varphi B_n(e_3; x) = \frac{n^3}{(n+1)^3(n+2)(n+3)(n+4)}\left[n^3x^3\frac{\varphi^{(3)}(nx)}{\varphi(nx)} + 9n^2x^2\frac{\varphi^{(2)}(nx)}{\varphi(nx)} + 18nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)} + 6\right] \qquad (2.8)$

5) $\varphi B_n(e_4; x) = \frac{n^4}{(n+1)^4(n+2)(n+3)(n+4)(n+5)}\left[n^4x^4\frac{\varphi^{(4)}(nx)}{\varphi(nx)} + 16n^3x^3\frac{\varphi^{(3)}(nx)}{\varphi(nx)} + 72n^2x^2\frac{\varphi^{(2)}(nx)}{\varphi(nx)} + 96nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)} + 24\right] \qquad (2.9)$

Proof.

1) $\varphi B_n(e_0;x) = \frac{n}{\varphi(nx)}\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \int_0^{\frac{n}{n+1}} p_{n,k}(t)dt$

$$= \frac{1}{\varphi(nx)}\sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k$$

$$= 1$$

2) $\varphi B_n(e_1;x) = \frac{n}{\varphi(nx)}\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \int_0^{\frac{n}{n+1}} p_{n,k}(t)t\,dt$

$$= \frac{n}{\varphi(nx)}\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \int_0^{\frac{n}{n+1}} \left(\frac{n+1}{n}\right)^n \binom{n}{k} t^{k+1}$$

$$\times\left(\frac{n}{n+1}-t\right)^{n-k} dt$$

$$= \frac{n}{\varphi(nx)}\left(n+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \frac{k+1}{n(n+2)}\left(\frac{n}{n+1}\right)^3$$

$$= \frac{n}{(n+1)(n+2)}\left[nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)}+1\right]$$

3) $\varphi B_n(e_2;x) = \frac{n}{\varphi(nx)}\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \int_0^{\frac{n}{n+1}} p_{n,k}(t)t^2\,dt$

$$= \frac{n}{\varphi(nx)}\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \int_0^{\frac{n}{n+1}} \left(\frac{n+1}{n}\right)^n \binom{n}{k} t^{k+2}$$

$$\times\left(\frac{n}{n+1}-t\right)^{n-k} dt$$

$$= \frac{n}{\varphi(nx)}\left(n+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \frac{(k+1)(k+2)}{n(n+2)}\left(\frac{n}{n+1}\right)^4$$

$$= \frac{n^2}{(n+1)^2(n+2)(n+3)}\left[n^2x^2\frac{\varphi^{(2)}(nx)}{\varphi(nx)}+4nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)}+2\right]$$

4) $\varphi B_n(e_3;x) = \frac{n}{\varphi(nx)}\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \int_0^{\frac{n}{n+1}} p_{n,k}(t)t^3\,dt$

$$= \frac{n}{\varphi(nx)}\left(1+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \int_0^{\frac{n}{n+1}} \left(\frac{n+1}{n}\right)^n \binom{n}{k} t^{k+3}$$

$$\times\left(\frac{n}{n+1}-t\right)^{n-k} dt$$

$$= \frac{n}{\varphi(nx)}\left(n+\frac{1}{n}\right)^2 \sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k \frac{(k+1)(k+2)(k+3)}{n(n+2)}\left(\frac{n}{n+1}\right)^5$$

$$= \frac{n^3}{(n+1)^3(n+2)(n+3)(n+4)}\left[n^3x^3\frac{\varphi^{(3)}(nx)}{\varphi(nx)} + 9n^2x^2\frac{\varphi^{(2)}(nx)}{\varphi(nx)} + 18nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)} + 6\right]$$

5) $\varphi B_n(e_4;x) = \frac{n}{\varphi(nx)}\left(1+\frac{1}{n}\right)^2\sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k\int_0^{\frac{n}{n+1}}p_{n,k}(t)t^4\,dt$

$$= \frac{n}{\varphi(nx)}\left(1+\frac{1}{n}\right)^2\sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k\int_0^{\frac{n}{n+1}}\left(\frac{n+1}{n}\right)^n\binom{n}{k}t^{k+4} \times\left(\frac{n}{n+1}-t\right)^{n-k}dt$$

$$=\frac{n}{\varphi(nx)}\left(n+\frac{1}{n}\right)^2\sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k\frac{(k+1)(k+2)(k+3)(k+4)}{n(n+2)}\left(\frac{n}{n+1}\right)^6$$

$$= \frac{n^4}{(n+1)^4(n+2)(n+3)(n+4)(n+5)}\left[n^4x^4\frac{\varphi^{(4)}(nx)}{\varphi(nx)} + 16n^3x^3\frac{\varphi^{(3)}(nx)}{\varphi(nx)} + 72n^2x^2\frac{\varphi^{(2)}(nx)}{\varphi(nx)} + 96nx\frac{\varphi^{(1)}(nx)}{\varphi(nx)} + 24\right]$$

Lemma 3.

For the operators $\varphi B_n(f;x)$ get the following relation:

1) $\varphi B_n\big((t-x);x\big) = \frac{n\left[nx\left(\frac{\varphi^{(1)}(nx)}{\varphi(nx)}-1\right)+(1-3x)\right]-2x}{(n+1)(n+2)}$ (2.10)

2) $\varphi B_n((t-x)^2;x) = \frac{n^2}{(n+1)^2(n+2)(n+3)}\Big[n^2x^2\left(\frac{\varphi^{(2)}(nx)}{\varphi(nx)} - 2\frac{\varphi^{(1)}(nx)}{\varphi(nx)} + 1\right) + n\left\{\left(7-8\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right)x^2 + \left(4\frac{\varphi^{(1)}(nx)}{\varphi(nx)}-2\right)x\right\} + \left((17-6\frac{\varphi(nx)}{\varphi(nx)})x^2 - 8x + 2\right)\Big]$

$+n(17x^2-6x)+$

$6x^2$ (2.11)

3) $\varphi B_n((t-x)^3;x) = o\left(\frac{1}{n}\right)$ (2.12)

4) $\varphi B_n((t-x)^4;x) = o\left(\frac{1}{n^2}\right)$ (2.13)

Proof.

By little calculations, we get required results (2.10) to (2.13).

3. Main Results

If the function φ verifies

$\lim_{y\to\infty} \frac{\varphi^{(1)}(y)}{\varphi(y)} = \lim_{y\to\infty} \frac{\varphi^{(2)}(y)}{\varphi(y)} =$
1 (3.1)

Then the following convergence theorem holds:

Theorem 1.

For $f \in C\left[0, \frac{n}{n+1}\right]$, the sequence of linear positive operators $\varphi B_n(f;x)$ is converges uniformly to f as $n \longrightarrow \infty$

Proof.

From (2.5), (2.6) and (2.7) we have:

$\varphi B_n(e_0;x) \to e_0$ Uniformly as $n \to \infty$

$\varphi B_n(e_1;x) \to e_1$ Uniformly as $n \to \infty$

$\varphi B_n(e_2;x) \to e_2$ Uniformly as $n \to \infty$

Then from Korovkin theorem, we get:

$\varphi B_n(f;x) \to f$ Uniformly as $n \to \infty$.

Remark. Taking (3.1) into account, for $x \in \left[0, \frac{n}{n+1}\right]$ we have

$$\lim_{n\to\infty}\left(\frac{\varphi^{(1)}(nx)}{\varphi(nx)} - 1\right) = 0$$

And

$$\lim_{n\to\infty}\left(\frac{\varphi^{(2)}(nx)}{\varphi(nx)} - 2\frac{\varphi^{(1)}(nx)}{\varphi(nx)} + 1\right) = 0$$

In the following, we assume there exist γ so that $0 < \gamma \leq 1$ and the function φ verifies the conditions:

$$\lim_{n\to\infty} n^{\gamma}\left(\frac{\varphi^{(1)}(nx)}{\varphi(nx)} - 1\right) = \sigma_1(x) \quad (3.2)$$

$$\lim_{n\to\infty} n^{\gamma}\left(\frac{\varphi^{(2)}(nx)}{\varphi(nx)} - 2\frac{\varphi^{(1)}(nx)}{\varphi(nx)} + 1\right) = \sigma_2(x) \quad (3.3)$$

Where σ_1, σ_2 are functions and $\sigma_1, \sigma_2 \colon [0,\mathbb{R}) \to \infty$

Theorem 2.

For f be an integrable bounded function on $\left[0, \frac{n}{n+1}\right]$ and for $f^{(2)}$ exist at a point

$x \in \left[0, \frac{n}{n+1}\right]$, then

$$\lim_{n \to \infty} n[\varphi B_n(f;x) - f(x)] = \left\{\sigma_2(x)x^2 + \left(7 - 8\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right)x^2 + \left(4\frac{\varphi^{(1)}(nx)}{\varphi(nx)} - 2\right)x\right\}\frac{f^{(2)}(x)}{2}$$

$$+\{\sigma_1(x)x + (1 - 3x)\}f^{(1)}(x)$$

Proof. By Tylor expansion for the function, $f(t)$ we get:

$$f(t) = f(x) + (t - x)f^{(1)}(x) + \frac{(t-x)^2}{2}f^{(2)}(x) + (t - x)^2\xi(t - x) \qquad (3.4)$$

Where $\xi(t - x) \to 0 \;\; as \;\; t \to x$

So to give $\varepsilon > 0$, there exist $\delta > 0$ such that $|\xi(t - x)| \le \varepsilon \quad whenever \;\; |t - x| \le \delta$

Weill, if $|x - t| > \delta$ then there exist $K > 0$ such that:

$$|\xi(t - x)| \le K \le K\frac{(t - x)^2}{\delta^2}$$

Then for any , $t \in \left[0, \frac{n}{n+1}\right]$, we have

$$|\xi(t - x)| \le \varepsilon + K\frac{(t - x)^2}{\delta^2} \qquad (3.5)$$

Now, applying φB_n on (3.2), we have

$$\varphi B_n(f;x) = f(x) + \varphi B_n\big((t - x),\big)f^{(1)}(x) + \varphi B_n((t - x)^2, x)\frac{f^{(2)}(x)}{2}$$

$$+\varphi B_n((t - x)^2\xi(t - x); x)$$

$$= f(x) + \frac{1}{(n+1)(n+2)}\left[n^2\left(\frac{\varphi^{(1)}(nx)}{\varphi(nx)} - 1\right) + n(1 - 3x) - 2x\right]f^{(1)}(x)$$

$$+\frac{1}{(n+1)^2(n+2)(n+3)}\left[n^4\left(\frac{\varphi^{(2)}(nx)}{\varphi(nx)} - 2\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right| + 1\right)x^2$$

$$+n^3\left\{\left(7-8\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right)x^2+\left(4\frac{\varphi^{(1)}(nx)}{\varphi(nx)}-2\right)x\right\}$$

$$+n^2\left\{\left(17-6\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right)-8x+2\right\}$$

$$+n(17x^2-6x)+6x^2]\frac{f^{(2)}(x)}{2}$$

$$+n\left(1+\frac{1}{n}\right)^2\frac{1}{\varphi(nx)}\sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k\int_0^{\frac{n}{n+1}}p_{n,k}(t)(t-x)^2\,\xi(t-x)dt$$

Multiplying by n, we get:

$$n[\varphi B_n(x)-f(x)]$$
$$=\frac{n}{(n+1)(n+2)}\left[n^2\left(\frac{\varphi^{(1)}(nx)}{\varphi(nx)}-1\right)+n(1-3x)-2x\right]f^{(1)}(x)$$
$$+\frac{n}{(n+1)^2(n+2)(n+3)}\left[n^4\left(\frac{\varphi^{(2)}(nx)}{\varphi(nx)}-2\frac{\varphi^{(1)}(nx)}{\varphi(nx)}|+1\right)x^2\right.$$
$$+n^3\left\{\left(7-8\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right)x^2+\left(4\frac{\varphi^{(1)}(nx)}{\varphi(nx)}-2\right)x\right\}$$
$$+n^2\left\{\left(17-6\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right)-8x+2\right\}$$

$$+n(17x^2-6x)+6x^2\,]\frac{f^{(2)}(x)}{2}+nE_n(t,x) \qquad (3.6)$$

$$E_n(t,x)=n\left(1+\frac{1}{n}\right)^2\frac{1}{\varphi(nx)}\sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k\int_0^{\frac{n}{n+1}}p_{n,k}(t)(t-x)^2\,\xi(t-x)dt$$

$$|nE_n(t,x)|=\left|n\left\{n\left(1+\frac{1}{n}\right)^2\frac{1}{\varphi(nx)}\sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k\int_0^{\frac{n}{n+1}}p_{n,k}(t)(t-x)^2\,\xi(t-x)dt\right\}\right|$$

$$n\left\{n\left(1+\frac{1}{n}\right)^2\frac{1}{\varphi(nx)}\sum_{k=0}^{\infty}\frac{\varphi^{(k)}(0)}{k!}(nx)^k\int_0^{\frac{n}{n+1}}p_{n,k}(t)(t-x)^2\,|\xi(t-x)|dt\right\} \qquad (3.7)$$

Then, from (3.3) we have:

$$|nE_n(t,x)|\le n\varepsilon\varphi B_n((t-x)^2;x)+\frac{K}{\delta^2}\varphi B_n((t-x)^4;)$$
$$\le n\varepsilon o\left(\frac{1}{n}\right)+\frac{nK}{\delta^2}o\left(\frac{1}{n^2}\right)$$
$$\le \varepsilon+\frac{K}{\delta^2}o\left(\frac{1}{n}\right)$$

Let $\delta = n^{-1/4}$ we get:

$$|nE_n(t,x)| \leq \varepsilon + Ko\left(\frac{1}{\sqrt{n}}\right)$$

Since ε is arbitrary and small, we get

$|nE_n(t,x)| \to 0 \qquad as\ n \to \infty \qquad (3.8)$

Then,

$$\lim_{n\to\infty} n\,[\varphi B_n(x) - f(x)] = lim_{n\to\infty} \frac{n}{(n+1)(n+2)}\left[n^2\left(\frac{\varphi^{(1)}(nx)}{\varphi(nx)} - 1\right) + n(1-3x) - 2x\right] f^{(1)}(x)$$

$$+ \lim_{n\to\infty} \frac{n}{(n+1)^2(n+2)(n+3)}\left[n^4\left(\frac{\varphi^{(2)}(nx)}{\varphi(nx)} - 2\frac{\varphi^{(1)}(nx)}{\varphi(nx)}| + 1\right)x^2\right.$$

$$+n^3\left\{\left(7 - 8\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right)x^2 + \left(4\frac{\varphi^{(1)}(nx)}{\varphi(nx)} - 2\right)x\right\}$$

$$+n^2\left\{\left(17 - 6\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right) - 8x + 2\right\}$$

$$\left. +n(17x^2 - 6x) + 6x^2\right]\frac{f^{(2)}(x)}{2} + lim_{n\to\infty} n\,E_n(t,x)$$

from (3.2) ,(3.3) and (3.8) ,we get :

$$\lim_{n\to\infty} n[\varphi B_n(f;x) - f(x)] = \left\{\sigma_2(x)x^2 + \left(7 - 8\frac{\varphi^{(1)}(nx)}{\varphi(nx)}\right)x^2 + \left(4\frac{\varphi^{(1)}(nx)}{\varphi(nx)} - 2\right)x\right\}\frac{f^{(2)}(x)}{2}$$

$$+\{\sigma_1(x)x + (1-3x)\}f^{(1)}(x)$$

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