Sensitivity of Initial Conditions and Parameters in A new Four-dimension Hyper Chaotic system

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Abstract

In this paper a chaotic behaviour for a new hyper chaotic system is explore through sensitivity, it is well known that the chaotic systems characteristics with high sensitivity to little difference in the initial conditions, the chaotic behaviour for the new dynamical system will search through its sensitivity by utilizing Mathematica Program, where small change to initial condition can grows and become great when time become large.

Keywords: **sensitivity, Initial conditions, chaos system, influence, parameters.**

المستخلص

في هذا البحث السوك الفوضوي سيتم بحثه لنظام الفوضوي المفرط الجديد من خلال الحساسية ، وكما هو معروف ان الانظمة الفوضوية المفرطة تكون شديدة الحساسية للتغيير الطفيف في الشروط الابتدائية والمعلمات ، حيث سيتم احداث تغيير بسيط في الشروط الابتدائية والمعلمات لنظام الفوضوي المفرط الجديد وبيان مدى تأثير ذلك على السلوك المستقبلي للنظام الفوضوي باستخدام برنامج الماثيماتيكا .

الكلمات المفتاحية : الحساسية ، الشروط الابتدائية ، نظام فوضوي ، التأثير ، المعلمات .

1.Introduction

Chaos is a periodic long-term behaviour in a deterministic dynamical system that show sensitive dependence on initial conditions, Chaos is statistically indiscernible from randomness, yet it is deterministic and not random at all [1], chaotic systems generate same outcomes for the same inputs, but for small change in initial conditions or input, we cannot predict the way in which the system behaviour will change and it is impossible to predict the future behaviour of a chaos system unless the initial conditions are completely know and precisely [2], the first chaotic system famous as Lorenz system, was find out in 1963 by Lorenz for solving equations depict atmospheric flows [3], in 1979 Rossler presented a new dynamical system with four first order differential equations, the Rossler system was considered the first hyper chaotic system and characteristics with complex dynamic behaviour due to two positive Lyapunov exponents that system have [4], according to high complexity and unpredictability of the hyper chaotic systems, such systems used in many application like lasers, encryption, nonlinear circuits, neural network [5], In this paper a new autonomous four-dimension hyper chaotic system is presented, the system employs six cross-product nonlinearities terms and eight positive constant parameters, and generate complex strange attractor with two scroll, where we study the chaotic behaviour of the new system due to its sensitivity to initial conditions change.

2. Construction of new six cross-product nonlinearities terms system

The fourth dimensional autonomous hyper chaotic system include ten terms with six cross-product nonlinearities terms

and capable to generating complex chaotic behaviour , where the attractor can describe by the following form

$$\begin{aligned}\frac{dx}{dt} &= ayz - bxz - cw\\ \frac{dy}{dt} &= dx - xz - y\\ \frac{dz}{dt} &= exy - fz\\ \frac{dw}{dt} &= gxz + hzy\end{aligned} \qquad (1)$$

By fixing a, b, c, d, e, f, g and h to the values $(18,3.1,2,10,3,2.6,5,13)$ respectively, and the initial values for (x_0,y_0,z_0,w_0) assumed as $(0.2,0.4,0.6,0.8)$ respectively, the system (1) exhibit chaotic behavior and the Lyapunov Exponents for the system generated as $L_1= 1.4904$, $L_2= 1.9342$, $L_3= -2.6706$, $L_4= -31.0164$, where Figures $1-4$ show the strange attractor

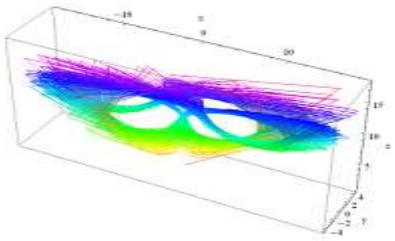

Figure 1. Phase portrait of system in(x,y,z).

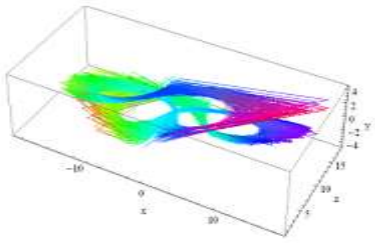

Figure 2. Phase portrait of system in(x,z,y).

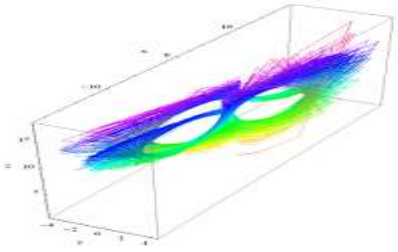

Figure 3. Phase portrait of system in(y,x,z).

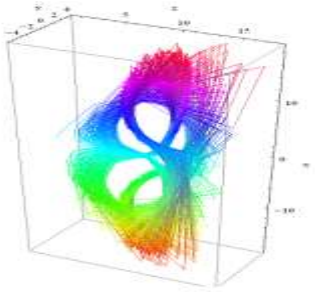

Figure 4. Phase portrait of system in (z,y,x).

3. Sensitivity of a new system to initial conditions

Chaotic system characteristic with its long term unpredictability, due to its sensitivity dependence on initial conditions, where with tiny change between two initial conditions it is impossible to predict the way in which the system can evaluated and will eventually become widely separated, and the state of system at future time cannot be prediction accurately, in Figures 5−9 show that the small difference in initial conditions will occur a great influence in evolution of the chaos trajectories [6]. The initial values of the system are selected as:$x_0=0.2$, $y_0=0.4$, $z_0=0.6$, $w_0=0.8$

For the solid line and x0=0.2, $y_0=0.400001$, $z_0=0.6$, $w_0=0.8$ for the dashed line.

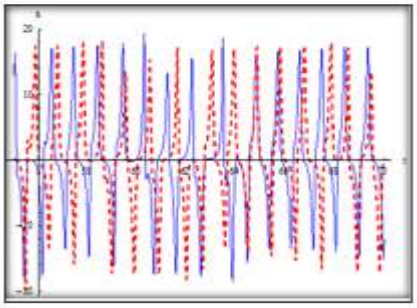

Figure 5.Sensitivity test of the new system x(t)

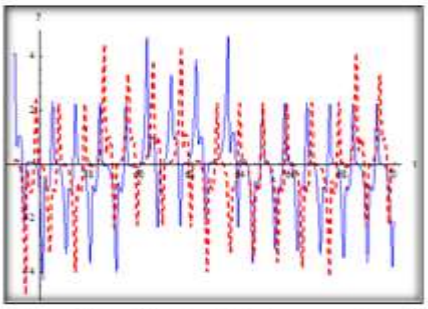

Figure 6.Sensitivity test of the new system y (t)

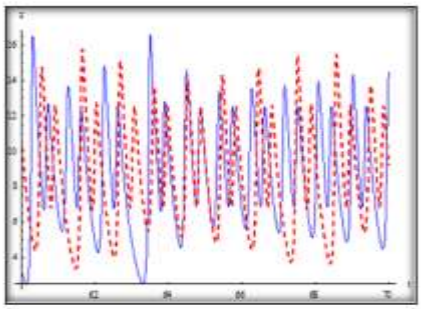

Figure 7.Sensitivity test of the new system z (t)

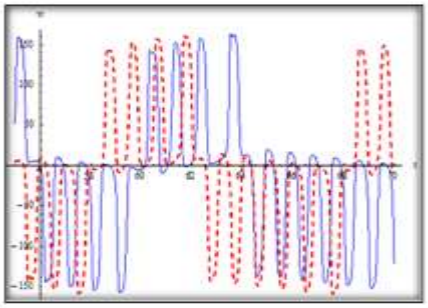

Figure 8.Sensitivity test of the new system w

Obviously, the waveform of system (1) is aperiodic and has much sensitivity to initial conditions and we call it, sensitive dependence on initial conditions.

4. The influence of system parameters

When change the system parameters the system state will change to different state and the stability for system equilibria also change, the numerical simulation utilized Mathematica program to study and analyzed the influence of change system parameters and system conditions on the system state which can show below.

The value of system parameter d increase, without change the other system parameters, with increase d parameter frequently, the system exhibit different behaviors such as fixed points, period-doubling loops, chaos and hyper chaotic as follow [7] :

- d = 0.1, the system is stable, as show in Figure 9.
- d= 1.5, the system exhibit a period-doubling behavior, as show in Figure 10.
- d= 2, the system exhibit also a period-doubling behavior, as show in Figure 11.
- d=3, the system exhibit chaotic behavior as show in Figure 12.
- d=5, the system exhibit hyper chaotic behavior as show in Figure 13.
- d=7, the system also exhibit hyper chaotic behavior as show in Figure 14.

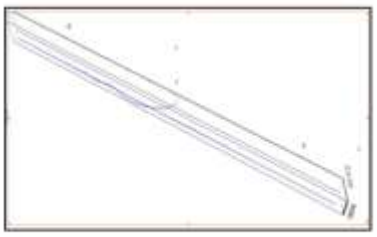

Figure 9. System behavior in x-y-z plane with d=0.1.

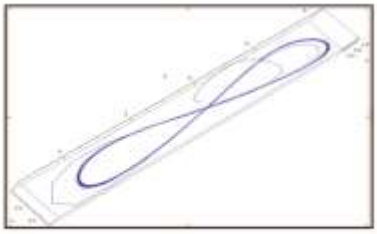

Figure 10. System behavior in x-y-z plane with d=1.5.

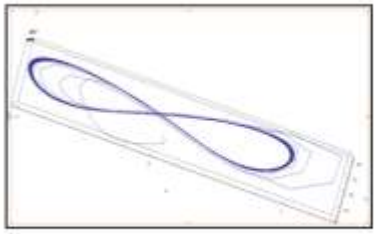

Figure 11. System behavior in x-y-z plane with d=2.

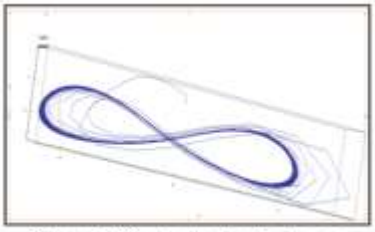

Figure 12. System behavior in x-y-z plane with d=3.

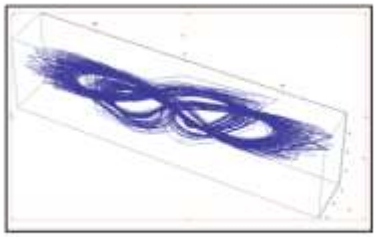

Figure 13: system behavior in x-y-z plane with d=5.

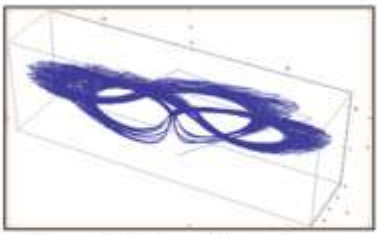

Figure 14. System behavior in x-y-z plane with d=7.

5. Conclusion

A new four dimensional autonomous nonlinear differential equation is presented in this paper, the phase portrait and strange orbit of the proposed hyper chaotic system is illustrate, the system have four Lyapunov Exponents include two positive and generated as $L_1 = 1.4904$, $L_2 = 1.9342$, $L_3 = -2.6706$, $L_4 = -31.0164$,, small value up to 10^{-6} add to initial conditions to prove the sensitivity of the new system for change of initial conditions, the small difference in initial conditions lead to great influence in evolution of the chaos trajectories and confirm the sensitivity for the new system, when the parameter d change this implies to change in a solution (bifurcations) of the nonlinear differential

equations, and great change of the topology for strange attractor and phase portrait where the system exhibit different behaviors such as fixed points, period-doubling loops, chaos and hyper chaotic behaviors.

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