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Stability Analysis of an Earth Dam Using GEO-SLOPE Model under Different Soil Conditions

Abstract Numerical models are used to check the stability of earth dams and to simulate the effects of all the parameters, which affect its safety. The physical properties of the materials used in the construction of an earth dam are one of these important parameters. The finite element analysis software used for numerical modeling in this study is GeoStudio 2007 (SEEP/W and SLOPE/W). The total number of finite elements used to simulate the standard model is 13508 (triangular elements with global size = 2m), while the total number of nodes is 6939. The selected case study is Al-Adhaim dam, which is an earth dam, located in Diyala governorate at the eastern part of Iraq, crossing Al-Adhaim River with a total length of (3.1 km). The major objective of this study is to derive an empirical equation to calculate the factor of safety for earth dams of similar geometries and materials without the need for sophisticated analysis, by assuming different soil conditions. In addition to the soil parameters (the total weight density γ , the angle of internal friction ϕ , the cohesive strength C) for the shell, core and filter within the dam, more parameters have been taken into consideration in predicting the critical factor of safety against slope failure to derive the empirical equation, which are: water depth H (m), coefficient of permeability k (m/s) and Seepage rate Q_s ($m^3/s/m$). The values of the computed factor of safety (F_s) using nine installed slope stability methods: Ordinary, Bishop, Morgenstern-Price, Janbu Generalized, Lowe-Karafiath, Corps of Engineers #1 and #2, Spencer and General Limit Equilibrium (GLE) are close to the safety factors values calculated using the general empirical equation which is a function of the soil and hydraulic parameters of the shell, core and filter within the dam.

Keywords: Slope Stability; GeoStudio 2007; Al-Adhaim Dam; Soil Conditions.

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1. Introduction

Dams can be classified in number of ways but they are mostly classified based on either their function such as storage dams, diversion dams, detention dams, debris dams and coffer dams or based on their design and structure such as gravity dams, buttress dams, arch dams, rock-fill dams and earth dams. In this study, this search will focus on earth dams. Since earth, dams are the most widely spread type of dams in the world; its stability is one of the major concerns in dams engineering. Recently, computer based numerical models are used to check the stability of earth dams and to simulate the effects of all the parameters that play a role in its safety. The materials used in constructing an earth dam are one of these important parameters. Thus, they are considered as the main subject of the study. Reference [1], used an efficient finite element procedure to identify the steady state flow free surface through an earth dam for evaluating the pressure distribution within the embankment. The seepage forces on each element in the finite

element mesh can be determined for stability analysis. Reference [2], studied several problems of Finite Element (FE) slope stability analysis compared with other solution method which is the stress-strain method, including the influence of a free surface on slope and dam stability. The conclusion of their study was that the FE method has been shown to be a reliable and robust method for assessing the factor of safety of slopes. The FE approach for determining the factor of safety of slopes has satisfied the criteria for effective computer-aided analysis. Therefore, it is argued that the finite element method of slope stability analysis is a more powerful alternative to traditional limit equilibrium methods. Reference [3], also used 2D finite element analyses to simulate pore-water pressure development by Geo-Studio software for saturated and unsaturated conditions. Results demonstrated that the presence of a chimney drain plays an important role in the dissipation of the pore-water pressure. Badush dam was chosen as a case study. A high pore-water pressure development was observed, in 8 days due to an

abrupt rise in water level, and it must be taken into consideration when designing a dam. This makes the construction time and the height of the dam are the most remarkable factors governing the pore-water pressure development. Reference [4], have performed in their study, seepage analysis in Yashigou earth dam using SLOPE/W software to evaluate slope stability of the dam. Analyses for each state and each slope with Morgenstern-Price method and finite element stress method are calculated so that the minimum safety factor in each of these methods is considered as a safety factor of slope stability. The two approaches of slope stability analyses, one based on Morgenstern-Price method and the other on the finite element stress method are widely used in geotechnical engineering. Reference [5], have introduced the results of stability analysis of an earth dam using finite element method. Stability study has brought out the importance of considering the coupled effects on the overall stability of the earth dam. It is concluded that the coupled analysis is a prerequisite for the design and performance evaluation of the earth dam under all conditions of seepage and stability. The study shows that increase in the Young's modulus of core and shell resulted in the decrease of the maximum crest displacement and the variation in angle of internal friction plays a vital role in the fulfillment of the overall stability criteria. The slope of 1V:2.5H was adopted for both the upstream and the downstream sides. The factor of safety (FS) was greater than 1.6 for both the full and low reservoir conditions whereas, the FS values were found to be less than the stipulated values for the other stability considerations. The current stability methods that are based on limit equilibrium do not take into consideration the changes in seepage flow and soil permeability in estimating the factor of safety against sliding. Thus, one of the major objectives here is to link these effects to the commonly used methods and produce modified empirical equations based on the principles of the original methods. Then, all the produced empirical equations can be summarized in one general empirical equation. As a result, a new method of calculating factor of safety will be obtained. The benefit of this process is to reduce time, effort and cost spent in the numerical modeling procedure into only one equation. Moreover, the effects of the soil properties used in the construction of earth dams on its stability will be studied so that the governing parameter can be identified.

2. Case Study

Al-Adhaim dam has been chosen as a case for this study. It is a multi-purpose earth dam located (34°33'54"N, 44°30'56"E) in the eastern part of Iraq (Diyala Governorate). The dam was constructed for the purposes of flood control, hydropower and irrigation. Al-Adhaim dam comprises of three fundamental layers, which are shell, core and filters as delineated in Figure 1. The shell is made out of sandy gravel soil, the width at top is (12 m), while the slope of the upstream is (1:2.5) and the downstream slope is (1:2). The core comprises of a silty clay soil sloping towards the upstream of the dam with a slope of (1:1.1) and (1:2) downstream slope. The thickness of the core is (8 m) at elevation (143.5 m) and it progressively increments until it reaches (33 m) at elevation (70 m). The thickness of the core at any point is half the water height at that point. The vertical filter comprises of two layers, which it used for core protection, the thickness of the layer filter (F) is (2 m) and the thickness of layer (T) is (2.5 m). In addition, the horizontal seepage comprises of three layers which have different thicknesses: filter F (0.3 m), filter T (2.5 m), and filter F (0.5 m) adjacent to the foundation of the earth dam, [6].

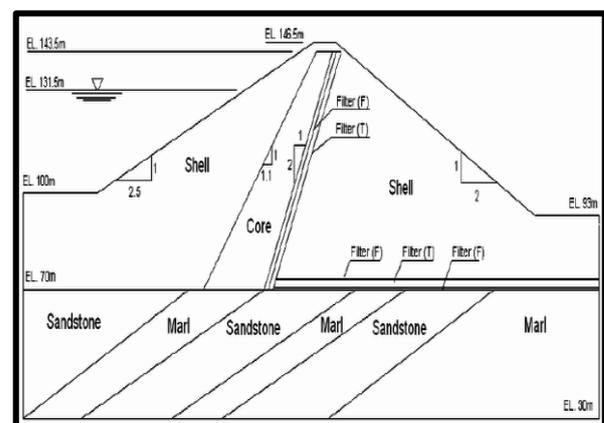


Figure 1: Al-Adhaim dam's cross-section, Al-Labban (2007).

3. Theories of Seepage and Slope Stability

SEEP/W is the software used in this study for simulating the water seepage from a reservoir through an earth dam, and SLOPE/W is the software utilized for computing the factor of safety against slope failure using different methods and under different soil conditions, [7]. The general governing differential equation for two-dimensional seepage can be expressed as:

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial H}{\partial y} \right) + Q = \frac{\partial \theta}{\partial t} \quad (1)$$

Where: H is the total head; k_x and k_y are the hydraulic conductivity in the x and y directions respectively; Q is the flow rate and Θ is the volume water content. This equation expresses that the difference between the input and output flow rates is equivalent to the change in storage of the soil systems. SLOPE/W solves two factors of safety equations (force and momentum). All the used methods of slices can be visualized as special cases of the General Limit Equilibrium solution. A factor of safety is characterized as that variable by which the soil shear strength must be reduced so that the mass of soil will bring into a condition of constraining balance along a chose slip surface.

The shear strength for an effective stress analysis is defined as:

$$S = c' + (\sigma_n - u) \tan \phi' \quad (2)$$

Where: S is the shear strength; C' is the cohesion of the soil; ϕ' is the angle of friction; σ_n is the total normal stress and u is the pore-water pressure. The stability analysis involves passing a slip surface through the earth mass and separating it into vertical slices. The slip surface may be circular, linear or comprises of any shape characterized by a progression of straight lines. The factor of safety with respect to horizontal force equilibrium is:

$$F_f = \frac{\sum(c' \beta \cos \alpha + (N - u\beta) \tan \phi' \cos \alpha)}{\sum N \sin \alpha + \sum kW - \sum D \cos \omega \pm \sum A} \quad (3)$$

And the factor of safety with respect to moment equilibrium is:

$$F_m = \frac{\sum(c' \beta R + (N - u\beta) R \tan \phi')}{\sum W_x - \sum N_f + \sum kW_e \pm \sum Dd \pm \sum Aa} \quad (4)$$

Where: W is the slice weight; N is the normal force on the base of the slice; D is an external load; kW is the horizontal seismic load; R is the radius for a circular slip surface; x and e are the horizontal and vertical distances from the centerline of each slice to the center of rotation respectively; d is the vertical distance from a point load to the center of rotation; a is the vertical distance from the resultant external water force to the center of rotation; A is the resultant external water forces and ω, α, β are the angle of the load from the horizontal, the angle between the tangent to the center of the base of each slice and the horizontal and slice base length respectively. The General Limit Equilibrium formulation and solution can be utilized to simulate most of the used methods of slices. The

methods of slices can be categorized in terms of the conditions of static equilibrium satisfied and the assumption regarding the inter-slice forces. Table 1 summarizes the conditions of static equilibrium utilized by different methods of slices. Table 2 summarizes the assumptions utilized in different equilibrium methods [8].

4. Methodology

In order to get the required results and meet the study objectives, the following steps were followed:

Step 1: Within a specified range of soil parameters for soils used in earth dams, six values deviating from the original design values were obtained, three of them are greater than the original values and the other three are lesser than the original values as shown in Table (3), [9].

Step 2: After drawing the geometry of Al-Adhaim dam's cross-section using SEEP/W component, specifying the regions of the dam's body and assigning the material for each region, the first seepage analysis was made. The standard values of the soil parameters were adapted to make a comparison between the simulated and the design seepage rates. This is necessary to check whether the model is giving an acceptable simulation or not.

Step 3: The results of the first seepage analysis which is based on the standard properties of the dam materials, are illustratively transferred from SEEP/W (as a primary analysis) into SLOPE/W to conduct the stability analyses. The nine methods which were selected to calculate the factor of safety against sliding only, since Sliding is expected to be the most dominant mechanism in earth dam slopes especially in the upstream. Thus, nine branching slope stability analyses from the primary SEEP/W analysis; were conducted under the standard soils conditions.

Step 4: The results of the stability analyses using SLOPE/W for the standard soils conditions are checked to see whether they match the predicted performances of each method or not. Therefore, if the results are as expected then it is safe to proceed with the next trials because it means that SLOPE/W model is giving realistic results and there are no errors in data input and/or calculations. The minimum acceptable value of factor of safety against sliding in earth dams is 1.5 which states that the resisting force is 1.5 times greater than the driving force in a sliding failure curve [10].

Step 5: The first water elevation used in the seepage analysis was 131.5m above mean sea level, which represents the water height at the spillway level (the maximum pool water level),

and it is expected to be considered as one of the critical cases regarding slope failures in earth dams. The second adapted W.L. is the minimum pool water level (MPWL) which represents the minimum water height at which the water can pass through the dam outlet from the reservoir side (Upstream) to the riverside (Downstream); in Al-Adhaim dam, it is 118m above mean sea level. Thus, the seepage and stability analyses are repeated and the results are checked similarly. The reason why these two heights have been chosen is that they contain all the possible ranges of water heights that can affect upstream slope's stability.

Step 6: To understand the influence of soil parameters on slope stability, the six soil case types have been used in addition to that of the case study, these values are deviated from the

standard values by fixed step sizes, see Table 3. Nevertheless, it is mandatory to check whether the changes in seepage rates will significantly affect the values of factor of safety or not; especially after assuming a different coefficient of permeability (k) for each case type and under both water levels as shown in Table 4.

Step 7: To start investigating the effect of seepage rates on safety factors values under both upper and lower water levels, it is logical to start with the two most critical cases Max3 and Min3 which represent the highest and lowest soil parameters values respectively. Each SEEP/W analysis is the primary analysis for nine SLOPE/W analyses, which is equal to the number of the used methods.

Table 1: Conditions of static equilibrium utilized by different methods of slices.

Method	Force Equilibrium		Moment Equilibrium
	Vertical Direction	Horizontal Direction	
Ordinary	utilized	Not utilized	utilized
Bishop	utilized	Not utilized	utilized
Janbu	utilized	utilized	Not utilized
generalized			
Spencer	utilized	utilized	utilized
Morgenstern-price	utilized	utilized	utilized
GLE	utilized	utilized	utilized
Corps of Engineers	utilized	utilized	Not utilized
Lowe-Karafiath	utilized	utilized	Not utilized

Table 2: Assumptions utilized in different equilibrium methods.

Method	Assumption
Ordinary	Interstice forces are neglected
Bishop	Resultant inter-slice forces are horizontal (interstice shear forces equal to zero)
Janbu generalized	Line of thrust is assumed for the definition of the location of the inter-slice normal force
Spencer	constant slope is utilized for calculating Resultant inter-slice forces
Morgenstern-price	Determination of the interstice forces are limited by utilizing an arbitrary function(Direction of the resultant)
GLE	Determination of the resultant inter-slice forces are limited by utilizing an arbitrary function
Corps of Engineers	Direction of the resultant interstice force is equal to the average slope or Parallel to the ground surface
Lowe-Karafiath	The direction of resultant inter-slice force is equal to the average of the ground surface and the slope at the base of each slice

Table 3: Assumed variations of soil parameters deviating from the standard design values of Al-Adhaim dam.

Case Type	Core			Shell			Filter		
	Weight Density (kN/m ³)	c(kpa)	φ(Degree)	Weight Density (kN/m ³)	C(kpa)	φ(Degree)	Weight Density (kN/m ³)	C(kpa)	φ(Degree)
Min3	11	15	10	14	0	27	13	0	32
Min2	13	25	15	15	0	30	14	0	33
Min1	15	35	20	16	0	33	15	0	34
Standard	17	45	25	17	0	37	16	0	35
Max1	18	50	26	18	0	38	17	0	37
Max2	19	55	28	19	0	39	18	0	39
Max3	20	60	30	20	0	40	19	0	41

Table 4: Variations in the coefficient of permeability (k) for each soil case, under 131.5m and 118m water levels in the reservoir.

W.L. (m)	Case Type	K _{filt} (m/s)	K _{sh} (m/s)	K _{core} (m/s)
131.5	Min3	0.001	0.000001	1E-11
131.5	Min2	0.000685	0.00000483	8.17E-11
131.5	Min1	0.000371	0.00000867	1.54E-10
131.5	Standard	0.000056	0.0000125	2.25E-10
131.5	Max1	0.0334	0.0000417	3.35E-08
131.5	Max2	0.0667	0.0000708	6.67E-08
131.5	Max3	0.1	0.0001	0.0000001
118	Min3	0.001	0.000001	1E-11
118	Min2	0.000685	0.00000483	8.17E-11
118	Min1	0.000371	0.00000867	1.54E-10
118	Standard	0.000056	0.0000125	2.25E-10
118	Max1	0.0334	0.0000417	3.35E-08
118	Max2	0.0667	0.0000708	6.67E-08
118	Max3	0.1	0.0001	0.0000001

Step 8: If the two seepage cases (Max3 and Min3) do not significantly change the values of safety factors, then it is safe to say that the effect of seepage rates are negligible. Otherwise, the trials shall proceed until no considerable differences in Fs are being noticed. Unlike the seepage cases, it is important to remember that all the slope stability analyses until this stage are based on the standard values of the soil parameters. Thus, two additional variables are taken into consideration when calculating the factor of safety (Fs) using the nine limit-equilibrium methods.

Step 9: SLOPE/W trials shall continue to include the other soil cases for each water level, and after the results are obtained the analyses process starts. The purpose of analyzing the data is to represent them in an understandable way (i.e. equations and figures). The method used to derive the empirical equations is “Dimensional Analysis”; it uses dimensionless variables

obtained from the variables, which represent the seepage and slope stability results. The factor of safety is a function of many variables results, which are going to be used in the dimensional analysis.

Step 10: The equations obtained from the dimensionless analysis are derived according to the results obtained from each stability method. In addition, a general equation, which correlates all other empirical equations, is going to be presented. This equation is useful in case all the values of safety factors are relatively close. The boundary conditions of this equation shall be determined.

5. Results and Discussion

At first, the seepage analysis was conducted using SEEP/W component for the standard soil case then it was considered to be as a primary analysis of SLOPE/W analyses, which means that the

results of seepage analysis are used in the stability analysis. Nevertheless, every SLOPE/W analysis is based on one of the nine slope stability methods, which are mentioned earlier, and all this has to be done under both upper and lower water levels. The soil cases are not only used in SEEP/W, it is also considered the input of each stability method, which is used in SLOPE/W. To sum up, 126 trials are branched from the standard seepage analysis (9 (stability methods) x 7 (soil cases) x 2 (water levels) = 126 SLOPE/W Analyses). Figure 2 and 3 illustrate the behavior of the stability models under different soil conditions (cases). Generally, it can be noted from both figures that the water level has an effect on the values of Fs as they increase when the water level increases. Although the increase is not significant, the effect of water level is still obvious for some methods considering the fact that each limit equilibrium method considers certain factor. The methods, which consider more variables, would give more results that are realistic. Therefore, the criterion for selecting the best method is not dependent on the high values of Fs. As previously explained, it is mandatory to check whether the seepage rate value is relatively close to the value of the seepage rate provided by the designer of Al-Adhaim dam. Table 5, shows that each water level gives different seepage rates (5.42 E-10 to 5.01E-07 m³/s), the average value of the seepage rates for all soil cases lies within the range of the design seepage rate value (5.78E-08 m³/s). This gives an indication that the seepage

model is acting in an acceptable way and the results are not extremely deviated from the design value. Figures 4 and 5, illustrate the seepage analyses conducted using the standard soil type by SEEP/W (pore water pressure contour lines), for the maximum and minimum reservoir water levels. It is noted that the values of seepage rates increases as the soil parameters increase in value. This is because different values of Coefficient of Permeability (k) are assumed for each soil case in an ascending manner (Table 4). The more permeable the soil is the higher seepage rate it gives. Thus, the seepage model is responding realistically. The standard soil case produces Fs values above the most common (min.) acceptable value of Fs which is 1.5, while the soil conditions below the standard design values of the case of study are less than 1.5, which means that the standard values of the soil parameters within the dam body are optimal. Figure 6, illustrates one of the critical values of the factor of safety obtained by using SLOPE/W. Dimensional analysis offers a method for reducing complex physical problems to the simplest (that is, most economical) form prior to obtaining a quantitative answer. The method is of great generality and mathematical simplicity [11]. The Buckingham Π theorem is one of the approaches that researches used in developing general factor of safety of the slope stability.

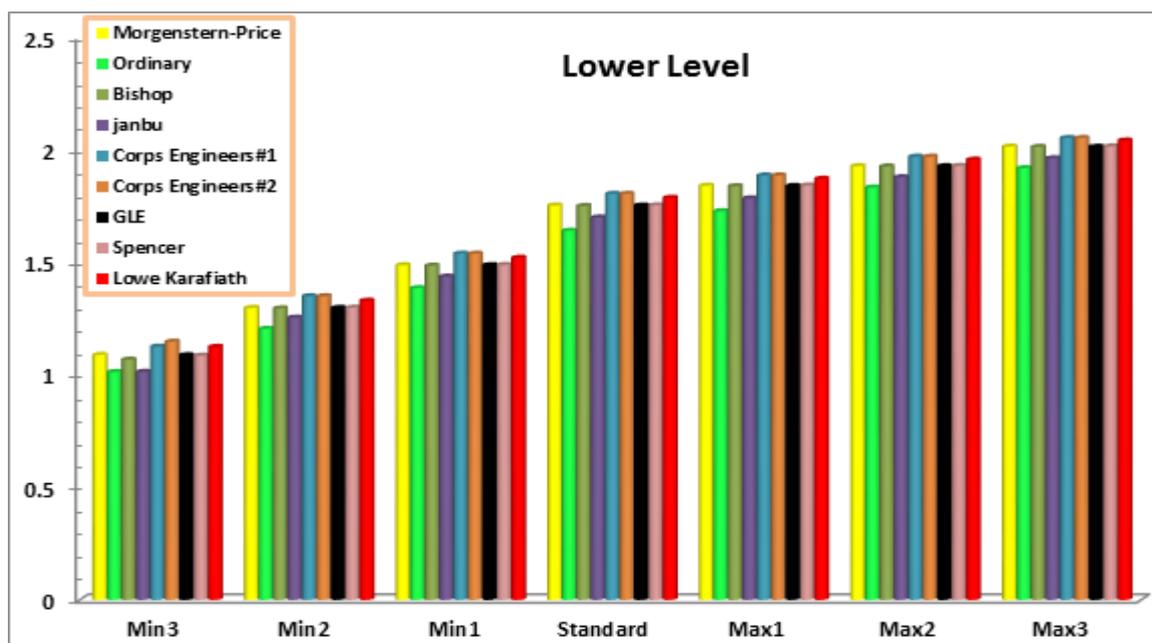


Figure 2: The relationship between calculated safety factors values verses the soil cases used for the nine stability methods under the lower water level (118m).

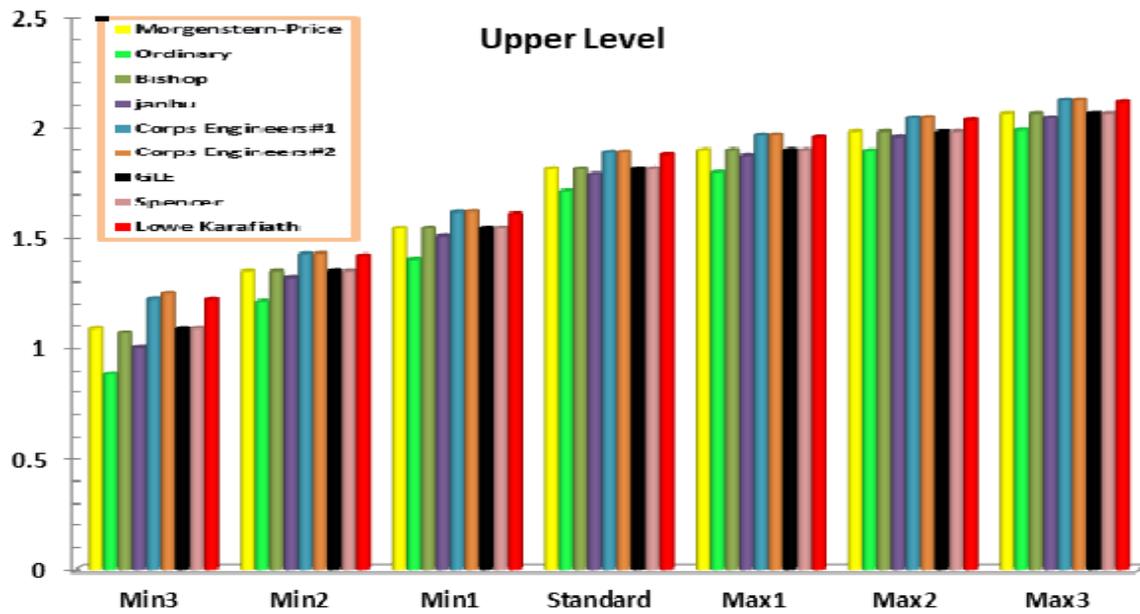


Figure 3: The relationship between calculated safety factors values verses the soil cases used for the nine stability methods under the upper water level (131.5m).

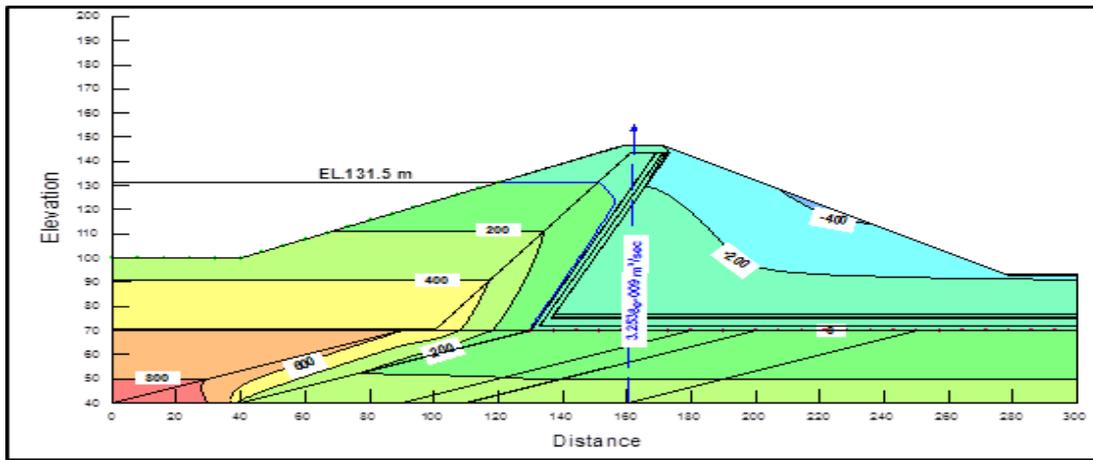


Figure 4: Seepage analysis using SEEP/W for Standard soil type (upper level) with pore-water pressure contour lines.

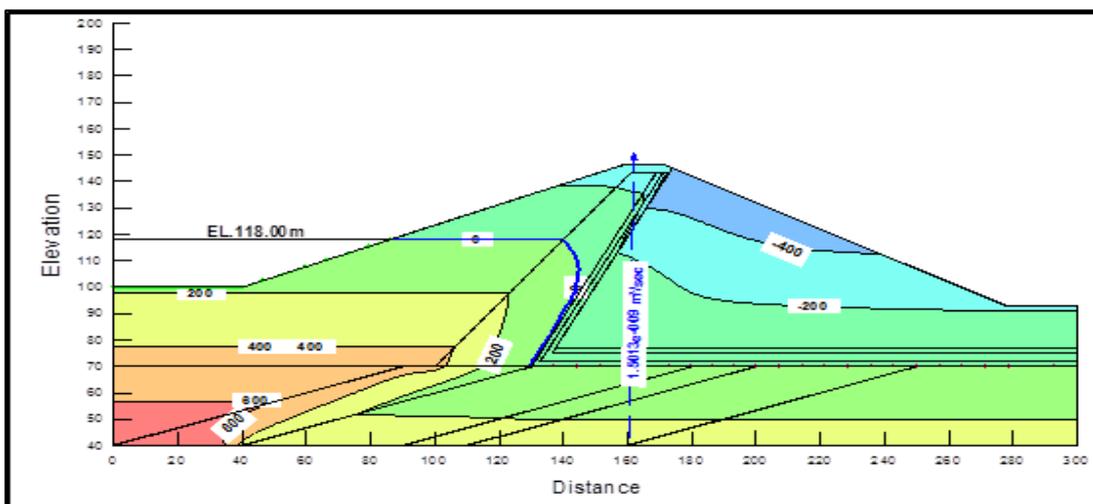


Figure 5: Seepage analysis using SEEP/W for Standard soil type (lower level) with pore-water pressure contour lines.

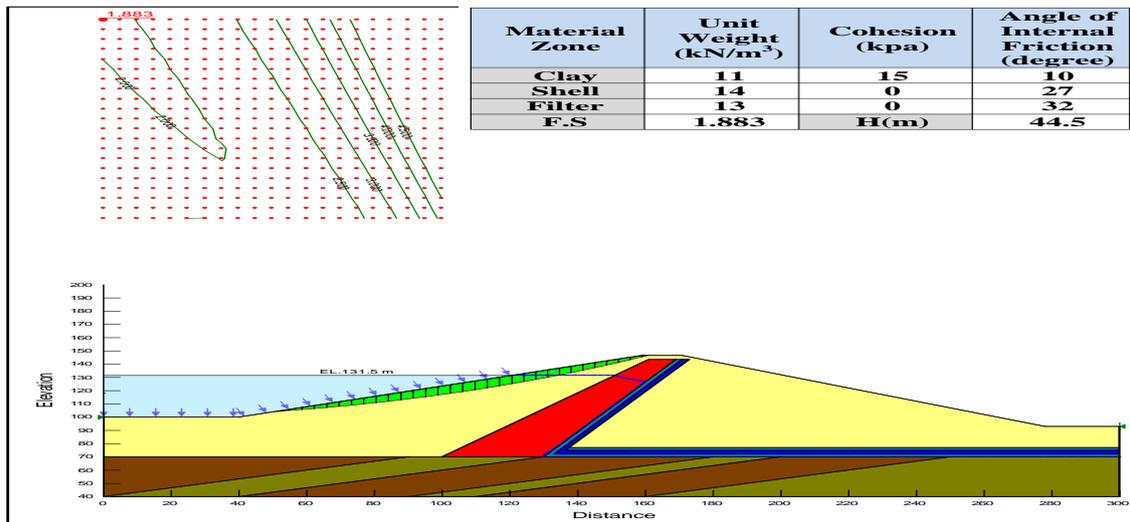


Figure 6: Slope stability analysis result using SLOPE/W (Corps of Engineers #2 method)

Table 5: Seepage rates for all soil types

Soil Case Type	Q(m ³ /sec)		
	At the upper water level (131.5m above m.s.l.)	At the lower water level (118m above m.s.l.)	Design value
Min3	7.34E-10	5.42E-10	5.78E-08
Min2	1.07E-09	7.47E-10	5.78E-08
Min1	2.32E-09	9.62E-10	5.78E-08
Standard	3.25E-09	1.5E-09	5.78E-08
Max1	1.74E-07	1.66E-08	5.78E-08
Max2	3.32E-07	2.68E-08	5.78E-08
Max3	5.01E-07	4.60E-08	5.78E-08

Based on theorem, the proposed influential parameter is the general form of the factor of safety Eq. (5):

$$F_s = f(\gamma^*, \phi^*, k^*, H_u, C_c, \rho_w, Q_s, k_{filt}) \quad (5)$$

Where:

$\gamma^* = \gamma_{sh} / \gamma_{core}$; $\phi^* = \phi_{sh} / \phi_{core}$; $k^* = k_{sh} / k_{core}$; H_u = Reservoir water level (upstream) and Q_s = seepage flow rate.

The equations resulted from the dimensional analysis process represents the relation between the calculated F_s values using SLOPE/W with all the variables, which have been taken into consideration. These equations are empirical equations derived from the results obtained by each limit equilibrium method. In addition to these empirical equations, a general equation, which can be considered as a representative to all other equations since it, takes into account all the variables previously used in deriving each empirical equation. The software used in deriving the following empirical equations is STATISTICA. The final form of the equation has been determined using the multiple regression

analyses for the observed data. The general multiple regression equation for all data and for all methods was:

$$F_s = (\gamma^*)^{-2.804} (\phi^*)^{0.857} (k^*)^{-0.111} \left(\frac{H^4 C_c}{\rho_w Q_s^2}\right)^{0.013} \left(\frac{H^2 K_{fl}}{Q_s}\right)^{-0.062} \quad (6)$$

The coefficient of determination R^2 of equation (7) was found to be equal to (0.982). The standard error of estimate was 0.0714. The resulted factor of safety values by using Eq.(6) are shown to be close to the simulated values of the factor of safety using SLOPE/W Figure 7).

To test which dimensionless parameter affects the stability of the dam, a sensitivity analysis was made. The partial correlations indicate that γ^* , ϕ^* , k^* and $\frac{H^4 C_c}{\rho_w Q_s^2}$ are the most important variables while $\frac{H^2 K_{fl}}{Q_s}$ is less important than the others. The partial correlations values were 0.96, 0.96, 0.79, 0.79 and 0.32 respectively, Table 6.

6. Conclusions and Recommendations

The conclusions of this study are summarized in the following points:

- 1- It is possible to create new methods of calculating the factor of safety against sliding by adding more variables to the commonly used limit equilibrium methods.
- 2- Empirical equations can be derived by assuming different soil conditions in the earth-dam's body.
- 3- A general empirical equation can represent all the derived empirical equations since the differences in calculated Fs values are not high.
- 4- There is no need to use the same soft wares (SLOPE/W and SEEP/W) in performing seepage and slope stability analysis for earth-dams, which have similar dimensions of the case study.
- 5- One simple equation is enough to give factor of safety results without using software's because it is already derived through numerical modeling (finite elements) so there is no need to repeat the numerical modeling procedures.
- 6-The empirical equation is going to reduce the effort, time and cost which are expected from performing seepage and stability analyses using numerical modeling. It is highly recommended for next researchers to derive a general empirical equation not only based on changing the soil conditions but also based on changing the geometry of an earth dam. This will add new variables, which influence the seepage flow patterns and the stability of the dam. Nevertheless, trying different well-known

commercial soft wares similar to GeoStudio for numerical modeling and comparing the results obtained by each one can add more sustainability to the selected equation.

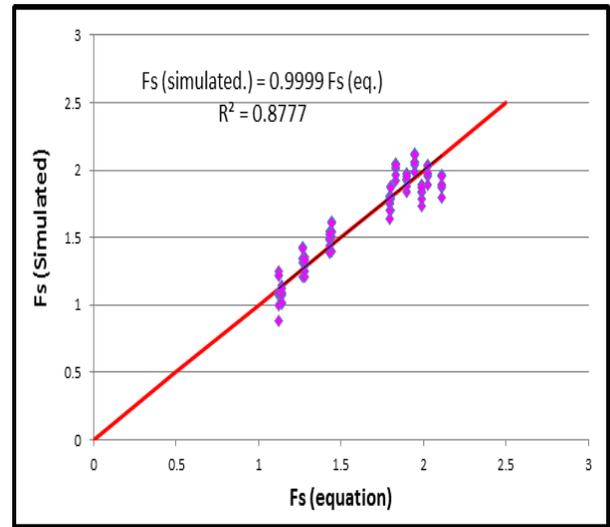


Figure 7: Simulated values of factor of safety against sliding verses calculated values using the general empirical equation.

Table 6: The sensitivity analysis for the dimensionless parameters.

Method	Morgenstern-price	Ordinary	Bishop	Janbu	Corps of Engineers #1	Corps of Engineers #2	GLE	Spencer	Lowe-Karafiath	General	Average
Π2	-0.97	0.97	0.98	0.9	-0.97	-0.96	0.97	0.9	0.96	0.9	0.96
Π3	-0.98	0.98	0.98	0.9	-0.97	-0.97	0.98	0.9	0.97	0.9	0.97
Π4	-0.81	-0.8	-0.8	-0.8	-0.81	-0.81	0.81	0.8	0.81	0.7	0.80
Π5	-0.81	-0.8	-0.8	0.8	-0.81	-0.81	0.81	0.8	0.81	0.7	0.80
Π6	-0.32	0.33	0.32	0.3	-0.32	-0.31	0.32	0.3	0.31	0.3	0.31

References

- [1]- T.K. Hnang, "Stability Analysis of an Earth Dam under Steady State Seepage," *Computer and Structures J.*, Vol. 58, pp. 1075-1082, 1996.
- [2]- D.V. Griffiths and P.A. Lane, "Slope Stability Analysis by Finite Elements," *Geotechnique J.*, Vol.49, pp. 387-403, 1999.
- [3]- A. Khalil, "Numerical Modeling of Pore Water Pressure Development within a Thin Clay Core in an Earth Dam," *Al-Rafdain Engineering Journal*, Vol. 20, pp. 93-109, 2012.
- [4]- H.T. Pham, "Stability of Slope and Seepage Analysis in Earth Dam Using Numerical Finite Element Model," *Study of Civil Engineering and Architecture (SCEA)*, Vol. 2, pp. 104-108, 2014.
- [5]- S.S. Athani, "Seepage and Stability Analyses of Earth Dam Using Finite Element Method," *International Conference on Water Resources, Coastal and Ocean Eng.*, Vol. 4, pp. 876-883, 2015.
- [6]-S.N. Al-Labban, "Seepage Analysis of Earth Dams by Finite Elements," M.Sc. Thesis, College of Engineering, University of Kufa, Iraq, 2007.
- [7]- GEO-SLOPE International Ltd., "Seepage Modeling with SEEP/W 2007 Version," *An Engineering Methodology*, Fourth Edition, 2009.
- [8]- GEO-SLOPE International Ltd., "Stability Modeling with SLOPE/W 2007 Version," *An Engineering Methodology*, Fourth Edition, 2008.
- [9]- T.M. Le, "Stability and Failure Mass of Unsaturated Heterogeneous Slopes," *Canadian Geotechnical Journal*, Vol. 11, pp. 1747-1761, 2015.
- [10]- D. Christopher and K. Prabir, "Embankment Slope Stability Analysis of Dwight Mission Mine Site Reclamation Project," *International Conference on Case Aistories in Geotechnical Eng.*, pp. 1-10, 2013.
- [11]- A.A. Sonin, "The Physical Basis of Dimensional Analysis," *Department of Mechanical Engineering*, second Edition, 2001.