



## Evaluating and Modeling of Parabolic Solar Cooker by RSM

Zain alabdeen Hussein Obaid<sup>1</sup>, Osama Ibrahim Abd<sup>2</sup>

<sup>1,2</sup>Renewable Energy Research Center, University of Anbar, Ramadi, Iraq.

<sup>1</sup>[zainobaid82@gmail.com](mailto:zainobaid82@gmail.com), <sup>2</sup>[osamameng21@gmail.com](mailto:osamameng21@gmail.com)

### Abstract

The present paper has dealt with two stages. In the first, an experimental set up of the parabolic cooker with direct use has been made-up and studied at Renewable Energy Research Center-University of Anbar. Based on experimental results, statistical studies by using Response Surface Methodology (RSM) have been done to identify the optimum influential parameters and estimate mathematical temperature model. In this study, parabolic collector parameters that can effect on the Parabolic Collector efficiency are studied in more details. Six amounts of water are used for measuring the increasing of temperature relative to the measuring time. Parabolic Collector parameters are optimised with the consideration of single-response; temperature of the working fluid (in terms of water). The achieved experimental results are analysed by the desirability functional analysis DFA approach, and optimal levels of input factors have been distinguished. ANOVA also has been utilized to recognize the assurance of powerful factors on the response.

**Keywords:** Solar parabolic cooker; RSM; Model; Performance; Optimization.

## تقييم و نمذجة طباخ شمسي بشكل قطع مكافئ باستخدام طريقة السطوح الإحصائية

زين العابدين حسين عبيد<sup>1</sup>، أسامة إبراهيم عبد<sup>2</sup>

<sup>2,1</sup> مركز بحوث الطاقة المتجددة، جامعة الانبار، الرمادي، العراق.

<sup>1</sup>zainobaid82@gmail.com, <sup>2</sup>osamameng21@gmail.com

### الملخص

يتضمن البحث الحالي مرحلتين، في الأولى اعداد الجانب العملي للطباخ الشمسي متضمنا التصنيع والاستخدام المباشر حيث تمت الدراسة في جامعة الانبار - مركز بحوث الطاقة المتجددة. اعتمادا على النتائج التجريبية يتم اجراء نمذجة باستخدام طريقة السطوح الإحصائية لتعيين الظروف المثلى وتخميين نموذج رياضي من الدرجة الثانية لدرجة حرارة الماء. درست متغيرات الطباخ الشمسي والتي تؤثر بدورها على أداء وكفاءة الطباخ الشمسي بالتفصيل. حيث استخدمت ست كميات من الماء لقياس ارتفاع درجة حرارة الماء بالنسبة لوقت القياس. تم اجراء الأمثلية للمتغيرات المدروسة اعتمادا على النتائج العملية بالنسبة لهدف مستقل وهو درجة حرارة المائع المستخدم (الماء في هذه الدراسة).

تم استخدام طريقة DFA لتحليل النتائج العملية وتحديد أفضل المتغيرات الداخلة في الدراسة والتي تعطي أفضل درجة حرارة للماء. تحليل المتغيرات أيضا اجري لمعرفة وتحديد العوامل المؤثرة على الهدف (درجة حرارة الماء في هذه الدراسة).

**الكلمات الدالة:** الطباخ الشمسي، طريقة السطوح، الاداء، نمذجة، الأمثلية.

## 1. Introduction

Solar cooker technology offers a wide variety of applications to exploit this source of renewable energy. In the middle of the thermal applications of solar energy, solar cooking is considered as one of the simplest, the most viable and promising opportunities in terms of the utilization of solar energy. Solar cooker utilises solar energy that is recognized as one of the most choices since it is free and offers clean and environmentally friendly energy. Therefore, it helps in the reduction of the level of greenhouse gas (GHG) emissions and fossil fuel prices, also in the solution of energy reduction in the remote areas such as desert regions [1-3]. Solar cookers can be classified into four types [4]: (a) concentrator cookers; (b) solar ovens; (c) box cookers; and (d) indirect solar cookers. The categories have been extra subdivided into different sub-categories. Many enhancements have been prepared to many kinds of solar cookers, but typically modifications have been developed to box type cookers & concentrator cookers owing to their major effectiveness & users approachable.

Response Surface Methodology (RSM), is an experimental strategy first described by Box and Wilson in 1951 for determining optimal conditions for multivariable systems, and is considered an efficient technique for process optimization [5]. RSM is useful in the solution of many types of engineering problems. Recently, one of these problems is optimization of the response in solar cookers. One of the earliest mathematical models to test the thermal performance of a Solar Cooker was obtained by Garg et al., [6]. Sinha and Sharma [7] defined a model for parabolic collector as solar Thermal Electric power system, which was established for high temperature uses. Kahrobaian & Malek Mohammadi [8] had presented a new procedure to optimize parabolic solar collectors by the exergy analysis. In this work, mathematical models of optical & thermal efficiencies are applied for simulation. Mohana Reddy & et. al. [9] estimated the optimum variables of a solar parabolic collector of multi responses; for example temperature, thermal & optical efficiency, using grey relational analysis. Also, P. Venkataramaiah et. al. [10] applied the desirability functional analysis (DFA) approach to optimize a solar parabolic collector process parameters. They are stated that the silvered mirror strip is the optimum option among the used reflective materials.

This study attempted to apply the RSM approach to evaluate, model and optimize of a solar parabolic collector parameters namely mass of water, and time of the measuring of working fluid temperature (time of trial) with the consideration of single response namely temperature of working fluid.

## 2. Solar Cookers Performance

For comparing the different forms of the solar cookers. The characteristic values essential to be well defined, The important values are the terms of power & efficiency. A mean heating power of a cooker is considered as follow:

$$\dot{Q}_{heat} = \frac{m_w \cdot c_p \cdot \Delta t_{\infty-95}}{\Delta t} \quad (1)$$

Where;

$m_w$  is the mass of water (kg)

$c_p$  is the specific heat capacity at constant pressure (J/kg K),

$\Delta t_{\infty-95}$  is the temperature difference (K)

$\Delta t$  is the duration of the measurement (sec)

Heating power typically is calculated from ambient temperature up to (95<sup>0</sup>C), to stay away the instability of the precise boiling point. The power of evaporation is determined through the water evaporation at the point of boiling. Solar cooker heatcapacity takes less effect on the thermal performance due to the system works at a const. temperature. The power is considered the measured mass of water evaporated.

Where;

$\dot{m}$ :an evaporation mass of water (kg/s).

$h_{fg}$ :latent heat of evaporation (J/kg).

$$\dot{Q}_{ev} = \dot{m} \cdot h_{fg} \quad (2)$$

The efficiency is calculated in according equation (3). For parabolic concentrators, it is the direct solar radiation on the aperture surface. For flat-plate collectors, the solar radiation is global radiation on the surface [5].

$$\eta = \frac{\dot{Q}}{I \cdot A} \quad (3)$$

Where;

I is the solar radiation (W/ m<sup>2</sup>).

A is the cooker surface area (m<sup>2</sup>).

The differences in the tracking mechanism and the surface of various cookers show that efficiencies are appropriate to compare the cookers at the same kind. To compare various kinds of cookers, further parameters, besides efficiency, are required [6].

### 3. Practical Set-up and Measurements

In this section, the practical results in [Table 1](#) are used for evaluating the performance of parabolic solar cooker. The parabolic solar cooker is shown in [Fig.1](#) with a diameter of 1.8m. In this test, 6 amounts of water are used for measuring the increasing of temperature relative to the measuring time. The amounts are 1-litre, 2-litres, 3-litres, 4-litres, 5-litres and 6-litres. During the test, we measure the temperature of water by using the thermometer sensor (k-type 2-channels thermometer). The temperature of water before the test is fixed around 25 degrees then the test started by one liter putted inside the vessel. The thermal wire sensor adjusted inside the vessel for reading the temperature. The measuring of temperature started from 5-seconds because the vessel needs time to be heated. After finishing the measurements, the water will replace by 2-liters and the previous process is repeated till 6-liters.



**Fig. 1:** Parabolic solar cooker used in the experiment.

**Table 1:** Practical temperature measurements for different amounts of water.

	Temperature (degree)							
	5min.	8min.	10 min.	12 min.	14 min.	16 min.	18 min.	20 min.
1-liters	55	67	75	83	88	94	100	105
2-litres	52	58	64	72	80	87	93	99
3-litres	48	54	61	68	75	82	89	94
4-litres	43	49	56	65	72	79	85	92
5-litres	39	44	52	60	68	75	82	89
6-litres	34	39	45	52	61	68	76	84

#### 4. Response surface methodology

Response surface methodology is a set of statistical and mathematical procedures that are useful for the analyzing and modeling engineering problems. In this route, the main objective is to optimize the response surface that is influenced by various process parameters. Response surface methodology also helps in determining the relationship between the controllable input parameters and the developed response surfaces [11]. The RSM design procedure is as follows:

- 1) Defining the independent input variables and desired responses with the design constraints.
- 2) Designing of experiments series for adequate & reliable measurement of the interest response.
- 3) Developing a mathematical model of the 2<sup>nd</sup> order response surface with best fittings.
- 4) Finding the optimal set of experimental factors that offer a (max or min) value of response (Maximum at this study).
- 5) Representing the direct & interactive effects of process factors through (2D and 3D) plots.

In the RSM, the quantitative form of response surface function stated as below:

$$Y = f(X_1, X_2, \dots, \dots, X_k) \quad (4)$$

The main objective is for optimization of the Response (Y). It's assumed that the independent variables are continuous and controllable by the experiments with errors are negligible. It is essential to get an appropriate approximation for a True-Functional Correlation between independent-input variables and the output response. Commonly a 2<sup>nd</sup> order model Eq. (5) applied in RSM procedure [11].

$$Y = \beta_0 + \sum_{i=1}^k \beta_i X_i + \sum_{i=1}^k \beta_{ii} X_i^2 + \sum_{i=j}^{k-1} \sum_{i=j}^k \beta_{ij} X_i X_j + E \quad (5)$$

Where;

$\beta$ : Coefficient of regression.

Y: Response.

K: Number of variables considered in each experiment.

$X_i, X_j$ : Process variables.

E: Random error.

Higher values of coefficient of regression ( $\beta$ ) specify a higher variable importance & vice-versa.  $\beta$ -coefficients, which should be determined in 2<sup>nd</sup> order model, are achieved by Least Square Method. The analysis of regression & Analysis of Variance ANOVA were directed for model fitting and to get statistically the significance of model terms. A software of DesignExpert

ver. 9.0 was utilized for data analyzing, performing RSM and to get 2D & 3D response graphs. Numerical optimization of input variables depending on the single response was achieved by software (DesignExpert 9.0). In this study, the temperature of the working fluid is considered as the performance characteristic (response) of parabolic solar cooker, maximize of temperature of the working fluid leads to maximize efficiency of solar cooker, so, the desired goal (maximization temperature of the working fluid) was utilized to achieve optimization of input variables & the response.

The levels were specified for each parameter as given in the Table 2. Two process parameters with low and high levels resulted in a total of 48 runs by RSM user defined design. The observations are presented in Table 3 for further analysis and studies.

**Table 2: Practical parameters and their levels.**

Name	Units	Type	Std. Dev.	Low	High
X1: Amount of water	Liter	Factor	0	1	6
X2: Time	Minute	Factor	0	5	20
Y: Temperature	°C	Response	1.85192	34	105

**Table 3: Design layout of practical parameters and response of parabolic solar cooker.**

Std	Run	Factor 1 X1	Factor 2 X2	Response Y
37	1	1	18	100
32	2	2	16	87
48	3	6	20	84
40	4	4	18	85
21	5	3	12	68
1	6	1	5	55
13	7	1	10	75
5	8	5	5	39
17	9	5	10	52
15	10	3	10	61
42	11	6	18	76
9	12	3	8	54
35	13	5	16	75
23	14	5	12	60

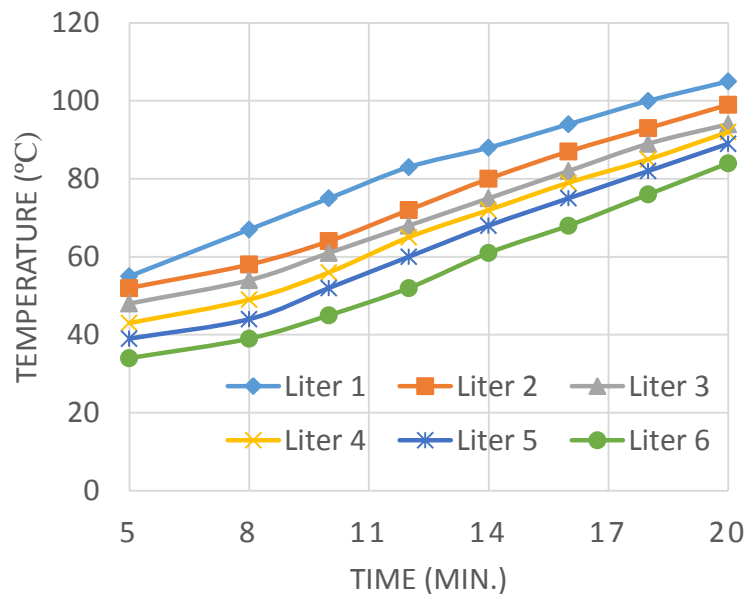


Std	Run	Factor 1 X1	Factor 2 X2	Response Y
25	15	1	14	88
29	16	5	14	68
16	17	4	10	56
18	18	6	10	45
46	19	4	20	92
33	20	3	16	82
2	21	2	5	52
14	22	2	10	64
12	23	6	8	39
3	24	3	5	48
47	25	5	20	89
19	26	1	12	83
24	27	6	12	52
36	28	6	16	68
41	29	5	18	82
30	30	6	14	61
11	31	5	8	44
10	32	4	8	49
38	33	2	18	93
22	34	4	12	65
39	35	3	18	89
31	36	1	16	94
4	37	4	5	43
26	38	2	14	80
8	39	2	8	58
28	40	4	14	72
43	41	1	20	105
34	42	4	16	79
45	43	3	20	94
44	44	2	20	99
27	45	3	14	75
20	46	2	12	72
6	47	6	5	34
7	48	1	8	67

## 5. Results and Discussion

### 5.1 Practical Measurements

Fig. 2 displays the relation between the temperature of the cooking fluid (water) and time of trial at different amounts of water. From this figure, it can be seen that the temperatures at five minutes are (55, 52, 48, 43, 39, and 34 °C) at mass of water (1, 2, 3, 4, 5, and 6 liter), respectively, these values are increased up to (105, 99, 94, 92, 89, and 84 °C) after 20 minutes. At the case, the water starts to be evaporated after 90 °C. The spot of the concentrated light spreaded by about 20cm on the vessel. At all cases, the relation of temperature is still approximately linear with time, i.e. there is an increment in temperature with time. As comparison all the cases, the increment in temperature is about (50, 47, 46, 49, 40, and 50) at mass of water (1, 2, 3, 4, 5, and 6 liter), respectively, after 20 minutes. With increase the liters of water up to 6 liters, the temperature difference is about (21 °C) after 20 minutes. Also, has to be noted that the increase liters of water led to a slow rise in temperature during the time fixed in all cases used, as is evident from the graph.



**Fig. 2:** The relation between the temperature of cooking fluid and time of trial at different amounts of water.

From Table 1 and equations (1) and (3), we can calculate heating-power of a solar cooker and the efficiency when solar radiation ( $I$ ) is constant at  $850 \text{ W/m}^2$  and  $A$  is the area in  $\text{m}^2$  ( $A = \pi r^2$ ). Figs. 3 and 4 show heating-power and efficiency of a solar cooker as a function of time at different amounts of cooking fluid, respectively. From graphs, we noted that at constant solar radiation the same manner for both relations. Higher values of heating-power and efficiency of a solar cooker are obtained at mass of water (6 liter), i.e., increase of heating-power and efficiency with increase the liters of water up to 6 liter. At constant amount of water, we noted that the values of heating-power and efficiency of a solar cooker decrease with increase the time due to the inverse relationship between heating-power and time (Eq. 2). With increase the liters of water up to 6 liter, the value of efficiency of a solar cooker at 20 minutes is 38.7%.

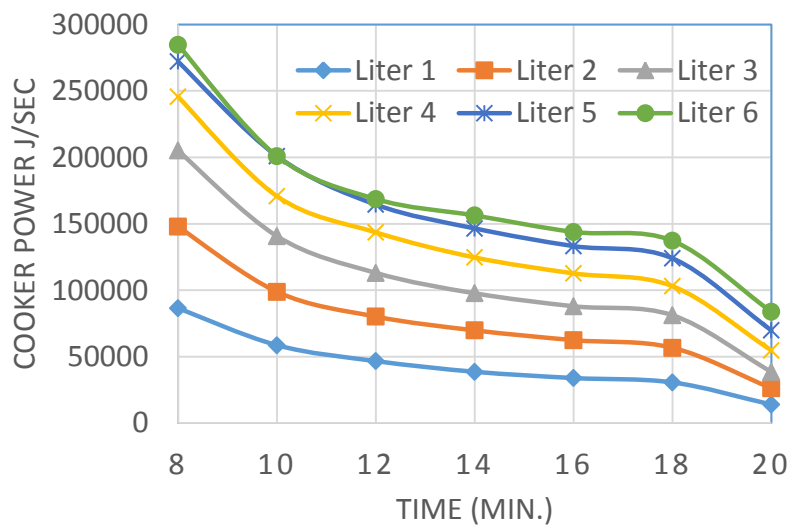
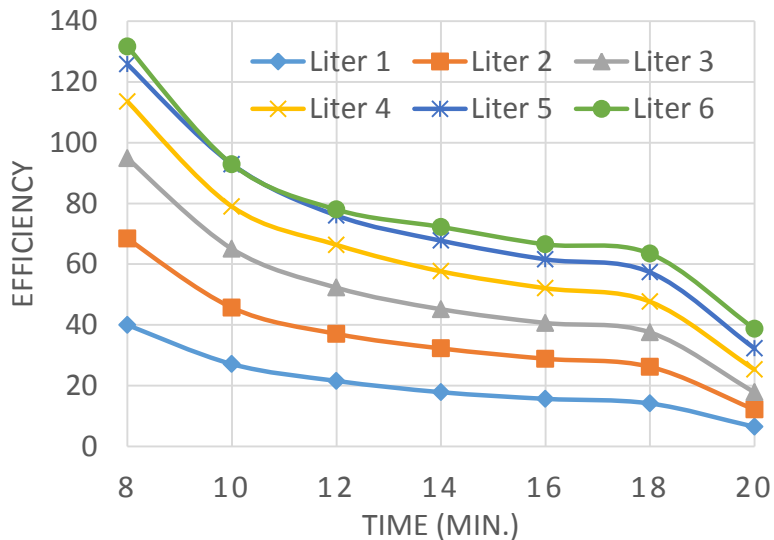


Fig. 3: Heating-power of a solar cooker as a function of the time.



**Fig. 4:** Efficiency of a solar cooker as a function of the time.

## 5.2 Mathematical Modelling

ANOVA and regression analysis of the experimental results was conducted by (Design-Expert 9.0 software). Estimated regression coefficients of the response surface quadratic model (Eq. 3) are presented in Table 4. Table 5 shows the results of ANOVA analysis for quadratic model. The corresponding values; coefficient of determination ( $R^2$ ), Coefficient of variation (CV), adequate precision and PRESS values are evaluated to check the suitability of the model. The value of  $R^2$  is an indicative of fitting degree and can be termed as a ratio of model variation to total variation. It can be observed that the model F-value of 921.89 indicates the model is significant. There is only a (0.01%) chance that F-value this large can happen because noise. The values of (Prob > F) less than 0.05 specify model terms are significant. The  $R^2$  values are high (>0.9) for the model of response. The predicted  $R^2$  of 0.9875 is in reasonable agreement with the adjusted  $R^2$  of 0.9899, i.e. the difference is less than 0.2. The value of coefficient of variation (CV) should be less than 10%, and was observed to be less than 3% for the model. This indicates that model is adequate. Adequate Precision measures a signal to noise ratio. A ratio more than 4 is desired. The 114.359 ratio shows a satisfactory signal. This model can be used to navigate the design space.

**Table 4:** Estimated Coefficients for Response Surface Quadratic model (in actual terms).

Factor	Coefficient	df	Standard Error	95% CI		VIF
	Estimate			Low	High	
Intercept	50.02	1	2.64	44.68	55.35	
A-X1	-6.51	1	0.87	-8.28	-4.75	31.22
B-X2	2.63	1	0.34	1.94	3.32	37.39
AB	0.047	1	0.033	-0.019	0.11	12.45
A <sup>2</sup>	0.15	1	0.11	-0.064	0.37	23.97
B <sup>2</sup>	0.023	1	0.013	-2.394E-003	0.048	33.19

**Table 5:** ANOVA Table for Response Surface Quadratic model.

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
<b>Model</b>	15808.62	5	3161.72	921.89	< 0.0001	significant
<b>A-X1</b>	3296.72	1	3296.72	961.25	< 0.0001	
<b>B-X2</b>	12430.49	1	12430.49	3624.46	< 0.0001	
<b>AB</b>	7.00	1	7.00	2.04	0.1606	
<b>A<sup>2</sup></b>	6.88	1	6.88	2.01	0.1640	
<b>B<sup>2</sup></b>	11.46	1	11.46	3.34	0.0747	
<b>Residual</b>	144.04	42	3.43			
<b>Cor Total</b>	15952.67	47				
<b>Std. Dev.</b>	1.85		<b>R-Squared</b>		0.9910	
<b>Mean</b>	69.83		<b>Adj R-Squared</b>		0.9899	
<b>C.V. %</b>	2.65		<b>Pred R-Squared</b>		0.9875	
<b>PRESS</b>	200.00		<b>Adeq Precision</b>		114.359	

The regression equation of temperature of working fluid (Y) relating to actual levels of input parameters of parabolic solar cooker was found as (Eq. 6):

$$Y = 50.02 - 6.51 * X1 + 2.63 * X2 + 0.047 * X1 * X2 + 0.15 * X1^2 + 0.023 * X2^2 \quad (6)$$

Fig. 5 is a graph that checks the normality distribution of the residuals. In this figure, experimental points are near the center cross line, which means the test follows the normal distribution relatively well.

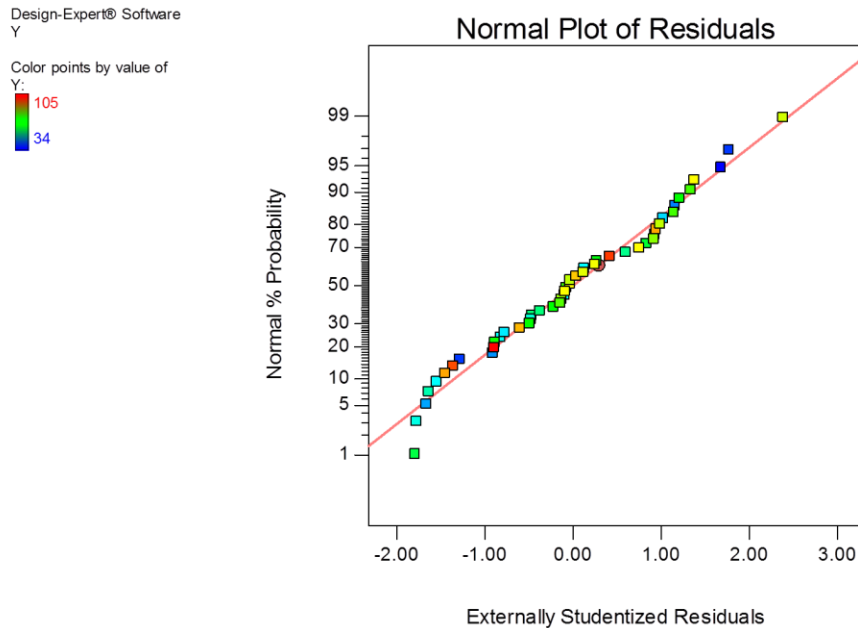
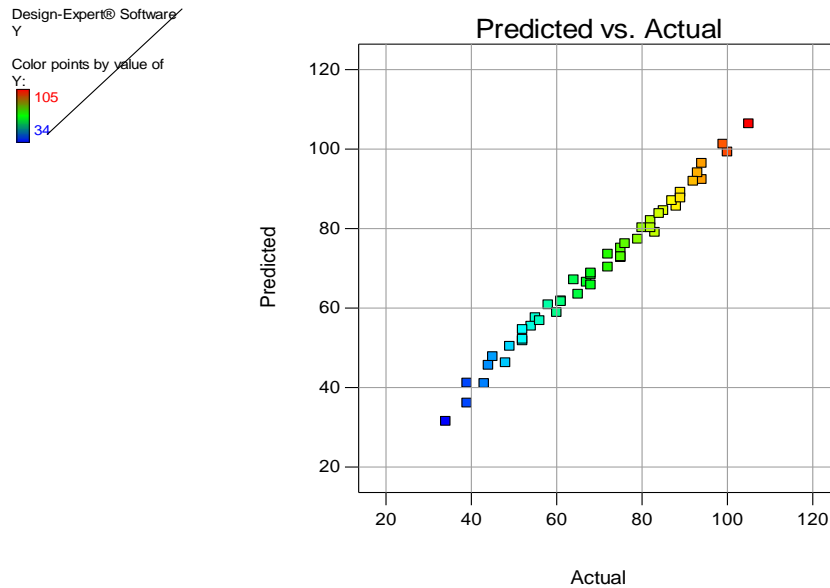


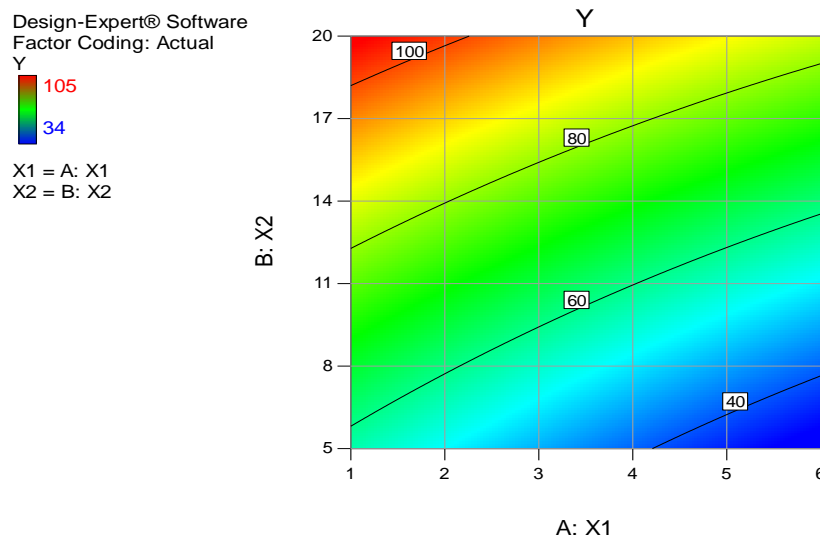
Fig. 5: Normal Probability Plot of residuals.

A graph of the observed (actual) response values versus the predicted response values is obtained using model equation Eq. (5), and presented in Fig. 6. Predicted values match with the experimental data points, indicating a good model fit ( $R^2$ ) value for this model was found as 0.9910).



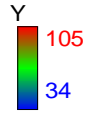
**Fig. 6:** A graph of the actual response values vs. the predicted values.

Figs. 7 & 8 display the 2D and 3D response surface plots, these model graphs show the effects on temperature of working fluid (Y) based on the input parameters of amount of water (A: X1), time (B: X2).

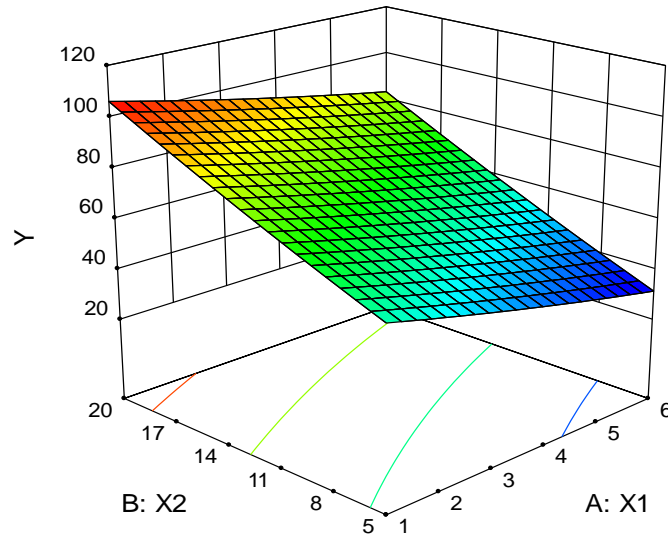


**Fig. 7:** 2D response surface graph showing the effect of input factors on temperature of working fluid (A: X1: Mass of water; B: X2: Time; Y: Temperature).

Design-Expert® Software  
Factor Coding: Actual



X1 = A: X1  
X2 = B: X2



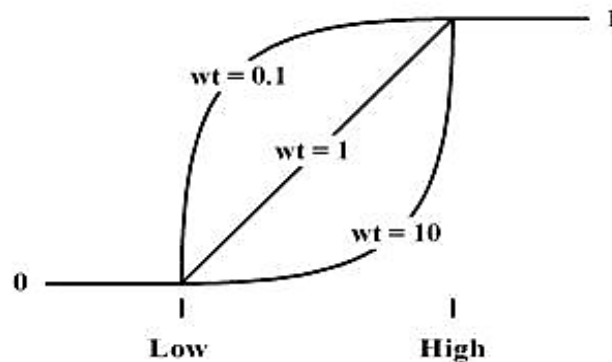
**Fig. 8:** 3D response surface graph showing the effect of input factors on temperature of working fluid (A: X1: Mass of water; B: X2: Time; Y: Temperature).

### 5.3 Response Optimization

The numerical optimization finds a point that maximizes the desirability function, where desirability is an objective function that ranges from zero outside of the limits to one at the goal. In this work, response optimization is performed by desirability function where the main goal is identified as maximizing of the temperature of the cooking fluid in parabolic solar cooker. The target value of the response is selected as 125 °C. For the goal of maximum, the desirability will be defined by the following formulas (Eq. 7), where desirability Curves for Goal are shown in Fig. 9 [12]. To optimize the parabolic solar cooker, constraints for desirability function were defined based on the temperature of cooking fluid with emphasis on the mass of water and time parameters as presented in Table 6.



$$\begin{aligned}
 d_i &= 0, & Y_i &\leq Low_i \\
 d_i &= \left[ \frac{Y_i - Low_i}{High_i - Low_i} \right]^{wt_i}, & Low_i &< Y_i < High_i \\
 d_i &= 1, & Y_i &\geq High_i
 \end{aligned} \tag{7}$$

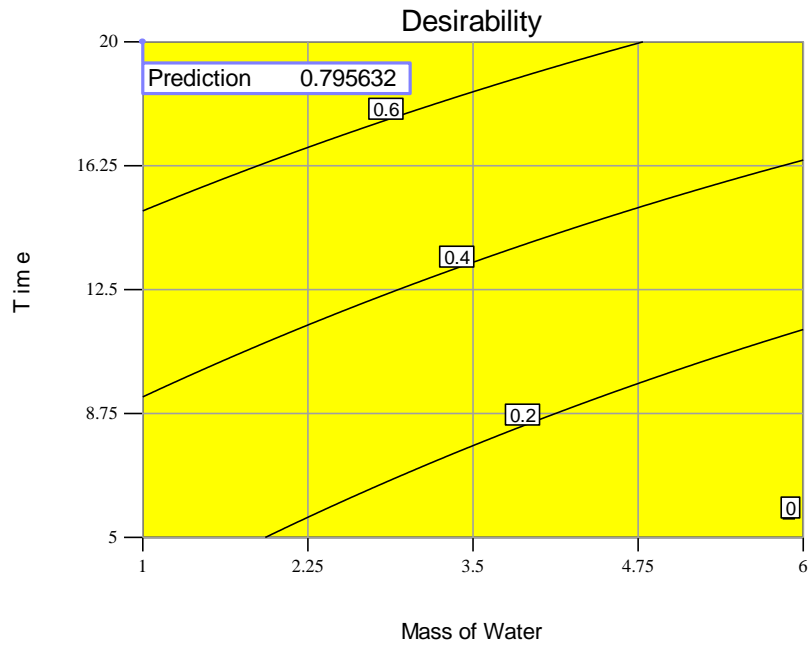


**Fig. 9:** Desirability Curves for Goal is Maximum.

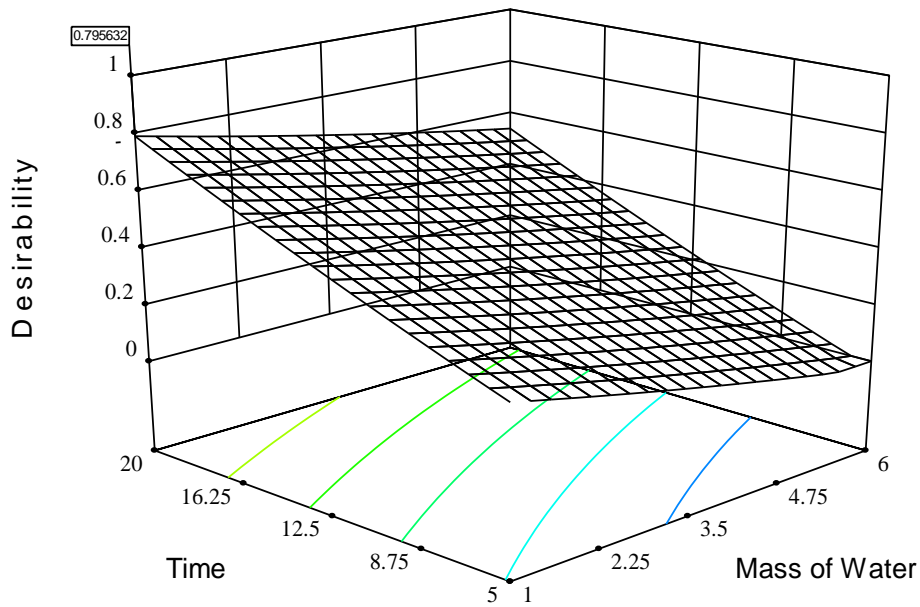
**Table 6:** Constrains in desirability function.

		Lower	Upper	Lower	Upper	
Name	Goal	Limit	Limit	Weight	Weight	Importance
A: X1: Mass of water	is in range	1	6	1	1	3
B: X2: Time	is in range	5	20	1	1	3
Y: Temperature	maximize	34	125	1	1	3

The 2D and 3D response surfaces curves for the single response are plotted in Figs. 10 and 11. The final solutions of desirability function for the response optimization are given in Table 7. As presented, a maximum desirability (selected) was equal to 0.7956 and the conditions to achieve it were the amount of water of 1 liter and time of 20 minutes.



**Fig. 10:** Contour plot for the effect of Mass of water and time on desirability function.



**Fig. 11:** A three-dimensional Response surface plot for the effect of Mass of water and time on desirability function.

**Table 7: Final solutions of desirability.**

Run	X1	X2	Y	Desirability	
1	1.000	20.000	106.402	0.796	Selected
2	2.000	20.000	101.278	0.739	
3	3.000	20.000	96.458	0.686	
4	4.000	20.000	91.941	0.637	
5	5.000	20.000	87.728	0.590	
6	6.000	20.000	83.818	0.547	

## 6. Conclusions

In this work, based on experimental data, both of mathematical modeling and response optimization have been done to estimate a mathematical model of cooking fluid temperature and identify the optimum influential parameters using Design-Expert 9.0 software. The maximum temperature was selected as the performance characteristic (quality target) of parabolic solar cooker to get the greatest effectiveness; max efficiency. It is identified that the increase in the liters of water up to 6 liters; the value of efficiency of a solar cooker at 20 minutes was 38.7%. Experimental data of response are analyzed by desirability function, & optimal levels of input variables have been recognized. A maximum desirability was equal to 0.7956 & the conditions to attain it were the amount of water of one liter and time of 20 minutes.

## Acknowledgements

The author would like to thank Renewable Energy Research Center, University of Anbar, Ramadi, Iraq for technical supporting of this work.



## References

- [1] Abhishek Saxena, et al. “A *technical note on performance testing of a solar box cooker provided with sensible storage material on the surface of absorbing plate*”, Int. J. Renewable Energy Technology, 3(2), 165 (2012).
- [2] S. B Riffat, E. Cuce, “A *review on hybrid photovoltaic/thermal collectors and systems*”. Int. J. Low – Carbon Technol; 6 (3), 212 (2011).
- [3] B. Halacy, C. Halacy, “*Cooking with the sun*”, California, USA, Jack Howel, Lafayette, (1992).
- [4] Abhishek Saxena, Varun, S.P. Pandey, G. Srivastav, “A *thermodynamic review on solar box type cookers*”, Renewable and Sustainable Energy Reviews 15, 3301 (2011).
- [5] Q. Kong, G.Q., Q. H. Chen, F. Chen, “*Optimization of medium composition for cultivating clostridium butyricum with response surface methodology*”, J. Food Sci. 69: M163-M168, (2004).
- [6] H. Garg, B. Bandyopadhyay, G. Datta, “*Mathematical-Modeling Of The Performance Of A Solar Cooker*”, Applied Energy 14 (3), 233 (1983).
- [7] U. K. Sinha, S. P. Sharma, “*Modelling for the Parabolic Collector for solar Thermal Electric Power*”, ARISER, 4, 205 (2008).
- [8] A. Kahrobaian, H. Malekmohammadi, “*Exergy Optimization Applied to Linear Parabolic Solar Collectors*”, Journal of Faculty of Engineering 42 (1), 131 (2008).



- [9] P. Mohana Reddy, P. Venkataramaiah, P. Sairam, “*Optimization of Process Parameters of a Solar Parabolic Trough in Winter Using Grey-Taguchi Approach*”, International Journal of Engineering Research and Applications, 2(1), 816 (2012).
- [10] P. Venkataramaiah, P. Mohana Reddy and P. Sairam, “*Simulation and optimisation studies on a solar parabolic collector: an experimental investigation*”, International Journal of Sustainable Energy, 33(4), 869 (2014).
- [11] N. Aslan, Y. Cebeci, “*Application of Box–Behnken design and response surface methodology for modeling of some Turkish coals*”, Fuel 86, 90 (2007).
- [12] Mohammad Jafar Dalvand, Seyed Saeid Mohtasebi, Shahin Rafiee, “*Optimization on drying conditions of a solar electrohydrodynamic drying system based on desirability concept*”, Food Science & Nutrition, 2(6), 758 (2014).