J. of university of anbar for pure science : Vol.4:NO.1 : 2010

ISSN: 1991-8941

Study of ∂-open function and Inductively ∂-open function in bitopological spaces

Nadia Ali Nadhim
College of Education. - Anbar university
Received: 22/4/2009 Accepted:2/12/2009

Abstract: A new definition of bitopological space is introduce in this paper with its ∂-open set ∂-open function , and inductively ∂-open function and on some theorems for its .

Key words: ∂-open function , Inductively , bitopological spaces

1- Introduction

A bitopological space (X, p_1, p_2) [J .C. Kelly " bitopological space " ,1963] is anon-empty set X with two topological $P_1$and P_2 on X and then definition of open set which is said to be ∂-open set ,also define ∂-open function ,and study some of properties for its , also introduce inductively ∂-open function and we study the relation between ∂-open function and inductively ∂-open function in bitopological space and then we write some of theorems for them.

2- Basic definitions and theorems

Definition 2-1 :.

Let (X, p_1, p_2) be bitopological space then asubset A of X is said to be ∂-open set iff ,there exists p_i- open set U ,such that U $\subseteq$ A ,and $\bigcap Cl_{p_i}(U) \subseteq$ A , I =1,2

Example 2-2 :.

Let X ={ a ,b, c, d }, p_1= {Φ ,X ,{a} ,{b}, {a,b}} , p_2={ Φ,X, {c},{a, c}}

then ∂-open sets ={ Φ ,X, {a, b, d},{b, c, d},{a, c, d}}.

Remark 2-3 :.

The intersection of tow ∂-open sets is not necessarly ∂-open .while the union is ∂-open set.

Proof :

Let $\{A_\lambda : \lambda \in \wedge\}$be any arbitrary collection of ∂-open sets ,then there exists P_i-open set U_λ such that $U_\lambda \subseteq A_\lambda$ and $\bigcap Cl_{P_i}(U_\lambda) \subseteq A_\lambda$, I= 1,2 , for each $\lambda \in \wedge$.

Since :

$$\bigcup_{\lambda\in\wedge}\left(\bigcap_{i=1,2} Cl_{P_I}(U_\lambda)\right) = Cl_{P_1}(\bigcup(U_\lambda)) \cap Cl_{P_2}(\bigcup(U_\lambda)))$$

$$= \bigcap_{i=1,2} Cl_{Pi}(\bigcup_{\lambda\in\wedge}(U_\lambda)) \subseteq \bigcup_{\lambda\in\wedge} A_\lambda$$

Remark 2-4 :

1- The set of all ∂-open sets is not atopological space.
2- If A is P_i -closed set for I =1,2 , then A is ∂-open set .
3- If A is P_i -open set for I =1,2 , then A is not necessary ∂-open sets .

Examples 2-5 :

Let X ={ a ,b, c, d }, p_1= {Φ ,X ,{a} ,{b, c }} , p_2={ Φ,X, {a},{c,d}}.

the ∂-open sets ={ Φ ,X, {a},{a, b},{a, c},{a, d},{a, b, c},{a, b, d}{b, c, d}} .

since {a, b, d}∩ {c, b, d} ={b, d} which is not ∂-open set .then the set of all ∂-open

Also {b, c} is open in P_1 , but not ∂-open .

{c, d} is open in P_2 , but not ∂-open .

Definition 2-6 :

A function f: (X , P_1, P_2) → (Y , W_1, W_2) is said to be ∂-open function iff f(U) is ∂-open in Y whenever Uis ∂-open set in X .

Definition 2-7 :

Let f: : (X , P_1, P_2) → (Y , W_1, W_2) be function we say that f is inductively ∂-open function iff there exists a subset $X^* \subseteq$ X .such that $f(X^*)$= f(X) and the function $f|_{X^*}$: (X^* , P_1, P_2) → (f(X), W_1, W_2) is ∂-open function .

Remark 2-8 :

1- every onto closed function is ∂-open function .

2- every onto ∂-open function is inductively ∂-open function .

Theorem 2-9 :

If f:(X , P_1, P_2) → (Y , W_1, W_2) is one to one function and on some $X_1 \subseteq X$ with $f(X_1) = f(X)$, f is inductively ∂-open function on X , then f is inductively ∂-open function on X_1 .

Proof :-

Since f: (X , P_1, P_2) → (Y , W_1, W_2) be inductively ∂-open one to one function then , there exists $X^* \subseteq X$, such that $f(X^*)= f(X)$ and $f|_{X^*}$: (X^* , P_1, P_2) → (f(X), W_1, W_2) is ∂-open now ,to prove f:(X_1 , P_1, P_2) → (f(X) , W_1, W_2) is inductively ∂-open function .

let $X_2 \subseteq X$ such that $X_2 = X^* \subseteq X_1$.

we need to show that $f(X_2)=f(X_1)$ and $f|_{X2}$: (X_2 , P_1, P_2) → (f(X), W_1, W_2) is ∂-open function .

now , $f(X_2)= f(X^* \cap X_1) = f(X^*) \cap f(X_1)$
$= f(X) \cap f(X)$
$= f(X)$
$= f(X_1)$

Let U be ∂-open set in (X_2 , P_1, P_2) , to show f(U) is ∂-open in (f(X), W_1, W_2) .

Since U is ∂-open in X_2 , then there exists U^* closed in X^*, such that $U = U^* \cap X_2$

$f(U) = f(U^* \cap X_2)$
$= f(U^*) \cap f(X_2) = f(U^*) \cap f(X) = f(U^*)$.

Since U^* is closed in X^* , then U^* is ∂-open in X^* and $f|_{X^*}$: (X^* , P_1, P_2) → (f(X), W_1, W_2) is ∂-open hence $f(U^*)$ is ∂-open in f(X).

Definition 2-10 :

Let f: (X , P_1, P_2) → (Y , W_1, W_2) be a function in bitopological space .

And $A \subseteq X$ a subset A is said to be an inverse set iff $A = f^{-1}(f(A))$.

Theorem 2-11 :

If f: (X , P_1, P_2) is inductively ∂-open function in bitopological space, and A inverse subset of X , then $f|_A$: (A , P_1, P_2) → (Y, W_1, W_2) is also inductively ∂-open function

Proof :

Since f: (X , P_1, P_2) → (Y , W_1, W_2) be inductively ∂-open function , so there exists asubset $X^* \subseteq X$, such that $f(X^*)= f(X)$and $f|_{X^*}$: (X^* , P_1, P_2) → (f(X), W_1, W_2) is ∂-open function .

Now to prove that $f|_A$: (A , P_1, P_2) → (Y, W_1, W_2) inductively ∂-open function .

Let $A_1 \subseteq A$, such that $A_1 = A \cap X^*$ and we need to show $f(A_1) = f(A)$ and

$f|_{A1}$: (A_1 , P_1, P_2) → (f(A) , W_1, W_2) is ∂-open function .

$f(A_1) = f(A \cap X^*)$
$f(A_1) = f(f^{-1}(f(A)) \cap X^*)$
$= f(A) \cap f(X^*)$
$= f(A) \cap f(X)$
$= f(A)$

Now ,let U be ∂-open in A_1 , so there exists closed set U^* in X^* , such that $U = U^* \cap A_1$.

Hence $f(U) = f(U^* \cap A_1)$
$= f(U^* \cap A \cap X^*)$
$= f(U^* \cap X^* \cap A)$
$= f(U^* \cap A)$
$= f(U^* \cap f^{-1}(f(A)))$
$= f(U^*) \cap f(A)$

Since U^* is closed in X^* , then U^* is ∂-open in X^* , and $f|_{X^*}$: (X^*, P_1, P_2) → (f(X), W_1, W_2) is ∂-open function .

hence $f(U^*)$ is ∂-open in $f(X^*)= f(X)$.

there fore $f(U^*) \cap f(A)$ ∂-open in f(A) .

thus $f|_{A1}$: (A_1 , P_1, P_2) → (f(A), W_1, W_2) is ∂-open function .

so $f|_A$: (A , P_1, P_2) → (Y, W_1, W_2) inductively ∂-open restriction of a function .

Proposition 2-12 :

If f: : (X , P_1, P_2) → (Y , W_1, W_2) is ∂-open function , let $T \subseteq Y$, then

f_T : ($f^{-1}(T)$, P_1, P_2) → (T , W_1, W_2) ∂-open function .

proof

Let V be ∂-open set in $f^{-1}(T)$.

So ,there exists closed set V^* in X ,such that $V = V^* \cap f^{-1}(T)$.

$f_T(V) = f(V) = f(V^* \cap f^{-1}(T)) = f(V^*) \cap T$.

since V^* is closed in X , then V^* is ∂-open and f:(X , P_1, P_2) → (Y , W_1, W_2) is ∂-open , then $f(V^*)$ ∂-open in Y .

so $f(V^*) \cap T$ is ∂-open in T .

hence f_T : ($f^{-1}(T)$, P_1, P_2) → (T , W_1, W_2) is ∂-open function .

Theorem 2-13 :

If f: (X , P_1, P_2) → (Y , W_1, W_2) is onto inductively ∂-open function , let $\Phi \neq T \subseteq Y$,then f_T : ($f^{-1}(T)$, P_1, P_2) → (T , W_1, W_2) be also inductively ∂-open function .

proof :

since f: (X , P_1, P_2) → (Y , W_1, W_2) onto inductively ∂-open function , then there exists a subset $X_1 \subseteq X$,such that $f(X_1) = Y$ and $f|_{X1}$: (X_1 , P_1, P_2) → (Y, W_1, W_2) is ∂-open .

now ,to prove f_T : ($f^{-1}(T)$, P_1, P_2) → (T , W_1, W_2) is inductively ∂-open function ,where $\Phi \neq T \subseteq Y$.

let X_1^* be a subset of $f^{-1}(T)$, such that $X_1^* = X_1 \cap f^{-1}(T)$ we need to show that

$f_T(X_1^*)$= T and $f_T\big|_{X_1^*} : (X_1^*, P_1, P_2) \rightarrow$ (T, W_1, W_2) is ∂-open function .

now , let U ∂-open set in X_1^* .

hence , thereexists U^* closed set in X_1 , such that $U = U^* \cap X_1^*$

$f(U) = f(U^* \cap X_1^*)$

$f(U) = f(U^* \cap X_1 \cap f^{-1}(T)) = f(U^* \cap f^{-1}(T)) = f(U^*) \cap T$

since U^* closed in X^*, then U^* is ∂-open in X^*, and

$f|_{X^*}: (X^*, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is ∂-open ,so $f(U^*)$ is ∂-open in Y .

there fore $f(U^*) \cap T$ is ∂-open in T .

so $f|_{X1}^*: (X_1^*, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is ∂-open function .

there fore $f_T : (f^{-1}(T), P_1, P_2) \rightarrow (T, W_1, W_2)$ inductively ∂-open function .

Proposition 2-14 :

If $f: (X, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is onto function , $Y = T_1 \bigcup T_2$ open cover of Y .

$f_{T_1} : (f^{-1}(T_1), P_1, P_2) \rightarrow (T_1, W_1, W_2)$ and $f_{T_2} :(f^{-1}(T_2), P_1, P_2) \rightarrow (T_2, W_1, W_2)$ are ∂-open , then $f: (X, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is ∂-open function .

proof :

to prove $f: (X, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is ∂-open .let U be ∂-open set in X , to show f(U) is ∂-open in Y .

$U = U \cap X$

$= U \cap f^{-1}(Y) = U \cap f^{-1}(T_1 \bigcup T_2)$

$= U \cap (f^{-1}(T_1) \bigcup f^{-1}(T_2))$

$= (U \cap (f^{-1}(T_1)) \bigcup (U \cap (f^{-1}(T_2))$

Since U is is ∂-open in X , so $U \cap f^{-1}(T_1)$ is ∂-open in $f^{-1}(T_1)$.

Also $U \cap f^{-1}(T_2)$ is ∂-open in $f^{-1}(T_2)$.

Now , $f(U) = f[U \cap f^{-1}(T_1) \bigcup (U \cap f^{-1}(T_2))]$

$= f(U \cap f^{-1}(T_1)) \bigcup f(U \cap f^{-1}(T_2))$

Since $f(U \cap f^{-1}(T_1))$ is ∂-open in T_1 and $f(U \cap f^{-1}(T_2))$ is ∂-open in T_2 ,

So $f(U \cap f^{-1}(T_1)) \bigcup f(U \cap f^{-1}(T_2))$ is ∂-open in Y .

Theorem 2-15:

If $f: (X, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is onto function , let $Y = T_1 \bigcup T_2$ be open cover of Y .let $f_{T_1} : (f^{-1}(T_1), P_1, P_2) \rightarrow (T_1, W_1, W_2)$ and $f_{T_2} :(f^{-1}(T_2), P_1, P_2) \rightarrow (T_2, W_1, W_2)$ are inductively ∂-open function , then $f: (X, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is also inductively ∂-open in function .

Proof :

Since $f_{T_1} : (f^{-1}(T_1), P_1, P_2) \rightarrow (T_1, W_1, W_2)$ inductively is ∂-open in function , then there exists a subset $X_1 \subseteq f^{-1}(T_1)$ such that $f_{T_1}(X_1) = f_{T_1}(f^{-1}(T_1)) = T_1$ and

$f_{T_1}|_{X1}: (X_1, P_1, P_2) \rightarrow (T_1, W_1, W_2)$ is ∂-open function .

similarly $f_{T_2} :(f^{-1}(T_2), P_1, P_2) \rightarrow (T_2, W_1, W_2)$ inductively ∂-open function , then there exists $X_2 \subseteq f^{-1}(T_2)$ such that :.

$f_{T_2}(X_2) = f_{T_2}(f^{-1}(T_2)) = T_2$ and $f_{T_2}|_{X2}: (X_2, P_1, P_2) \rightarrow (T_2, W_1, W_2)$ is ∂-open function .

now , to prove $f: (X, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is inductively ∂-open function .

let $X_3 \subseteq X$, such that $X_3 = X_1 \bigcup X_2$.

we need to show that $f(X_3) = Y$ and $f|_{X3}: (X_3, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is ∂-open function .

$f(X_3) = f(X_1 \bigcup X_2)$

$= f(X_1) \bigcup f(X_2)$

$f(X_3) = T_1 \bigcup T_2 = Y$

Now , let U_3 be ∂-open set in X_3 .

Hence $U_3 \cap X_1$ is ∂-open in X_1 , and $U_3 \cap X_2$ is ∂-open in X_2 .

$f(U_3) = f(U_3 \cap X_3)$

$= f(U_3 \cap (X_1 \bigcup X_2)) = f((U_3 \cap X_1) \bigcup (U_3 \cap X_2)) = f(U_3 \cap X_1) \bigcup f(U_3 \cap X_2)$

Since $f(U_3 \cap X_1)$ is ∂-open in T_1 , and $f(U_3 \cap X_2)$ is ∂-open in T_2 .

$f(U_3 \cap X_1) \bigcup f(U_3 \cap X_2)$ is ∂-open in Y .

hence $f|_{X3}: (X_3, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is ∂-open function .

there fore $f: (X, P_1, P_2) \rightarrow (Y, W_1, W_2)$ is inductively ∂-open function .

Theorem 2-16:

Let $f: (X, P_1, P_2)$ be a function in bitopological space , $X = U_1 \bigcup U_2$ with $f(U_1)$ and $f(U_2)$ are closed in f(X) , if $f|_{U1}: (U_1, P_1, P_2) \rightarrow (Y, W_1, W_2)$ and $f|_{U2}: (U_2, P_1, P_2) \rightarrow (Y, W_1, W_2)$ are inductively ∂-open function , then $f: (X, P_1, P_2) \rightarrow (Y, W_1, W_2)$ inductively ∂-open function .

proof :

$f|_{U1}: (U_1, P_1, P_2) \rightarrow (Y, W_1, W_2)$ inductively ∂-open function .

then there exists $X_1 \subseteq U_1 \ni f(X_1) = f(U_1)$ and $f|_{X_1}: (X_1, P_1, P_2) \to (f(U_1), W_1, W_2)$ is ∂-open function .
also $f|_{U_2}: (U_2, P_1, P_2) \to (Y, W_1, W_2)$ inductively ∂-open function .
then ,there exists a subset $X_2 \subseteq U_2$,such that $f(X_2) = f(U_2)$ and
$f|_{X_2}: (X_2, P_1, P_2) \to (f(U_2), W_1, W_2)$ is ∂-open function .
now , to show $f: (X, P_1, P_2) \to (Y, W_1, W_2)$ inductively ∂-open function .
let $X^* = X_1 \bigcup X_2 \subseteq X$

$f(X^*) = f(X_1 \bigcup X_2) = f(X_1) \bigcup f(X_2) = f(U_1) \bigcup f(U_2) = f(U_1 \bigcup U_2)$
$= f(X)$

So $f(X^*) = f(X)$, and to show $f|_{X^*}: (X^*, P_1, P_2) \to (f(X), W_1, W_2)$ is ∂-open function .
Let T ∂-open in X^*

$T = T \cap X^* = T \cap (X_1 \bigcup X_2) = (T \cap X_1) \bigcup (T \cap X_2)$

$f(T) = f[(T \cap X_1) \bigcup (T \cap X_2)]$
$= f(T \cap X_1) \bigcup f(T \cap X_2)$

Since T ∂-open in X^* , so $T \cap X_1$ is ∂-open in X_1 and $f|_{X_1}: (X_1, P_1, P_2) \to (f(U_1), W_1, W_2)$ is ∂-open function .
then $f(T \cap X_1)$ ∂-open in $f(U_1)$ and $f(U_1)$ is closed in f(X) then $f(U_1)$ is ∂-open in f (X) , hence $f(T \cap X_1)$ ∂-open in f(X) .
similarly $f(T \cap X_2)$ is ∂-open in f(X)
$f(T) = f(T \cap X_1) \bigcup f(T \cap X_2)$ is ∂-open in f(X) $f|_{X^*}: (X^*, P_1, P_2) \to (f(X), W_1, W_2)$ is ∂-open function .
there fore $f: (X, P_1, P_2) \to (Y, W_1, W_2)$ inductively ∂-open function .

Theorem 2-17 :
If $f: (X, P_1, P_2) \to (Y, W_1, W_2)$ is a function in bitopological space , $X = \bigcup_{\alpha \in \wedge} U_\alpha$ with $f(U_\alpha)$ ∂-open in f(X) , for each $\alpha \in \Lambda$, $f|_{U_\alpha}: (U_\alpha, P_1, P_2) \to (Y, W_1, W_2)$ inductively ∂-open function , then $f: (X, P_1, P_2) \to (Y, W_1, W_2)$ also inductively ∂-open function .

Proof :
$f|_{U_\alpha}: (U_\alpha, P_1, P_2) \to (Y, W_1, W_2)$ inductively ∂-open function .
then , there exists $X_\alpha \subseteq U_\alpha$, such that $f(X_\alpha) = f(U_\alpha)$ and
$f|_{X_\alpha}: (X_\alpha, P_1, P_2) \to (f(U_\alpha), W_1, W_2)$ ∂-open function for each $\alpha \in \Lambda$.
let $X^* = \bigcup_{\alpha \in \wedge} X_\alpha$ be a subset of X .
now ,to show $f: (X, P_1, P_2) \to (Y, W_1, W_2)$ inductively ∂-open function .
we need to show $f(X^*) = f(X)$ and $f|_{X^*}: (X^*, P_1, P_2) \to (f(X), W_1, W_2)$ ∂-open .

$f(X^*) = f(\bigcup_{\alpha \in \wedge} X_\alpha) = \bigcup_{\alpha \in \wedge} f(X_\alpha) = \bigcup_{\alpha \in \wedge} f(U_\alpha) = f(\bigcup_{\alpha \in \wedge} U_\alpha) = f(X)$

Now ,let T ∂-open in X^* .

$T = T \cap X^* = T \cap (\bigcup_{\alpha \in \wedge} X_\alpha) = \bigcup_{\alpha \in \wedge} (T \cap X_\alpha)$

$f(T) = f(\bigcup_{\alpha \in \wedge} (T \cap X_\alpha))$

since $T \cap X^*$ ∂-open in X_α and $f|_{X_\alpha}: (X_\alpha, P_1, P_2) \to (f(U_\alpha), W_1, W_2)$ ∂-open .
then $f(T \cap X^*)$ ∂-open in $f(U_\alpha)$ and since $f(U_\alpha)$ ∂-open in f(X) , for each $\alpha \in \Lambda$.
then $f(T) = \bigcup_{\alpha \in \wedge} f(T \cap X_\alpha)$ ∂-open in f(X) .
so $f|_{X^*}: (X^*, P_1, P_2) \to (f(X), W_1, W_2)$ is ∂-open .
there fore $f: (X, P_1, P_2) \to (Y, W_1, W_2)$ inductively ∂-open function .

References: .

1- J. N. Sharma ," Topology" Meerut college , Meerut . 1977.
2- J. C. Kelly , " Bitopological spaces " , proc. London Math . Soc .13 (1963) , 71-89 .
3- Yiezi .K. M. and sukaina . A.L; " Study of some topological concepts in bitopological spaces " . department of mathematics , college of education , Babylon university , 2007 .

دراسة الدالة المفتوحة ∂ في الفضاء ثنائي التوبولوجي

نادية علي ناظم
E.mail: anbsci@yahoo.com

الخلاصة : (X , P1, P2) يقدم هذا البحث تعريف جديد للمجموعة المفتوحة في الفضاء ثنائي التبولوجي في هذا الفضاء عرفنا المجموعة المفتوحة – ∂ والدوال المفتوحة– ∂ وكذالك الدوال المفتوحة– ∂ استقرائيا والمبرهنات المتعلقة بهذه المواضيع ودراسة بعض الخواص المرتبطة فيه