On Weak Forms of Regular Generalized Some Separation Axioms In Intuitionistic Topological Spaces

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Abstract: Our goal in this paper is to give new definition of regular generalized T1 and regular generalized T2 separation axioms in intuitionistic topological spaces and study relations among several types of regular generalized separation axioms with some basic properties and counter examples.

Key words: Weak Forms, Regular Generalized, Axioms, Intuitionistic, Topological Spaces

Introduction:

The concept of "Intuitionistic fuzzy sets" was introduced by Atanassov in 1983[1] (IFS for short), on the other hand Coker[4] introduced the notions of intuitionistic fuzzy First, we present the fundamental definitions, topological spaces.

In this paper, we introduced the concept of regular generalized T1, locally Let X be an empty fixed set. An intuitionistic fuzzy set (IFS, for short) A is an object having the form A = <x, A1, A2>, which A1 and A2 are subset of X and satisfying the following Definitions: 2.2[4].

An intuitionistic fuzzy topology (IFT, for short) on an empty set X is a family T containing \( \emptyset \) and X closed under finite intersection and arbitrary union.

In this case the pair (X, T) is called an intuitionistic fuzzy topological spaces (IFT, for short) and each IFS in T is known as an intuitionistic fuzzy open set (IFOS, for short) in X. The complement of an IFOS A in an IFTS (X, T) is called an intuitionistic fuzzy closed set (IFCS, for short), in X.

Definitions 2.3[4]

Let X be an empty set and let the IFS's A and B be in the form A = <x, A1, A2>, B = <x, B1, B2> and let \( \{ A_i : i \in I \} \) be a finite intersection and arbitrary union.

i. \( A = B \iff A_1 \subseteq A_1 \land A_2 \supseteq A_2 \);
ii. \( A \subseteq B \iff A \subseteq A \land B \subseteq B \);
iii. \( \overline{A} = \overline{A} \land A_1 \supseteq A_1 \land A_2 \supseteq A_2 \);
iv. \( \bigcup A_i = \bigcup A_1 \cup A_2 \);
\( \bigcup A_i = \bigcup A_1 \cup A_2 \).
Definition : 2.4[4]
An intuitionstic fuzzy point in X ( IP for short ) is defined by \( \tilde{p} = \langle x, \{ \phi \}^c \rangle \) and the IS \( \tilde{p} = \langle x, \phi, \{ p \}^c \rangle \) in called avanishing intuitionstic point (VIP for short ) in X.

Definition : 2.5[1]
Let A be an IFS , then the interior and closure of an IFS A is defind by :
\[
\text{Int} A = \bigcup \{ G : G \in T, G \subseteq A \}, \\
\text{CLA} = \cap \{ k : k \in T, A \subseteq k \}
\]

Definition: 2.6
Let ( X , T ) be IFS , A subset A of (X , T ) is called regular generalized closed set ( RgcS for short ) if CLA \( \subseteq A \), whenever \( \bigcup A \subseteq U \) and U is regular open.

The complement of RgcS in X is called regular generalized open set ( RgoS for short ) in X.

Proposition : 2.7
Let ( X , T ) be IFS , A is RgoS in X if and only if for each regular closed set F such that F \( \subseteq A \), then F \( \subseteq \text{Int} A \).

Proof :
Suppose that A is RgoS in X , then A is Rgc , so for each RoS in X and A \( \subseteq U \), then CLA \( \subseteq U \). Put A \( \subseteq = F \) and U \( \subseteq \text{Int} A \), then F \( \subseteq \text{Int} U \).

Definition: 2.8
Let ( X , T ) be IFS , A is RgoS in X if and only if for each regular closed set F such that F \( \subseteq A \), then F \( \subseteq \text{Int} A \).

Remark : 2.9
i)Intersection of any family of RgcS is Rgc.
ii)Any Union of RgoS is RgoS.

Remark : 2.10
i)Every open set is RgoS but the converse is not true.
ii)Every closed set is RgcS , but the converse is not true.

Proof
Suppose that A is an open set , then for each RCS F \( \subseteq A = \text{Int} A \Rightarrow \text{Int} \Rightarrow A \) is Rgo and let A be closed set , so for each RgoS U , A \( \subseteq U \), CLA=A \( \subseteq U \Rightarrow A \) is Rgc.

Example : 2.11
Let X = \( \{ 1,2,3 \} \) and define T by T = \( \{ \phi, \tilde{X}, A \} \) weher A<\( \phi \), \{1\} , \{2,3\} > so RC(x) = \{ \phi, \tilde{X} \} and Rgo (x) = \{ \phi, \tilde{X}, A, B \} where B<\{1,2\} , \{3\} > , then B is Rgo but not open set and C = <\{X\}, \{3\}, \{1,2\}> is Rgc but no closed set.

For each U is ROS and B \( \subseteq U \) we have prove that CL B \( \subseteq U \).

Definition: 3.1
Let (X , T ) be an ITS, (X,T) is said to be :
\( a) \text{RGT1 (i)} \) if for each X,x \( \cdots y \in X, x \neq y \) there exists U,V where U,V are Rgo (X) s.t \( \tilde{x} \in U, \tilde{y} \in V, \tilde{x} \neq \tilde{y} \neq U \).

\( b) \text{RGT1 (ii)} \) if for each x,y \( \in X, x \neq y \), there exists U,V where U,V are Rgo (X) s.t
\[ \tilde{x} \in U, \tilde{y} \not\in U \quad \text{and} \quad \tilde{y} \in V, \quad \tilde{x} \not\in \tilde{X} \quad \text{and} \quad \tilde{y} \not\in V. \]

e) RGT1(iii) if for each \( x, y \in X, x \neq y \), there exists U,V where U,V are Rgo(X) s.t:

\[ \tilde{x} \in U \subseteq \tilde{y} \quad \text{and} \quad \tilde{y} \in V \subseteq \tilde{x}. \]

Proof: RGT1(vi) ⇒ RGT1(v)

Suppose \( \forall x, y \in X, x \neq y \) so there exists U=\(<x,u1,u2>_x\) and V=\(<y,v1,v2>_y\) are Rgo(X) s.t:

\[ \tilde{x} = <x, \{x\}, \{x\}^c > \in U \quad \text{and} \quad \tilde{y} = <y, \{y\}, \{y\}^c > \in U \]

\[ \tilde{y} \in V, \tilde{x} \not\in V, \tilde{x} \not\in U \quad \text{and} \quad \tilde{y} \not\in U \]

there for RGT1(v) holds.

RGT1(ii) ⇒ RGT1(vi)

Let \( x, y \in \tilde{X}, x \neq y \), since RGT1(ii) hold so there exists U,V are Rgo(X) s.t:

\[ \tilde{z} \subseteq U, \tilde{z} \not\in U \quad \text{and} \quad \tilde{z} \in V \quad \text{where} \]

\[ \tilde{y} = \{y, \phi, \{y\}^c \} > \in U \quad \text{and} \quad \tilde{y} \not\in U \]

so \( y \not\in U \), there for \( U_1 \subseteq \{y\}^c \) \( y \not\in U \)

\( U_1 \quad \text{and} \quad y \not\in U \) \( y \not\in U \)

RGT1(iii) ⇒ RGT1(i)

RGT1(i) + RGT1(ii) ⇒ RGT1(iii)

is direct

\[ \text{RGT1(vi) ⇒ RGT1(v)} \]

Suppose there exists U,V∈Rgo(X) s.t:

\[ \tilde{x} \not\in U, \tilde{y} \not\in U \quad \text{and} \quad \tilde{y} \not\in V \quad \text{this implies} \]

\[ \tilde{y} \not\in U \quad \text{and} \quad \tilde{x} \not\in V. \]

f) RGT1(vi) if for each \( x, y \in X, x \neq y \) there exists U,V where U,V are Rgo(X) s.t:

\[ \tilde{y} \not\in U \quad \text{and} \quad \tilde{x} \not\in V. \]

g) RGT1(vii) if for each \( x \in X, \tilde{x} \) is Rgc(X).

h) RGT1(viii) if for each \( x \in X, \tilde{x} \) is Rgc(X).

Theorem 3.2

Let \( (X, T) \) be an ITS, then the following implication are valued

\[ \text{RGT1(vi)} \quad \text{RGT1(v)} \]

\[ \tilde{z} = <x, \phi, \{x\}^c > \not\in \tilde{x} \quad \text{<x, \{x\}, \{x\}^c >} > \in V \quad \text{from this we get} \]

\[ \tilde{x} \not\in V \quad \text{and} \quad \tilde{y} \not\in U \quad \text{there for} \]

\[ \text{for RGT1(vi) hold.} \]

\[ \text{RGT1(v)} \]

RGT1(i) + RGT1(ii) ⇒ RGT1(iii)

Let \( x, y \in X, x \neq y \) since RGT1(i)+RGT1(ii) holds so there exists U=\(<x,U_1,U_2>_x\) and V=\(<y,V_1,V_2>_y\) are Rgo(X) s.t:

\[ \tilde{x} \subseteq U, \tilde{y} \subseteq \tilde{x} \quad \text{and} \quad \tilde{y} \subseteq U, \tilde{y} \subseteq \tilde{x} \quad \text{and} \quad \tilde{y} \subseteq U, \tilde{y} \subseteq V \]

First we have to prove \( \tilde{y} \subseteq U \subseteq \tilde{x} \) and \( \tilde{y} \subseteq \tilde{x} \) we have to prove that \( U \subseteq \tilde{x} \) and \( \tilde{y} < y, \{y\}, \{y\}^c > \), since

\[ \text{that} \quad \tilde{x} \not\in V \quad \text{and} \quad \tilde{y} \not\in V \quad \text{there for implies that} \quad U \subseteq \tilde{x}. \]

In similar way we can prove \( V \subseteq \tilde{x} \) Hence RGT1(iii) holds.
RGT1(iii) \implies RGT1(i) + RGT1(ii)

We have to prove RGT1(iii) \implies RGT1(i) and RGT1(iii) \implies RGT1(ii) we prove that RGT1(iii) \implies RGT1(i) , let, x, y \in X, x \neq y. Since RGT1(iii) hold so there exists U, V \in R.g.o(x) s.t \bar{x} \subseteq \bar{y} and 
\bar{y} \subseteq \bar{X} \implies \bar{x} \subseteq U \subseteq \bar{y} and \bar{y} \subseteq V. Since \bar{y} \subseteq \bar{V} \implies \bar{x} \subseteq U \subseteq \bar{y} \subseteq V, hence RGT1(i) holds.

We can use similar argument to prove that RGT1(iii) \implies RGT1(ii). RGT1(iii) \implies RGT1(vii).

Suppose RGT1(iii) hold . take, x, y \in X s.t 
\bar{x} \subseteq \bar{y} \subseteq \bar{X} \implies \bar{x} \subseteq U \subseteq \bar{y} \subseteq V. Since \bar{y} \subseteq \bar{V} \implies \bar{x} \subseteq U \subseteq \bar{y} \subseteq V. We have to prove that \bar{x} is R.G.o.

That is to prove that \bar{x} is R.G.o(X) for if \bar{x} = U \setminus V \in R.G.o(X), \bar{x} is union of R.g.o set so it is R.g.o therefore \bar{x} is R.G.C. RGT1(iv) \implies RGT1(viii).

Suppose that RGT1(iv) hold and let \bar{x} \subseteq \bar{y} \subseteq \bar{X}. Then we have to prove that \bar{x} is R.G.c , that is to prove is R.g.o (X) for if \bar{x} = U \setminus V \in R.O(X), \bar{x} is union of ROS so it is R.g.o therefore \bar{x} is R.G.C.

RGT1(viii) \implies RGT1(vii).

Suppose that RGT1(viii) hold and let \bar{x} \subseteq \bar{y} \subseteq \bar{X}. Then we have to prove that \bar{x} is R.G.o(X).

RGT1(iii) \implies RGT1(i) \implies RGT1(ii).

We have to prove RGT1(iii) \implies RGT1(i) and RGT1(iii) \implies RGT1(ii). First we prove that RGT1(iii) \implies RGT1(i), let, x, y \in X, x \neq y. Since RGT1(iii) hold so there exists R.g.o(x) s.t \bar{x} \subseteq \bar{y} and \bar{y} \subseteq \bar{X}. Now \bar{x} \subseteq \bar{Y} \subseteq \bar{V}, \bar{y} \subseteq \bar{U} \subseteq \bar{X} and \bar{y} \subseteq \bar{V}. Hence \bar{x} \subseteq \bar{Y} \subseteq \bar{V} \subseteq \bar{X}. Since \bar{y} \subseteq \bar{V} \subseteq \bar{X} \implies \bar{x} \subseteq \bar{Y} \subseteq \bar{V} \subseteq \bar{X}. RGT1(i) holds.

We can use similar argument to prove that RGT1(iii) \implies RGT1(ii), RGT1(iii) \implies RGT1(vii).

Example 3.3

1-Let X = \{a, b\} and define T = \{\bar{x}, \bar{X}, A, B\}, where A = x, B = \bar{x}, \bar{X}, A, B \subseteq \bar{x}, B = \bar{x}, B = \bar{X}, A, B, C \subseteq \bar{X}.

2-let X = \{a, b\} and define T = \{\bar{x}, \bar{X}, A, B, C\}, where A = x, B = \bar{x}, \bar{X}, A, B, C \subseteq \bar{X}.

3-Take X = \{a, b\} and define T = \{\bar{x}, \bar{X}, A, B, C\}.
where $A=<x, \phi, [a]>$, $B=<x, \phi, \phi>$, $C=<x, [a], \phi>$, so $R.C(X)=T$ and $R.g.o(X)=T \cup \{E, G\}$, where $E=<x, [b], \phi>$, $G=<x, [b], [a], \phi>$, so the IT(X,T) satisfies RGT1(viii) but does not satisfies RGT1(iv) and satisfies RGT1(vii) but not satisfies RGT1(iii).

4. Take $X=\{a, b\}$ and defined $T=\{\phi, \tilde{x}, A, B, C\}$, where $A=<x, \phi, \phi>$, $B=<x, [b], \phi>$, $C=<x, \phi, [a], \phi>$, so $R.C(X)=\{\phi, \tilde{x}, A, B, C\}$ and $R.g.o(X)=\{\phi, \tilde{x}, A, B, C, E\}$ where $E=<x, [a], [b], \phi>$, then the IT(X,T) satisfies RGT1(i) but not satisfies RGT1(ii).

5. Let $X=\{a, b\}$ and defined $T=\{\phi, \tilde{x}, A, B, C\}$, $A=<x, [a], [b], \phi>$, $B=<x, [b], \phi>$, $C=<x, \phi, [a], [b], \phi>$, so $R.C(X)=\{\phi, \tilde{x}, A, B, C\}$ and $R.g.o(X)=\{\phi, \tilde{x}, A, B, C, E\}$ where $E=<x, [a], \phi>$ then the IT(X,T) satisfies RGT1(i) but not satisfies RGT1(ii).

4. The Separation axiom Regular generalized T2:

In this section we recall the definition of weak forms of the separation axiom namely regular generalized T2(ki) (RGT2(k)) for short), where $k \in \{i, ii, iii, iv, v, vi\}$ in ITS.

Definition : 4.1

Let (X,T) be an ITS,(X,T) is said to be :

a) RGT2(i) if for each $x, y \in X, x \neq y$, there exists $U, V \in R.g.o(X)_{s.t}$ $\tilde{x} \in U$ and $\bar{y} \in V$ and $U \cap V = \phi$.

b) RGT2(ii) if for each $x, y \in X, x \neq y$, there exists $U, V \in R.g.o(X)_{s.t}$ $\tilde{x} \in U$ and $\bar{y} \in V$ and $U \cap V = \phi$.

c) RGT2(iii) if for each $x, y \in X, x \neq y$, there exists $U, V \in R.g.o(X)_{s.t}$ $\tilde{x} \in U$ and $\bar{y} \notin V$ and $U \subseteq \bar{V}$.

d) RGT2(iv) if for each $x, y \in X, x \neq y$, there exists $U, V \in R.g.o(X)_{s.t}$ $\tilde{x} \in U$ and $\bar{y} \in V$ and $U \subseteq \bar{V}$.

e) RGT2(v) if for each $x, y \in X, x \neq y$, there exists $U, V \in R.g.o(X)_{s.t}$ $\tilde{x} \in U$ and $U \cap V = \phi$.

\[ \tilde{x} \in U, \tilde{y} \in V, \text{ and } U \cap V = \phi \]

The following in the main theorem it gives relations of the several kinds of RGT2 separation axioms.

Theorem : 4.2

Let (X,T) be an ITS.Then the following implications are valid :

\[ \text{RGT}_2(v) \Rightarrow \text{RGT}_2(vi) \]

\[ \text{RGT}_2(i) \Rightarrow \text{RGT}_2(iii) \]

\[ \text{RGT}_2(vii) \Rightarrow \text{RGT}_2(iv) \]

\[ : \text{RGT}_2(v) \Rightarrow \text{RGT}_2(vi) \]

Let (X,T) be an ITS satisfy RGT2(v) and satisfy RGT2(vi).

Proof

\[ \text{RGT}_2(v), \text{for if let } x, y \in X, x \neq y, \text{there exists } U, V \in R.g.o(X)_{s.t} \tilde{x} \in U \text{ and } \tilde{y} \notin U \]

\[ \text{and } U \cap V = \phi. \text{Since } \tilde{x} \in U \text{ and } \tilde{y} \notin U \]

\[ \text{then we can get easily that } \tilde{x} \in U \text{ and } \tilde{y} \notin U \]

\[ \text{therefore } \tilde{x} \in V \text{ and } \tilde{y} \notin U \text{ and } U \subseteq \bar{V} \text{ and } U \cap V = \phi. \text{So we get that (X,T) is satisfy RGT2(v)} \]

\[ \text{RGT}_2(v) \Rightarrow \text{RGT}_2(ii). \]

Let (X,T) be an ITS satisfy RGT2(ii) so take $x, y \in X, x \neq y$, there exists $U, V \in R.g.o(X)_{s.t} \tilde{x} \in U, \bar{y} \notin V$ and $U \cap V = \phi$. Since $\tilde{x} \in U$, then we can get easily that $\tilde{x} \in U$ and $\bar{y} \notin V$, then

\[ \text{(X,T) satisfy RGT2(ii).} \]

Therefore RGT2(ii) holds.

\[ \text{RGT}_2(ii) \Rightarrow \text{RGT}_2(iii). \]

Let (X,T) be ITS satisfy RGT2(ii), for if $x, y \in X, x \neq y$. Since RGT2(ii) holds, this implies that there exists $U, V \in R.g.o(X)_{s.t} \tilde{x} \in U, \bar{y} \notin V$ and $U \cap V = \phi$. Since $\tilde{x} \in U$, then $\bar{y} \notin V$, this implies that $\tilde{x} \in V$. This prove for every $x$ in X if $\tilde{x} \in U$, then $\tilde{x} \notin V, \text{i.e } U \subseteq \bar{V} \text{ Therefore (X,T) satisfies RGT2(iii).} \]
RGT2(iii) $\Rightarrow$ RGT2(ii)

Let (X,T) be an ITS satisfies RGT2(iii) so there exists $U,V \in R.g.o(X)$ such that $\tilde{x} \in U$, $\tilde{y} \in V$ and $U \subseteq \tilde{V}$. To prove that $U \cap V = \tilde{\phi}$. Since $U \subseteq \tilde{V}$ and $\tilde{x} \in U$ so $x \in \tilde{V}$, this implies that $x \notin V$. Therefore $U \cap V = \tilde{\phi}$, so (X,T) satisfies RGT2(ii).

RGT2(ii) $\Rightarrow$ RGT2(iv)

Since RGT2(ii) hold, so let $x,y \in X, x \neq y$, there exists $U,V \in R.g.o(X)$ s.t $\tilde{x} \in U \subseteq \tilde{V}$, $\tilde{y} \in V \subseteq \tilde{\tilde{V}}$ and $U \cap V = \tilde{\phi}$ from this we directly that there exists $U,V \in R.g.o(X)$, s.t $\tilde{x} \in U \subseteq \tilde{V}$, $\tilde{y} \notin V$ and $U \cap V = \tilde{\phi}$, therefore RGT2(ii) holds.

RGT2(iv) $\Rightarrow$ RGT2(iii) is clear

RGT2(iii) $\Rightarrow$ RGT2(iv)

Let (X,T) be an ITS satisfies RGT2(iii), to prove that (X,T) satisfies RGT2(iv), for if $x,y \in X, x \neq y$, since RGT2(iii) holds, this implies that there exists $U,V \in R.g.o(X)$ such that $\tilde{x} \in U \subseteq \tilde{V}$, $\tilde{y} \notin V$ and $U \cap V = \tilde{\phi}$, so we get directly that $\tilde{x} \in U$. $\tilde{y} \notin V$ and $U \subseteq \tilde{V}$. Therefore (X,T) satisfies RGT2(iv).

RGT2(v) $\Rightarrow$ RGT2(i)

Let (X,T) be an ITS satisfies RGT2(v) for if $x,y \in X, x \neq y$, so there exists $U,V \in R.g.o(X)$ such that $\tilde{x} \in U \subseteq \tilde{V}$, $\tilde{y} \subseteq \tilde{\tilde{V}}$ and $U \cap V = \tilde{\phi}$ from this we get directly that $\tilde{x} \in U$, $\tilde{y} \subseteq \tilde{V}$ and $U \cap V = \tilde{\phi}$. Therefore (X,T) satisfies RGT2(i).

RGT2(i) $\Rightarrow$ RGT2(v)

Let (X,T) be an ITS satisfies RGT2(i), to prove that (X,T) satisfies RGT2(v), for if $x,y \in X, x \neq y$. Since RGT2(i) holds, this implies that there exists $U,V \in R.g.o(X)$ such that $\tilde{x} \in U$, $\tilde{y} \subseteq \tilde{V}$ and $U \subseteq \tilde{V} \Rightarrow \tilde{\tilde{V}}$. we have to prove $U \subseteq \tilde{\tilde{V}}$ and $\tilde{\tilde{V}} \subseteq \tilde{\tilde{V}}$ i.e $U \subseteq \{y\} c$ and $\{y\} c \subseteq \tilde{\tilde{V}}$ also $V_1 \subseteq \tilde{\tilde{Y}}$. Let $U_1 = x, U_1 \cap V_1$ and $V_2 \subseteq \{x\} c$ and $\{x\} c \subseteq \tilde{\tilde{V}}$.

Firstly it is prove $U \subseteq \tilde{\tilde{V}}$. Let $U = x, U_1 \cap V_1$ and $\tilde{\tilde{Y}} = x \{y\} c, \{y\} c >$. let $\tilde{z} \in U$ this implies that $\tilde{z} \in \tilde{\tilde{Y}}$ and so $\tilde{z} \in \{y\}$. Therefore $U \subseteq \{y\}$, if $\tilde{z} \in \tilde{\tilde{Y}}$ this implies that $\tilde{z} \in \{y\}$ and so $\tilde{z} \in \{y\}$. Therefore $U \subseteq \{y\}$. Now we have to prove $\{y\} \subseteq \tilde{\tilde{V}}$ and $\{y\} \subseteq \tilde{\tilde{V}}$. So $\{y\} \subseteq \tilde{\tilde{V}}$. In a similar way, we can prove

$V \subseteq \tilde{\tilde{V}}$. Therefore (X,T) satisfies RGT2(v).

RGT2(ii) $\Rightarrow$ RGT2(iv)

Let (X,T) be an ITS satisfies RGT2(ii) this implies that there exists $U,V \in R.g.o(x)$.

\[
\text{such that } U \subseteq \tilde{V} \text{ and } U \cap V = \tilde{\phi} \Rightarrow \tilde{\tilde{V}} \text{ we have to prove that } U \subseteq \tilde{V} \text{ and } V \subseteq \tilde{V} \text{ i.e. } V_1 \subseteq \tilde{\tilde{V}} \text{ and } \phi \subseteq \tilde{\tilde{V}} \text{ also } V_1 \subseteq \{x\} c \text{ and } \phi \subseteq \{x\} c . 
\]

Firstly it is prove $U \subseteq \tilde{\tilde{V}}$. Let $U = x, U_1 \cap V_1$ and $\tilde{\tilde{Y}} = x \{y\} c, \{y\} c >$. Since $\phi \subseteq \{x\} c$ we have to prove $\{y\} \subseteq \tilde{\tilde{V}}$.

Let $\tilde{z} \in U$ this implies that $\tilde{z} \notin \tilde{\tilde{V}}$, so $\tilde{z} \notin \tilde{\tilde{V}}$. (U_1 \cap V_1 = \tilde{\phi}) since $\tilde{\tilde{Y}} \subseteq \tilde{\tilde{V}}$, from the theorem 2.2 by transitivity.

RGT2(v) $\Rightarrow$ RGT2(ii)

The following implications followed from theorem 2.2 by transitivity.

RGT2(ii) $\Rightarrow$ RGT2(iv)
In general the converse of the diagram appears in the theorem 4.2 is not true in general. The following counter example shows the cases.

Example: 4.3

1. Let X={a,b} and T={∅, X, A, B, C}, where
   A=<X, φ, {a}> B=<X, φ, φ> so Rc(X)={φ, X, C}
   and R.g.o(X)=T, so the IT(X,T) satisfies RGT2(vi), but not satisfies RGT2(ii).

2. Let X={a,b,c} and define
   T={φ, X, A, B, C, D, E, F, G, H} where
   A=<X, {a}, {b,c}>, B=<X, {a}, {b,c}>, C=<X, {a}, {a}>, D=<X, φ, φ>, E=<X, {a}, {c}>,
   F=<X, {b}, φ>, G=<X, φ, φ>, H=<X, φ, φ>.
   R.C(X) = {φ, X, φ, E, H} and
   R.g.o(X)=TU{J,N,O,Q,V} where J=<X, {b}, {a}>, N=<X, φ, c>, O=<X, {b}, {a}>, Q=<X, {b}, φ>,
   V=<X, φ, φ>, so that IT(X) satisfies RGT2(i), but not satisfies RGT2(ii) and not satisfies RGT2(vi).

**Corollary: 4.4**

Let (X,T) be ITS, then if (X,T) satisfies RGT2(k), then it satisfies RGT1(k) where
k ∈ {i, ii, iii, iv, v, vi}.

**Remark: 4.5**

The converse of corollary 4.4 is not true in general. The following examples in example 3.3 show these cases.

**References:**


