

Study Surface Roughness and Friction of Synovial Human Knee Joint with Using Mathematical Model

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Abstract

In this paper lubrication and friction of normal and diseased knee joint has been introduced. Various surface roughness of articular cartilage have been determined during the gait cycle (stance phase- swing phase) using squeeze film characteristics in knee joint, this paper aims to study the effects friction and roughness on stride length for male and female during daily activities. The mathematical model depends on the basis of the Stokes micro continuum theory, as well as we used lubricant (Newtonian - non Newtonian) in case of non-Newtonian (micro-macro) roughness effect has been studied and we compared effect film thickness of squeeze film lubrication in cycle time and roughness on pressure film, load carry capacity, in the elderly and young people for each sexes. [DOI: [10.22401/ANJS.00.1.15](https://doi.org/10.22401/ANJS.00.1.15)]

Keywords: Lubrication, friction, knee joint, squeeze film.

1. Introduction:

The knee is composed of the tibiofemoral joint and patellofemoral joint. Each joint has cartilage subchondral bone and soft tissues in and around junctions. Cartilage a wear bearing material, covers the end of bones in the knee joint. Knee joint acts as a journal bearing in mechanical system, [4]. The synovial fluid, contained by the fibrous membrane which is around the margins of the menisci, lubricates the articulations. The synovial membrane provides low friction in the movement of the joint The knee joint sustains wear, functioning like a hinge, and can be flexed and extended when transporting loads from the femur to the tibia in the human normal walking process. Being the largest and most complex synovial joint in the human body, [5], [7]. Factors affecting on the knee-joint performance of its daily functions efficiently increase roughness of the surface of the articular cartilage. During stance phase surface roughness effect on sliding motion through the contact between the surface texture of articular cartilage and this leads to deformation these peals during sliding motion, [3]. Roughness division to micro-roughness and macro-roughness. Micro-roughness where texture surface then increase the load hydrodynamic as well as increasing the film thickness of squeeze lubrication, this

type of roughness appear clearly in phase (childhood and young) where joint is healthy, reach value ranges micro- roughness (0.1-0.4) either macro-roughness occurs speed the process of wear and damage surfaces articular cartilage this is occur with person progresses with age, movement decreases, and the joint's performance becomes low, where value ranges micro- roughness (2-4) roughness agrees with boundary-mixed lubrication so film thickness is reduce. Normal walk must into account the health status and factors affecting on gait cycle (stance-swing) phase. When the structure of surfaces of line and moderate then synovial fluid appears as a clear, pale yellow fluid present in small amounts at synovial joint thus increases the life of surfaces and reduce the wear. Karpenko and Akay [6] have been studied the effect of roughness between two surfaces using an algorithm to calculate the coefficient of friction between them. They concluded that there is a flexible deformation and shearing resistance depends on external loads, mechanical properties and topography surfaces to give the approximate limits of influence. However, the influence of surface roughness and hydrodynamic lubrication on the performance articular cartilage of synovial human knee joint during gait cycle (stance phase-swing phase) has not been studied so

far. Hence, in the present paper, a theoretical investigation is made to study the effect of **Nomenclature**

surface roughness on performance knee joint with hydrodynamic lubrication.

P	Pressure in film region, N/m^2	x , y	Rectangular coordinate
H	Film thickness, m	h_o	Minimal film thickness, m
U	Sliding motion, m/s	R	Equivalent radius ,m
V	Squeeze action, m/s	η	Viscosity , $\frac{N}{m^2 s}$
R_a	Surface roughness, μm	β	Stride length in stance phase, m
ρ	Density ,	$\bar{\beta}$	Stride length in swing phase, m

1.1 The Gait Cycle, [9]:

Though walking patterns can be distinctly individualized and varied all normal walking consists of the same repeating series of event. One full series of events is referred to as a gait cycle. While there is no specific starting or ending point, the most easily identified points are the instant the foot contacts the ground. Therefore, a gait cycle is most often defined as the time interval between two successive instants of foot to ground contact or initial

contact, for the same foot The gait cycle is separated into two distinct periods of stance and swing see Fig.(1.1). Functional tasks include weight acceptance and single limb support during stance and limb advancement during the swing. The stance period of the gait cycle includes initial contact, loading response, mid- stance, terminal stance, and pre-swing. The swing period of the gait cycle includes initial swing, mid-swing and terminal swing.

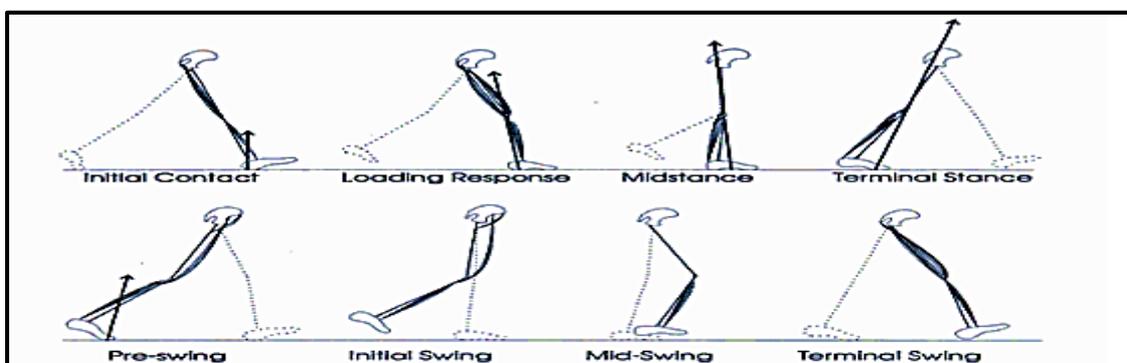


Fig.(1.1) The active muscles are shown during each phase of the gait cycle.

1.2 Gait parameters [8] [10]:

1.2.1 Cycle time. The time required in seconds to complete one walking cycle .To calculate the cycle time the following relations

knee is used: cycle time = time/number of steps. Fig.(1.2) illustration method measure by using the sensor from the right foot.



Fig.(1.2) Shows the method measure gait cycle.

1.2.2 Stride Length:

It is the distance between successive points of heel contact of the same foot. A stride include two steps, right and left but stride length is not always equal to length of two steps as there may be unequal steps ,stride length greatly varies among individuals because it is affected by length, sex, age and speed. Some studies depend in the calculation of step length on distance and number of steps. These can be expressed in the following relation [**stride length = 2×(distance/number of steps)**]. In this way the step length will be: (**stride length/2**).

Table (1.1)

Shows the step length and stride length for male and female at normal walking speed.

Age	Sex	Stride length (m)	Sex	Stride length (m)
20-29	F	1.61	M	1.66
30-39	F	1.51	M	1.56
40-49	F	1.47	M	1.51
50-59	F	1.42	M	1.47
60-70	F	1.35	M	1.38

2. Analysis

Based upon Reynolds equation ,effects of macro-roughness on the squeeze film characteristics between articular cartilage surface is analyzed. Comparing with the micro- roughness case. So we can calculate the pressure in synovial film.

$$\underbrace{\frac{\partial}{\partial x} \left(\frac{h^3}{\eta} \cdot \frac{\partial p}{\partial x} \right)}_{\text{Poiseuille}} + \underbrace{\frac{\partial}{\partial y} \left(\frac{h^3}{\eta} \cdot \frac{\partial p}{\partial y} \right)}_{\text{Physical wedge}} = 6R_a(U_1 - U_2) \frac{\partial(hp)}{\partial x} + \underbrace{12\rho(V_2 - V_1)}_{\text{Squeeze film effect}} \dots (1.1)$$

The first and two terms of the equation (1.1) are the Poiseuille flow expresses the relations knee between the rate of flow of a lubricant in a film thickness and the Pressure gradient in the film. The three and four terms physical wedge and squeeze film effect are the two major Pressure-generating devices in hydrodynamic or self-acting fluid film bearings. For hydrodynamic lubrication the motion is pure sliding, so that V is zero,

applying the assumptions hydrodynamic lubrication theory equation (1.1) becomes

$$\frac{\partial}{\partial x} \left(\frac{h^3}{\eta} \cdot \frac{\partial p}{\partial x} \right) = 6R_a U \frac{\partial(h)}{\partial x} \dots (1.2)$$

where $U = U_1 - U_2$

This equation can be integrated with respect to x to give

$$\frac{h^3}{\eta} \frac{dp}{dx} = 6UR_a h + Z \dots (1.3)$$

where Z denotes constant.

In order to solve the Reynolds equation the integral constants must be determined based on dimensionless pressure boundary conditions.

Natural boundary condition

$$\frac{dp}{dx} = 0 \text{ at } x = x_o, h = h_o$$

Hence $Z = -6UR_a h_o$

Substituting this into equation (1.3) gives

$$\frac{dp}{dx} = 6U \eta R_a \left(\frac{h-h_o}{h^3} \right) \dots (1.4)$$

Where

h_o =minimum film thickness when $dp/dx = 0$

$$h = h_o + \frac{x^2}{2R} \text{ m} \dots (1.5)$$

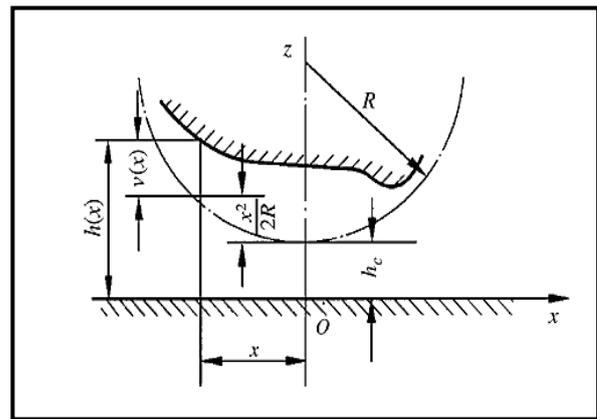


Fig. (1.3) Shape film.

Now substitute equation (1.5) in equation (1.4)

$$\frac{dp}{dx} = 6U \eta R_a \frac{h_o(1 + \frac{x^2}{2Rh_o}) - h_o(1 + \frac{x_o^2}{2Rh_o})}{h_o^3(1 + \frac{x^2}{2Rh_o})^3} \dots (1.6)$$

$$\tan \beta = \frac{x}{\sqrt{2Rh_o}} [1,2] \dots (1.7)$$

Substituting equation (1.7) in equation (1.5) yields to

$$h = h_o(1 + \tan^2 \beta) \dots (1.8)$$

$$h = \frac{h_o}{\cos^2 \beta} = h_o \sec^2 \beta \dots (1.9)$$

Now differentiating x and p with respect to β

$$\frac{dx}{d\beta} = \sqrt{2Rh_o} \sec^2 \beta \dots (1.10)$$

$$dp = 6U \eta R_a \left(\frac{h_o \sec^2 \beta - h_o \sec^2 \beta_o}{h_o^3 \csc^6 \beta} \right) \cdot \sec^2 \beta \sqrt{2Rh_o} d\beta \dots\dots\dots (1.11)$$

2.1 Film dimensionless pressure of the roughness journal bearing representing the knee joint

Introducing the following non-dimensionless quantities of the governing equation of pressure

$$p^* = \frac{p H_0^2}{\eta UR} \quad h_0^* = \frac{h_o}{H_0} \quad \beta = \frac{s}{h_o} \quad r^* = \frac{r}{R}$$

Applying the above dimensionless equations in equation (1.11). Thus the final dimensionless form of the modified Reynolds becomes :

$$\partial p^* = \frac{6\sqrt{2}R_a}{(h_0^*)^{3/2}} \left[\frac{d\beta}{\sec^2 \beta} - \frac{\sec^2 \bar{\beta} d\beta}{\sec^4 \beta} \right] \dots\dots (1.12)$$

The dimensionless. Pressure equation become

$$P^* = \frac{8.4R_a}{(h_0^*)^{3/2}} \left\{ \frac{\beta}{2} + \frac{\sin[2\beta]}{4} \right\} + \sec^2 \bar{\beta} \left\{ \frac{3\beta}{8} + \frac{\sin[2\beta]}{4} + \frac{\sin[4\beta]}{32} \right\} + K \dots\dots\dots (1.13)$$

Applying the boundary conditions $P = 0, \beta = 1.5$ on equations (1.13) hence it was obtained the integration constant $K = + \frac{6.614}{(h_0^*)^2} R_a + 0.58 \sec^2 \bar{\beta}$. Substituting for K

into the equation (1.13) to get general form:

$$P^* = \frac{8.4R_a}{(h_0^*)^{3/2}} \left\{ \frac{\beta}{2} + \frac{\sin[2\beta]}{4} \right\} + \sec^2 \bar{\beta} \left\{ \frac{3\beta}{8} + \frac{\sin[2\beta]}{4} + \frac{\sin[4\beta]}{32} \right\} + \frac{6.614}{(h_0^*)^2} R_a + 0.58 \sec^2 \bar{\beta} \dots\dots\dots (1.14)$$

Reynolds condition:

$$P = 0, \frac{dP}{d\beta} = 0, \beta = 1.5$$

$$0 = \frac{8.4R_a}{(h_0^*)^{3/2}} \left\{ \frac{\beta}{2} + \frac{\sin[2\beta]}{4} \right\} + \sec^2 \bar{\beta} \left\{ \frac{3\beta}{8} + \frac{\sin[2\beta]}{4} + \frac{\sin[4\beta]}{32} \right\} + \frac{6.614}{(h_0^*)^2} R_a + 0.58 \sec^2 \bar{\beta}$$

β can be found equation from equation (1.15)

$$\sec^2 \bar{\beta} = \frac{\left\{ \frac{8.4\beta}{2} + \frac{8.4\sin[2\beta]}{4} + 6.6 \right\}}{\frac{3\beta}{8} + \frac{\sin[2\beta]}{4} + \frac{\sin[4\beta]}{32} + 0.58}$$

With the Reynolds condition $\beta = 1.5$ and the solution of this equation must the done by successive approximations give $\bar{\beta} = 1.8$, so that in the hydrodynamic region, the

dimensionless pressure at any point can be found from:

$$P^* = \frac{8.4R_a}{(h_0^*)^{3/2}} \left\{ \frac{\beta}{2} + \frac{\sin[2\beta]}{4} \right\} + 1.8 \left\{ \frac{3\beta}{8} + \frac{\sin[2\beta]}{4} + \frac{\sin[4\beta]}{32} \right\} + \frac{6.614}{(h_0^*)^2} R_a + 1.44 \dots\dots\dots (1.16)$$

2.2 The load carrying capacity

The load carrying capacity of the porous partial articular cartilage is evaluated by integrating the dimensionless pressure film action on knee joint:

$$W = 2\pi \int_0^R PrhRdr \dots\dots\dots (1.17)$$

Introduce the non-dimensionless load carrying capacity in consideration to roughness porosity cartilage

$$W^* = \frac{W H_o}{\mu.U.R^2} \dots\dots\dots (1.18)$$

Substituted equation (1.17) into (1.18), and we integrate dimensionless pressure with respect to the dimensionless radial and thus we obtain

$$W^* = 2\pi \int_0^1 p^* h_0^* r^* dr^* \dots\dots\dots (1.19)$$

$$W^* = \frac{8.4 \pi R_a}{(h_0^*)^{1/2}} \left\{ \frac{\beta}{2} + \frac{\sin[2\beta]}{4} \right\} + 1.8 \left\{ \frac{3\beta}{8} + \frac{\sin[2\beta]}{4} + \frac{\sin[4\beta]}{32} \right\} + \frac{6.614}{(h_0^*)^2} R_a + 1.44 = g(h^*, \beta, R_a) \dots\dots\dots (1.20)$$

2.3 Friction force

Coefficient of friction between surface cylinder and surface plane is a very important factor in synovial knee joint, to find coefficient of friction depended on shear stress action on the ball surface.

$$\tau = \mu \left(\frac{U}{H} - \frac{1}{2\mu} \frac{\partial p}{\partial h_0} \right) \dots\dots\dots (1.21)$$

The frictional force is given by :

$$F = \int_0^2 \tau \cdot H \cdot dh_0 \dots\dots\dots (1.22)$$

$$F = \int_0^2 \mu \left(\frac{U}{H} - \frac{1}{2\mu} \frac{\partial p}{\partial h_0} \right) \cdot H^2 \cdot dh_0^* \dots\dots\dots (1.23)$$

where U is sliding motion, μ is dynamic viscosity. Introduce non-dimensionless frictional force by form:

$$F^* = \frac{F}{\mu u H} \dots\dots\dots (1.24)$$

Hitting the two parties by the formula above to convert the equation (1.22) to the following formula:

$$F^* = \frac{F}{\mu U} = \int_0^2 \left(1 - \frac{1}{2} \frac{\partial p^*}{\partial h_0^*} \right) dh_0^* \dots\dots\dots (1.25)$$

Substitute dimensionless pressure (p^*) from equation (1.16) in equation (1.25) to obtain the final dimensionless friction force in the synovial fluid pass through the synovial knee joint.

$$F^* = 2 + \frac{6.3 R_a}{h_0^{*3/2}} \left[\frac{\beta}{2} + \frac{\text{Sin}[2\beta]}{4} \right] + \frac{4.9 R_a}{h_0^{*3/2}} \dots\dots\dots (1.26)$$

3. Result and discussion:

On the basis of Reynolds equations, this paper examines the effective of roughness on synovial human knee joint in case (normal-disease) joint during normal walk. To take into account the friction force effects due to the number stride length for male and female during cycle time.

3.1 Squeeze film Pressure

"Fig.(1.4) illustrates dimensionless the pressure film generated by squeeze film action

during the stride length in a normal walk with different values of film thickness for hydrodynamic lubrication. It is found film dimensionless pressure (p^*) increases and becomes more with increasing value of film thickness parameters (h_0^*) during swing phase, comparing with stance phase such decreases film thickness lead to decreases in dimensionless pressure film. Further, it is seen from table (1. 2). Percentage rate of increase in dimensionless pressure in swing phase was approximate 86% at ($S = 0.7, h_0^* = 1.5$), while it was found the percentage rate of decrease in dimensionless pressure (p^*) in stance phase was approximate 63% at ($S = 0.7, h_0^* = 0.72$). Fig. (1.5) shows the variation of dimensionless Pressure film (p^*) as a function of stride length parameter for various values of micro-roughness and macro-roughness for (one cycle, two cycles,..., ten cycles), t is observed that the effect of macro-roughness value is to decrease the dimensionless. Pressure film as compared to micro-roughness value, so high value for each macro-roughness and micro-roughness lead to decrease in dimensionless pressure film.

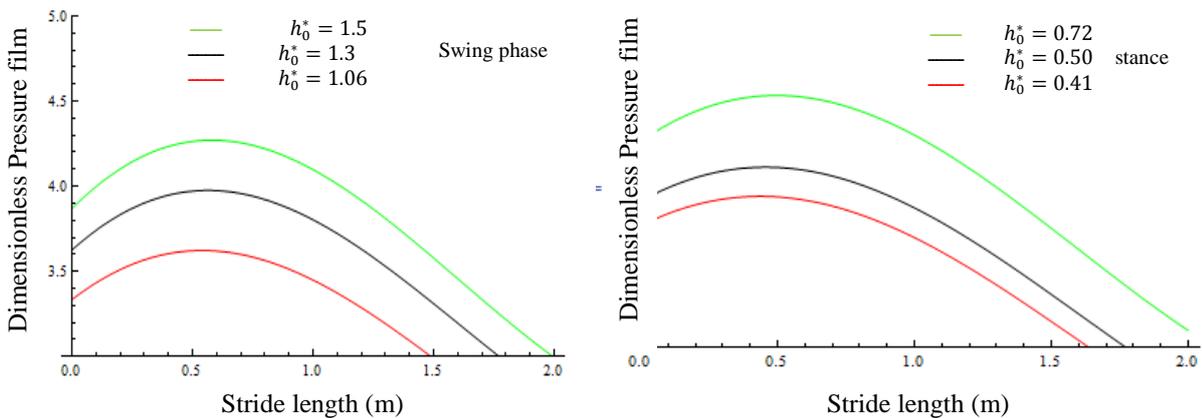


Fig.(1.4) Variation of dimensionless pressure film with stride length for different values of cycle time.

Table (1.2)
Relation between dimensionless pressure film and film thickness.

Stride length in young			Stride length in older		
Film thickness	Time(s)	dimensionless Pressure film	Film thickness	Time (s)	dimensionless Pressure Film
0.72	1.2	2.6	0.72	1.2	3.3
0.50	2.4	2.3	0.50	2.4	2.09
0.41	3.6	2.2	0.41	3.6	1.98
0.35	4.8	2.1	0.35	4.8	1.90
0.32	6	2.09	0.32	6	1.86
0.29	7.2	2.05	0.29	7.2	1.82
0.27	8.4	2.02	0.27	8.4	1.80
0.25	9.6	1.99	0.25	9.6	1.77

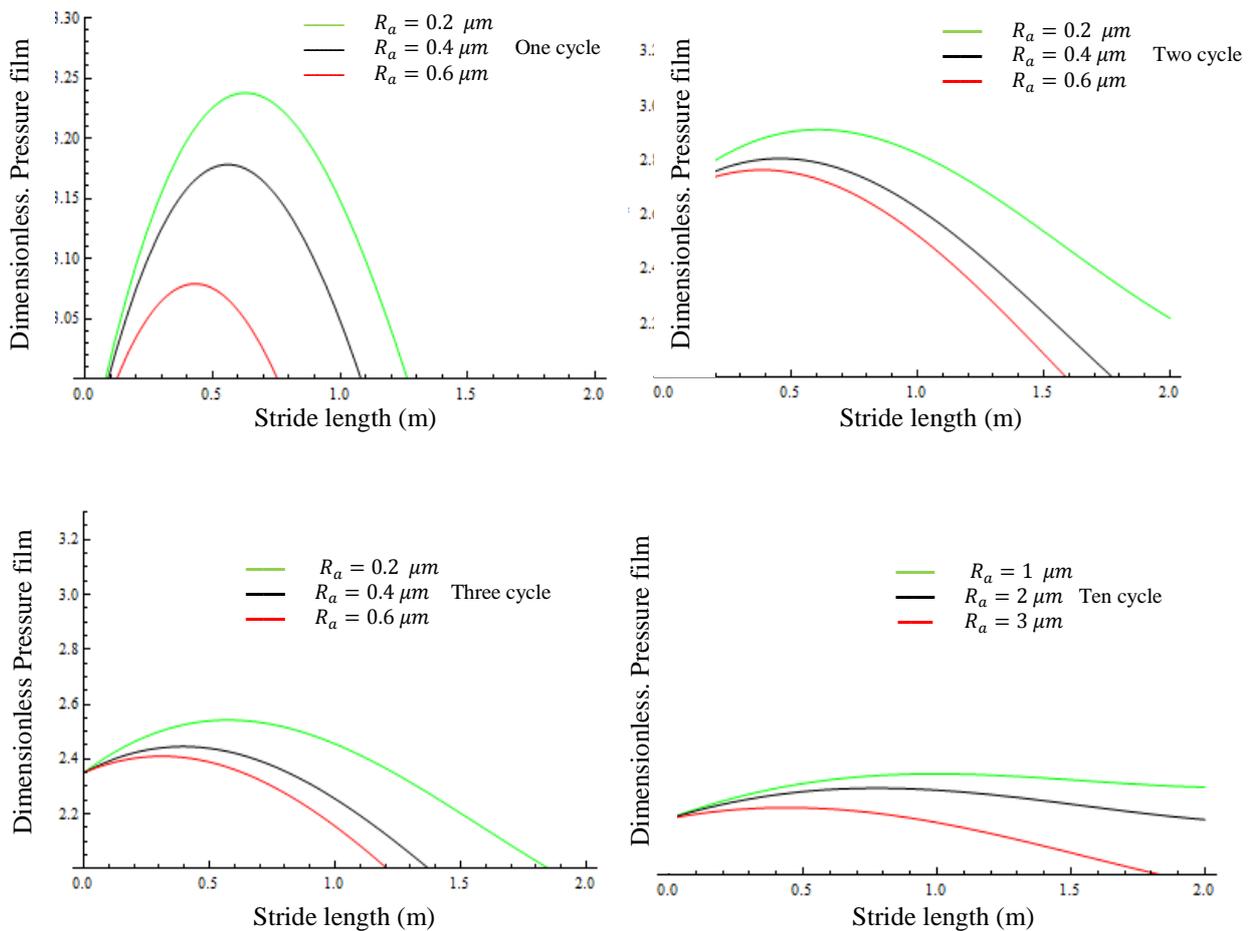


Fig.(1.5) Variation of dimensionless pressure film with stride length for different values of micro-roughness and macro- roughness.

Table (1.3)

Relation between dimensionless pressure film and roughness parameter during daily active.

<i>Film thickness with respect to (stance –swing) phase, $R_a = 0.4, S=1.66$</i>						
<i>Gait cycle</i>	<i>Initial contact</i>	<i>Weight acceptance</i>	<i>Mid-stance</i>	<i>Toe off</i>	<i>Initial swing</i>	<i>Terminal swing</i>
dimensionless Pressure film	6.54	5.87	5.58	5.39	5.28	5.12
<i>Film thickness, $R_a = 0.2$</i>						
dimensionless Pressure film	3.88	3.21	2.9	2.74	2.62	2.47
<i>Film thickness, $R_a = 2$</i>						
dimensionless Pressure film	3.55	2.88	2.06	2.40	2.2	2.1

Fig.(1.6) depicts the variation of dimensionless pressure film as a function of stride length parameter. It is observed that the effect of stride length in young (male-female) is to increase flow lubrication through the

porosity of articular cartilage and therefore increases in dimensionless pressure film. After the age of fifty stride length decreases with time since disease or lack of movement this results educe in dimensionless pressure film.

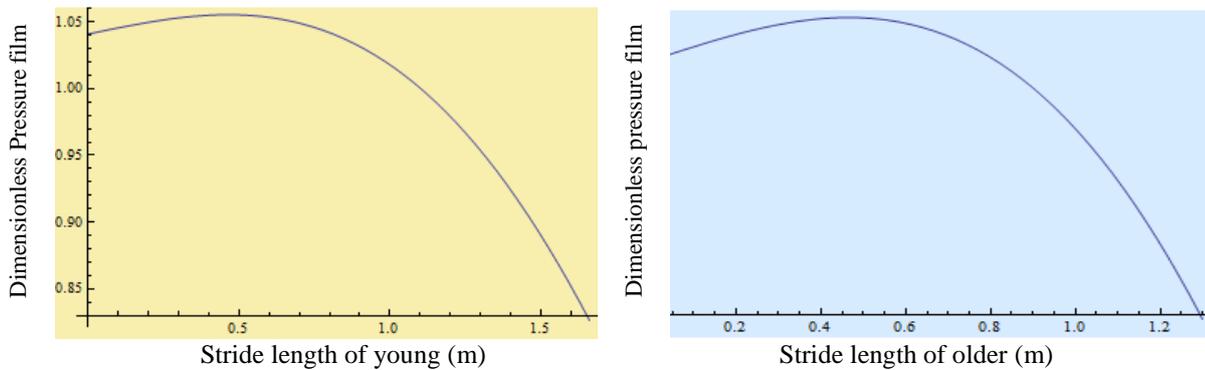


Fig. (1.6) Variation of dimensionless pressure film with different values of stride length.

3.2 Load carrying capacity

This is shown in the Fig.(1.7) stride length parameter for male and female during normal walking, dimensionless load carrying capacity increasing rapidly with stride length, we seen a range of stride length is dependent on the film thickness in hydrodynamic lubrication under effect micro- roughness parameter and macro-roughness parameter. But in all cases the knee joint has very little load carrying capacity for, the joint has high load carrying capacity for, also indicates one-cycle become then load carrying capacity is very little in micro-roughness and macro-roughness inversely when in a three -cycle. The percentage rate of increase in load carrying capacity in three-cycle was approximate 90%, while it was found the percentage rate of decrease in load

in one- cycle was approximate 45%. Fig. (1.8) shows the variation of dimensionless load carrying capacity as a function of surface roughness parameter for various values of Stride longing for (young-old). It is observed that the effect of stride length of young is to lifts dimensionless load as compared to elderly human, it is seen from Table (1.4).

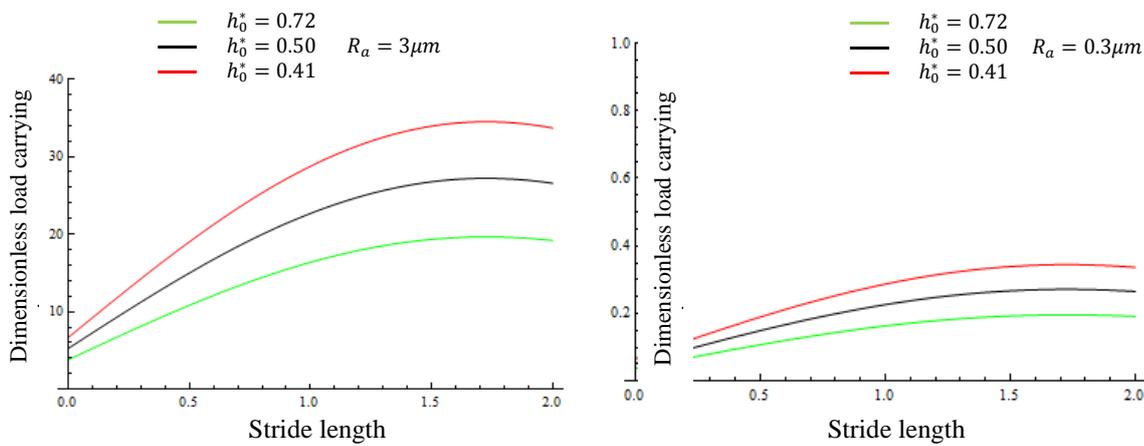


Fig.(1.7) Variation of dimensionless load carrying capacity with Stride length for different values of micro- roughness and macro-roughness.

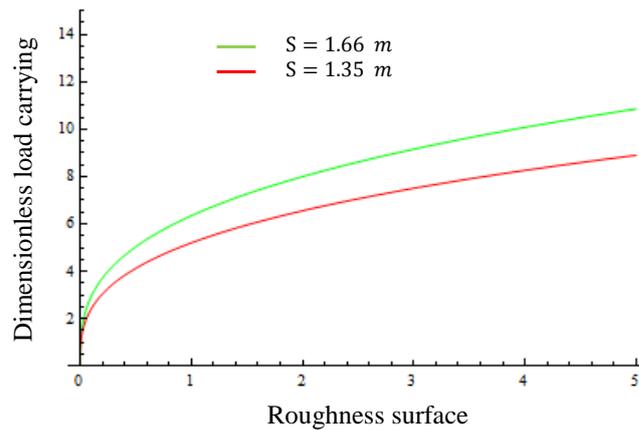


Fig. (1.8) Variation of dimensionless. load carrying capacity with surface roughness parameter for different values of stride length.

Table (1.4)

Relation between dimensionless load carrying capacity and type lubrication with different stride length parameter during daily active.

Type Lubrication	Film thickness	S = 1.66	S = 1.51	S = 1.41	S = 1.35
	h_0^*	w^*	w^*	w^*	w^*
Weeping	0.35	17.07	16	15.9	15
elasto – hydrodynamic	0.5	14.6	13.9	13.3	12.6
elasto- hydrodynamic	1	10.10	9.4	9	8.9
Squeeze	2	7.14	6.9	6.6	6.3
Squeeze	3	5.83	5.6	5.4	5.14
Hydrodynamic	6	4.12	4.02	3.8	3.6

3.3 Friction force

In Fig. (1.9) (a) and (b) we note that the micro-roughness parameter and macro-roughness parameter increases dimensionless. Friction force for all dimensionless. Stride length for male and female, where in stance phase it increased from 35% to 85 % at (respectively and other values are also higher friction force compared with swing phase

where increase is very little. Fig.(1.10) shows the variation of dimensionless. Friction force with (micro-macro) roughness parameter for different values of dimensionless. Stride longing for (young - old) at one -cycle and a ten-cycle. It is observed that the effect of stride length of order is to increase the dimensionless. Friction force as compared to stride length of young. The reason with age the

amount of leakage of lubrication pass articular has been reduced since (osteoarthritis- arthritis) then level dimensionless pressure film reduced. Further, it is noticed that difference in film thickness during daily activities where one-cycle after a ten-cycle film It gradually starts to decline until it reaches which leads to increased friction force.

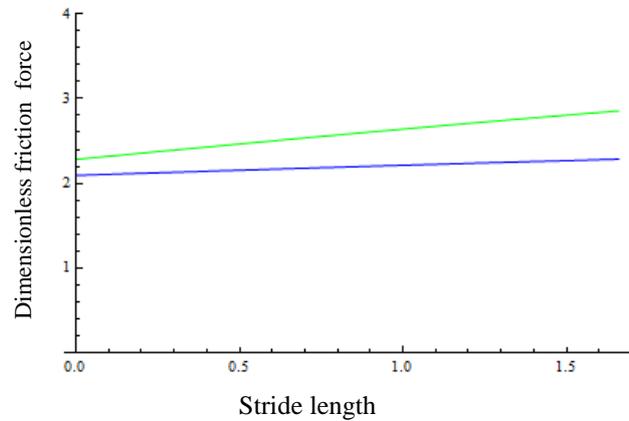
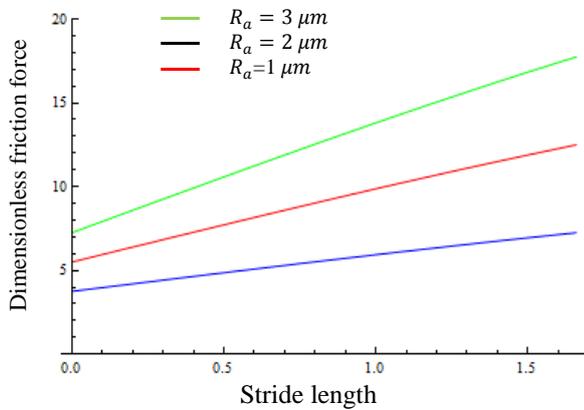


Fig.(1.9) Variation of dimensionless friction force with Stride length for different values (a) micro roughness parameter (b) macro roughness parameter.

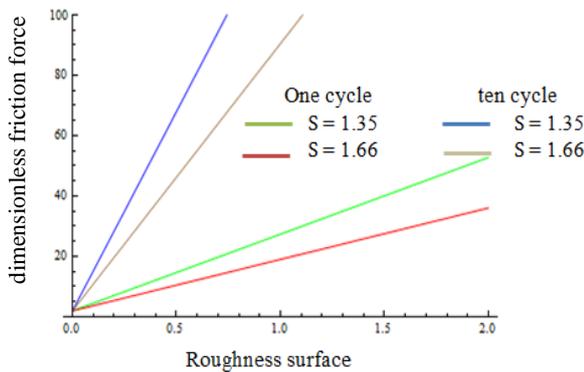


Fig.(1.10) Variation of with roughness parameter for different values of Stride length.

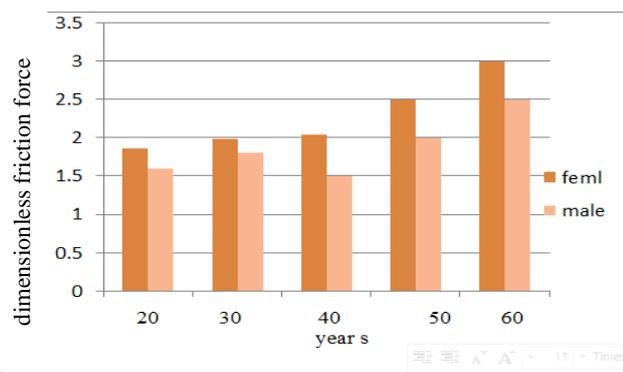


Fig.(1.11) Comparison of effective dimensionless friction force for male and female.

Fig.(1.11) shows the variation of dimensionless friction force with different sexes for age. The strength of friction increases with age and this increase varies between female and male for reasons of endurance and health status where men are more resilient and less prone to arthritis and all age groups.

4. Conclusion

In paper, using the investigated results, we concluded the following:

1. The use of squeeze film lubrication for knee joint during gait cycle led to higher level pressure film between articular cartilage that is clearly in swing phase compared with stance phase where low surface roughness.
2. roughness in (one-two-three) cycle very little a result, we find that level pressure film between articular cartilage is higher due to flow lubricant after ten cycle roughness increasing as well as loads on the joint this correlates inversely with pressure film.
3. In older person become stride length very small since diseases joint (osteoarthritis- arthritis) and increase in weight this led to movement obstruction, therefore bone of the knee joint become weaken and pressure film reduce compare as health joint.
4. since stress is less on the joint in swing phase where the roughness of articular is micro making load carrying capacity with different minimal film thickness in (one-two-three) cycle time is less as compared to stance phase that increase weight so load carrying capacity.

5. The big stride of young led to double the weight and increase in motion resulting from it increase in load carrying capacity this correlates inversely with stride length for elderly.
 6. The friction increases in the surface macro-roughness more than micro-roughness for all stride length in healthy and diseases.
 7. The higher value of friction is attributed to the higher value of surface roughness in stance phase.
 8. Stride length is different between male and female from youth to elderly during the gait cycle thus the friction force be more increase with respect to female from male.
 9. In the elderly the length of the stride less thus it increases friction and wear on the one- cycle to reach the highest level after ten cycles where less pressure and increase friction and roughness. When the young where the joint is normal and durability is high, resulting in a decrease in friction.
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