

CFD Study of Cuttings Transport through Vertical Wellbore

دراسة حركيات المائع الحسابية لقطع الصخور المنتقلة خلال بئر عمودي

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Abstract

Cuttings transportation from the bit up the annulus to the surface is one the essential functions that are performed by drilling fluid. Predication of drilling fluid efficiency to transport cuttings in the annulus is very complicated due to numerous parameters that have affected drilling operations. Computational Fluid Dynamics (CFD) is widely used as a numerical technique in handling complex multiphase flow problems in different operational conditions.

The present work has taken the advancement of CFD to computationally analyse the influence of the effects of various parameters like drilling fluid rheology, flow rate, pipe rotation, cuttings density, shape, concentration and drilling fluid- cuttings particle coupling regimes on the cuttings transport in a vertical wellbore. The CFD simulation was carried out by using transient solver of ANSYS-FLUENT CFD commercial code.

The dense discrete phase model (DDPM) is suggested in this work to overcome the main shortcomings of Eulerian–Eulerian and CFD-DEM approaches in simulating drilling fluid-cuttings flow. Also, some of the experimental investigations were involved in determining the fluid physical properties and essential input data to perform the CFD simulations. Regarding the results validation and verification, well agreement has been achieved between results obtained in this study with those reported in other studies.

Keywords Computational fluid dynamics (CFD), dense discrete phase model (DDPM), cuttings transportation, drilling fluid, Eulerian model, multiphase flow.

Introduction

The drilling fluid cycle is rising from the bottom of the wellbore to transport the cuttings to the surface. The circulating speed of the drilling fluid should rapidly enough to overcome cuttings tend to overwhelm through the rookie fluid due to the gravitational force. Other

factors are also affecting the cuttings removal involve drilling mud density, rheology, hole angle, angular velocity, drilled cuttings size and their shape [1].

A great effort has been devoted to analyse the main parameters effect on cutting transport behaviour through experimental studies [2-4]. The researchers have arrived at the conclusion that cuttings transport is controlled by a number of factors including the drilling fluid rheology, cutting characterizations (size, density, shape, and concentration), fluid velocity, and etc. Using of CFD technique can lead to better observe for cuttings transportation process inasmuch as measurement and visualization of downhole drilling fluid parameters during the drilling operation is not easily possible. Furthermore, the most existing theoretical relations are not having the ability of considering most parameters simultaneously.

CFD equips a qualitative analysis beside quantitative prediction of fluid flows as well as an insight into flow patterns that are hard, costly or difficult to consideration using experimental methods. Moreover, CFD does not displace the experimental technique entirely but the number of experimentation and the overall cost can be remarkably reduced.

The simulation of communication between the drilling fluid and the cuttings is particularly useful. As, the cuttings removal is advantageous since it decreases fluid loss and mechanical pipe sticking [5]. Nevertheless, if the cuttings cannot be removed from the wellbore, they will soon impede drilling. Therefore, it is important computationally analysed the influence of the effects of various parameters like drilling fluid rheology, flow rate, pipe rotation, cuttings density, shape, concentration and drilling fluid- cuttings particle coupling regimes on the cuttings transport in a vertical wellbore.

The cutting-drilling fluid flow can simulate as solid-liquid flow; the cutting as solid particles while the drilling fluid as non-Newtonian fluid. The literature on cuttings transportation shows a variety of modelling approaches. The choosing of the suitable model is a real serious concern. Drilling fluid-cuttings flow modelling is a challenging responsibility, but it is a handy tool to infer more about these flows. Recently, several authors have proposed a Eulerian-Eulerian approach to model drilling fluid-cuttings flow [6, 7]. This strategy usually requires much less computational resources corresponded to Eulerian-Lagrangian schemes. Thus, it can be employed to model pilot scale [8, 9]. However, the discrete nature of the solid phase is missed in the Eulerian-Eulerian strategy due to the continuous representation of the dispersed phase. This weakness can be

surmounted with discrete element method [10-12], in this approach, the solid particles are tracked separately based on Newton's laws of motion besides particle–particle and particle-wall collisions. Not only Eulerian approach has limitations, but also Lagrangian approach has, one of the main shortcomings of DEM technique is the cost computational demands that reduce its applications to small-scale [13].

Discrete phase model was used in the simulation of cuttings to eliminate the previous restrictions [1], which is two ways coupling between the cutting and fluid. As a result of using DPM, the volume fraction of the discrete phase should not be exceeded 10%. Otherwise weak accurate predictions will be obtained. To avert all previous approaches limitation, the dense discrete phase model (DDPM) has been established in which the details of particle-particle and particle-wall collisions are not overtly tracked anymore; as an alternative, a force is employed to represent these collisions [14]. Moreover, the hypothesis of the parcel is utilised to lessen the amounts of particles included in the computations, resulting in an essential acceleration of the speed of simulations. Therefore, the DDPM looks promising considering the advantages of Lagrangian methods and applies to large scales.

The DDPM model is suggested in this work to overcome the main shortcomings of Eulerian–Eulerian and CFD-DEM approaches. DDPM has the powers of smooth implementation of realistic particle size distribution and tracking the discrete character of particles. It's also less computational cost than Eulerian-Eulerian approach [15] since coarse grid can be employed to perform grid-size-independent simulations and the application of the idea of the parcel.

Although several studies have been done in recent years, no attention has been paid to the effect of cuttings concentration and coupling way. Nonetheless, it is possible to account the impact of these parameters with propose approach. With this goal, this work seeks to investigate the impact of cuttings concentration and coupling way as well as the drilling fluid rheology, cutting characterisations (size, density, shape, and concentration), fluid velocity.

Experimental work

In general, the preliminary investigations are complementary to the numerical simulations. The accuracy of statistical predictions is directly affected by, how far, the simulation

parameters and boundary conditions are realistic and accurate. Therefore, in the experimental work, the essential inputs for simulation have been determined. The methodology of the experimental work includes the following procedures:

- 1- **Drilling Fluid Formulation:** A stable formula of water-based non-Newtonian fluid has been formulated using the conventional components. The components those have been used to prepare the samples, and their functions are listed in Table (1). The preparation of the drilling fluid was performed by adding bentonite to the total amount of water and stirred using Hamilton Beach Mixer until no bentonite particulates are present in a stirrer container. After that, barite was added to the under-mixing bentonite slurry. Lastly, the other ingredients were added to the mixture and mixed until all solid particulates disappear.

Table (1) Water-based drilling fluid components and its functions

Component	Function	Quantity
Water	Base fluid	335 ml
Barite	Weight (density) addition	10 gm
Bentonite	Thixotropic property	20 gm
Caustic soda	pH regulation	0.2 gm
Soda Ash (Na_2CO_3)	pH regulation	0.2 gm

- 2- **Estimation of input parameters for simulation:** The drilling fluid is a non-Newtonian, the viscosity changes per shear rate method of the non-Newtonian power-law model (PL). The constants for PL model and the fluid physical properties have been determined from the experimental investigations.

Computational Simulation Methodology:

The present work is using fluent (Ansys 15) to simulate the drilling fluid process. The fluent consists of Five consecutive steps; geometry, mesh, setup, solution and results. Firstly, design modeller is used to constructing model geometry. The geometry of the simulation consists of two overlapped cylinders: the internal cylinder is drilling pipe (100 cm length and 20 cm diameter), and external cylinder wellbore (120 cm length and 50 cm diameter), as illustrated in Figure (1). The second step generates a mesh to the observed

model. The grids of the base case flow geometry have been produced using *ICEM CFD 15* pre-processor. Three different sizes of meshes have been used to eliminate the dependency on the mesh size (see Table (2)). Figure (2) displays a good agreement between the grids in term of velocity magnitude, turbulent kinetic energy (k), wall shear stress, and turbulence intensity. The refined medium grid has been selected for the further simulation to make a balance between the computational time and the accuracy of the prediction. The three meshes size and characteristics are presented in Table (2).

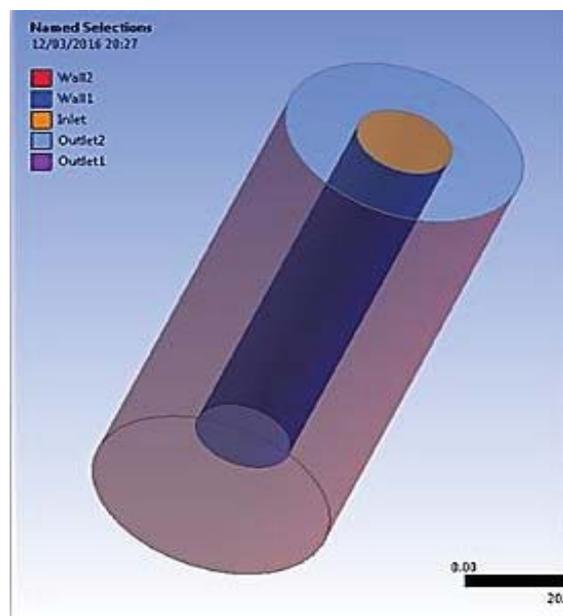


Fig. (1) Base-case flow geometry

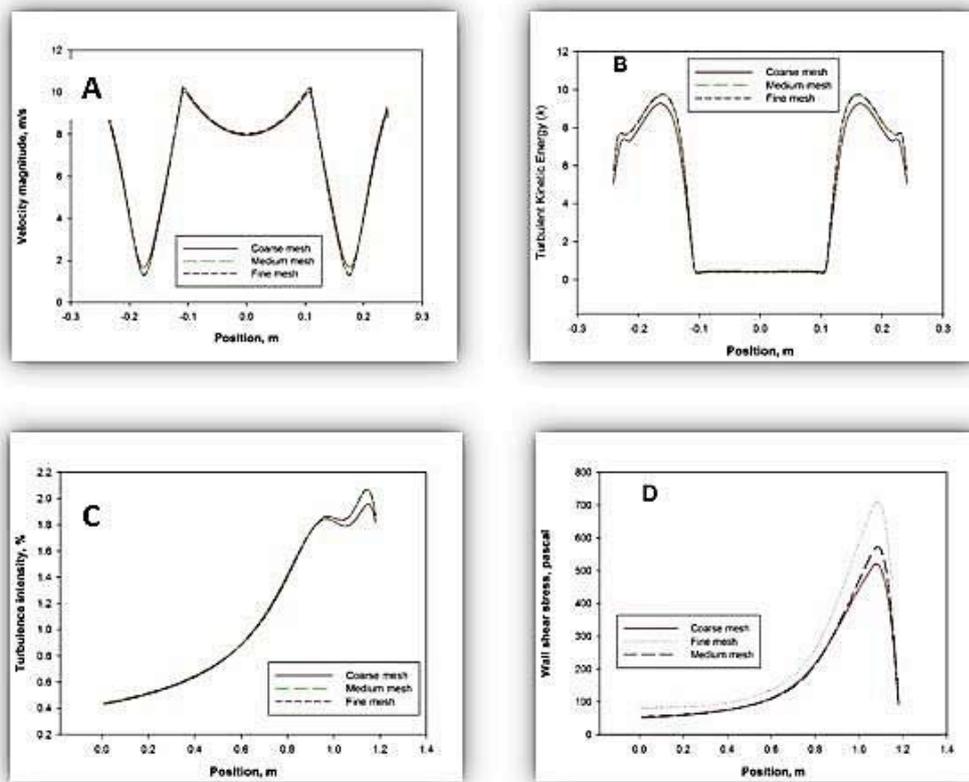


Fig. (2) (A) Velocity magnitude (m/s), (B) Turbulent kinetic energy (k), (C) Turbulent dissipation rate (Epsilon), and (D) Wall shear stress. A, B, and C are based on distance along the external pipe diameter (y=-0.24, 0.24). D is based on the distance.

Table (2) Meshes size and characteristics

Mesh	Cells	Faces	Nodes	Cell zone	Face zones	Max Aspect Ratio	Min. Orthogonal Quality
Coarse	77520	237460	82477	1	7	34.8	0.7
Medium	127182	388003	133712	1	7	23	0.715
Fine	172250	525239	180820	1	7	20.8	0.716

Thirdly, setup is employed to select the appropriate model, materials, phases, cell zone conditions and other specifications The Eulerian multiphase flow model is linked to dense

discrete phase model (Lagrangian Frame) to simulate cuttings transport with drilling fluid as the continuous phase. The volume fraction equation and conservation of momentum, mass, and energy equations included in Eulerian-Lagrangian model. The equations of volume fraction for each phase are individually solved using implicit time discretisation.

$$V_q = \int_V \alpha_q dV \quad (1)$$

$$\text{Where:} \quad \sum_{q=1}^n \alpha_q = 1 \quad (2)$$

The general form of mass conservation or continuity equation for multiphase incompressible flows can be written as follows:

$$\frac{\partial}{\partial t} (\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q) = \sum_{p=1}^n (\dot{m}_{pq} - \dot{m}_{qp}) + S_q \quad (3)$$

The conservation of momentum or (Newton's Second Law) can be written as follows:

$$\begin{aligned} \frac{\partial}{\partial t} (\alpha_q \rho_q \vec{v}_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q \vec{v}_q) &= -\alpha_q \nabla p + \nabla \cdot \overline{\tau}_q + \alpha_q \rho_q \vec{g} \quad (4) \\ &+ \sum_{p=1}^n (\overrightarrow{R}_{pq} + \overrightarrow{v}_{qp} \dot{m}_{pq} - \overrightarrow{v}_{qp} \dot{m}_{qp}) \\ &+ (\overrightarrow{F}_q + \overrightarrow{F}_{lift,q} + \overrightarrow{F}_{wl,q} + \overrightarrow{F}_{vm,q} + \overrightarrow{F}_{td,q}) \end{aligned}$$

In Lagrangian approach, *Ansys Fluent* predict the trajectory of particles by integrating the force balance on the particles. The force balance equates the particle inertia with forces applying on particles like drag force, gravity force, etc. (FLUENT, 2013). Particle force balance can be written as follows:

$$\frac{d\vec{u}_p}{dt} = F_D (\vec{u} - \vec{u}_p) + \frac{(\rho_p - \rho) \vec{g}}{\rho_p} + \vec{F} \quad (5)$$

Where F is an additional acceleration (force/unit particle mass), $F_D (\vec{u} - \vec{u}_p)$ is the drag

force per unit particle mass. The mud physical boundary conditions are listed in the table 4. The cuttings particulates have been injected from the bottom of the wellbore with size distribution using the Rosin-Rammler method. The distribution of particles diameter has been displayed in figure (3). The simulations involve two schemes of coupling of continuous-dispersed phases: two-way coupling, and four-way coupling. The parameters for particles (cuttings) modelling and size distribution that used in the simulation have been listed in Table 3. The cuttings material has been assumed to be Sandstone rock.

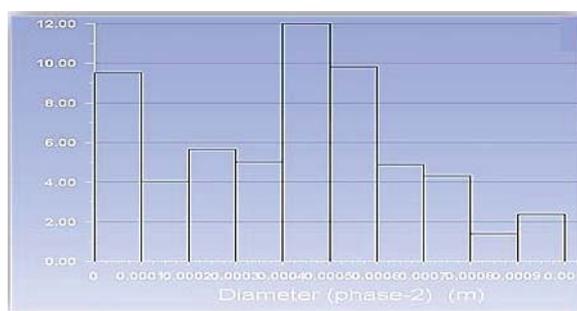


Fig. (3) Diameter distribution by Rosin-Rammler method

Table (3) Particle (cutting) parameters

Parameter	Values
Max. diameter, m	0.001
Min. diameter, m	10^{-6}
Mean diameter, m	0.0004
Spread parameter	3
Particle density, kg/m^3	2300
Mass flow rate, kg/s	0.4
Particle volume, m^3	$5.24 \times 10^{-10} - 5.23 \times 10^{-19}$
Particle weight, gm	1.95 – 0.0019
Temperature, K	300
X-velocity, m/s	0
Y-velocity, m/s	0.008

Table (4) Boundary conditions.

Zone	Parameters	Value
Inlet	Type	Velocity inlet
	Velocity magnitude, m/s	10
	Hydraulic diameter, m	0.25
	Temperature, K	298
	Turbulent intensity, %	5
	DPM	Escape
Outlet	Type	Pressure Outlet
	Outlet backflow turbulent intensity, %	5
	Backflow hydraulic diameter	0.25
	DPM	Escape
Wellbore Wall	Type	Wall
	Wall motion	Stationary
	Wall roughness constant	0.5
	Roughness height, m	0.0003
	Material of construction	Basalt Rock
	Thermal condition	UDF
	Thickness of wall, m	0.05
	DPM	Reflect
Wellbore Bottom	Type	Wall
	Wall motion	Stationary
	Wall roughness constant	0.5
	Roughness height, m	0.0003
	Material of construction	Basalt Rock
	Temperature, K	370
	Thickness of wall, m	0.05
	DPM	Reflect
Wellbore Bottom	Type	Wall
	Wall motion	Rotational motion
	Wall roughness constant	0.5
	Roughness height, m	0
	Material of construction	Carbon steel
	Thermal condition	Coupled
	Thickness of wall, m	0.01
	DPM	Reflect

Discussion of results and diagrams:

Power-Law Index (n), Consistency Index (k) Yield Stress Estimation:

A flow test for the non-Newtonian fluids can be defined by estimating the viscosity as a function of deformation rate (shear rate). This test may be regarded as a fundamental test for the drilling muds to evaluate a viscosity profile under a deformation rate. Various deformation rates are produced from a drilling pipe to drill different layers, which are mostly equivalent to 3, 6, 100, 200, 300, and 600 rpm. The mud has been tested to determine Power law index (n) and Consistency Index (K), as shown in Figure 4. These constants have been calculated by approximating the viscosity curve to PL model equation using curve fitting method. The constants have been listed in Table (5).

Table (5) Base mud physical properties

Properties and Constants	Values
Flow index (n)	0.631
Consistency index (k)	4

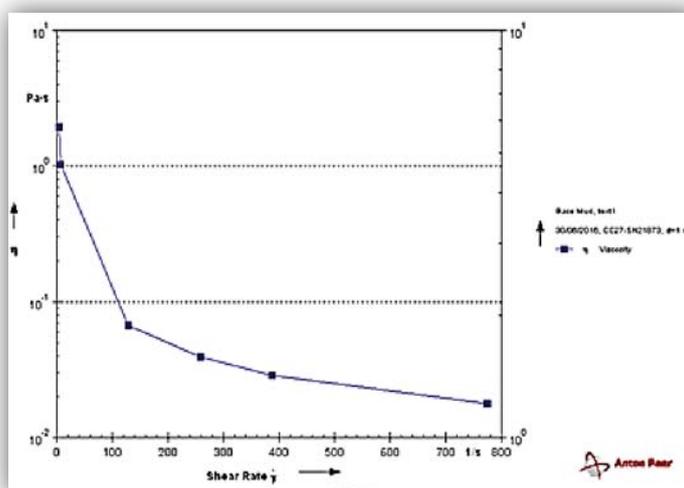


Fig. (4) Flow curve for the base mud.

Drilling fluid shows both viscous and elastic behaviour depending on the deformation. Viscoelasticity property of a drilling fluid can be tested through oscillatory amplitude test that includes calculating the storage modulus (G') and loss modulus (G'') as a function of shear strain. As mentioned earlier, G' and G'' curves contain several parameters related to

the fluid viscoelasticity, such as yield stress point. Figure (5) illustrates the storage modulus (G') and loss modulus (G'') profiles for the mud, and the red circle refers to the yield point, which is 5.75 pascal.

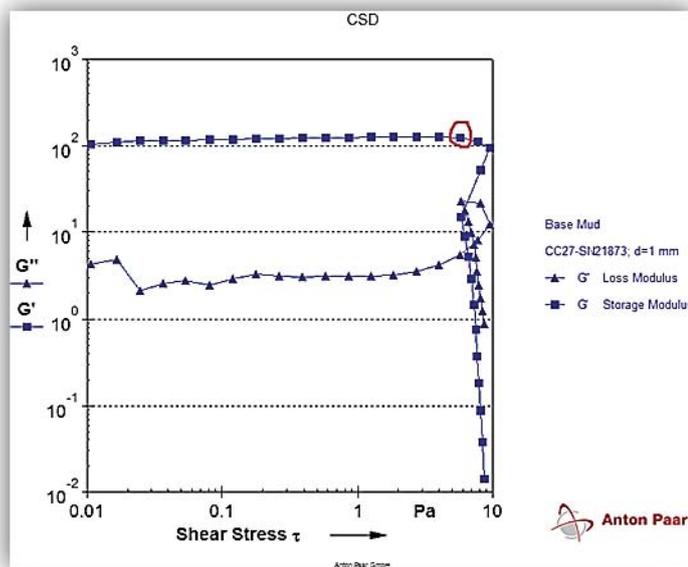


Fig. (5) Viscoelasticity curve for base mud.

Effects of Cuttings Concentration on the Cuttings Transport

The in-situ concentration of cuttings particulates in the wellbore bottom can influence the carrying capacity of the fluid. Three in-situ concentrations have been employed to examine the cuttings transport. Each concentration was simulated based on 600 rpm rotational speed of the drilling pipe. Figure (6) displays the volume fraction of leaving spherical cuttings for the three concentrations at 600 rpm rotational speed. When the in-situ concentration of cuttings is high, the cuttings carried by the fluid raises.

Effects of Cuttings Shape on the Cuttings Transport

As the drilling bit encroaches through the earth layers and crushing rocks starts, non-uniform cuttings shapes are produced. The proportion of particular shape relies on the formation type, and penetration rate and method. Three shape factors have been employed, namely 1, 0.9, and 0.8, with total cuttings concentration about 38 wt. %. The analysis result reveals the transportation of spherical cuttings is more than the transportation of the non-spherical cuttings, as shown in figure (7). As the cutting particle shape approaching spherical shape, the cuttings transport likely to be improved.

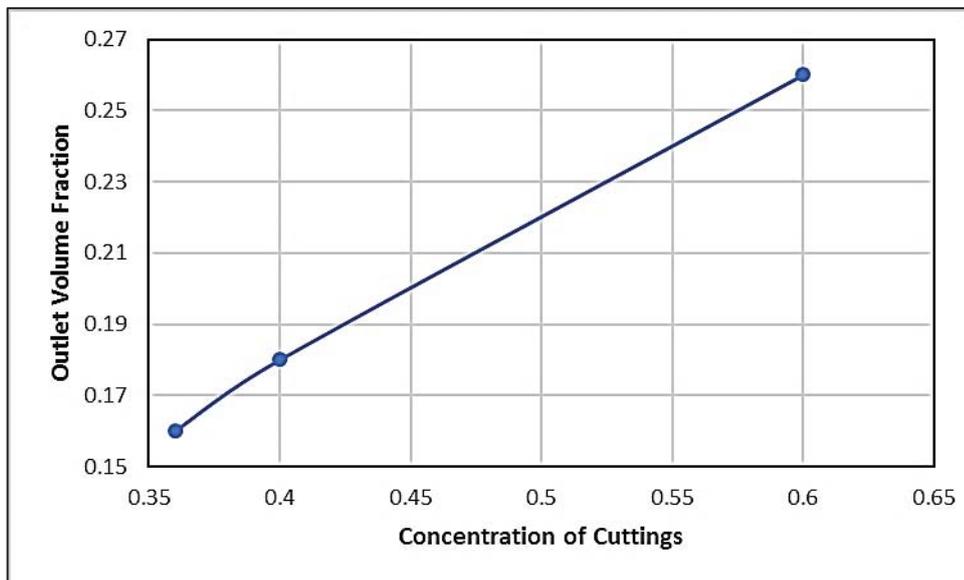


Fig. (6) Effects of cuttings concentration on the cuttings transport at 600 rpm

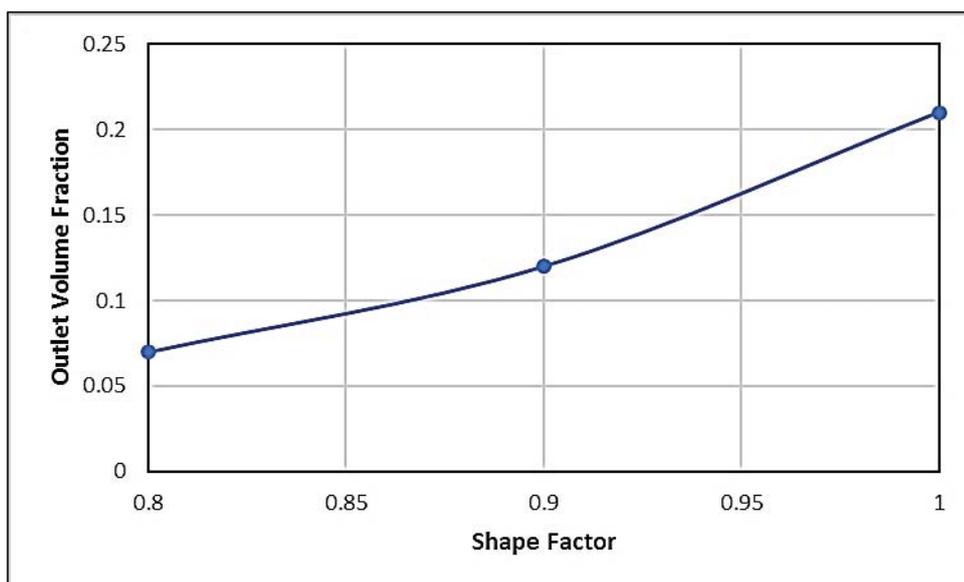


Fig. (7) Effects of cuttings shape on the cuttings transport at 600rpm

This trend can be explained according to Haider and Levenspiel correlation for calculating drag coefficient for non-spherical particles [16]. This correlation points out that increasing the shape factor of particle reduces the drag coefficient and drag force. Hence, the resistance to particles transport in the wellbore more likely to be lower. In terms of the

results validation, Akhshik et al. (2015) investigated the effects of particle shape on the hole cleaning process using CFD-DEM model. They found that the in-situ concentration of particles in the wellbore decreases with fluid flow rate. Also, the transportation of the spherical particle is better than the non-spherical ones for particular fluid velocity. Well agreement, regarding the concept, is obtained between the present results and Akhshik et al. results [17].

Effects of Cuttings Density on the Cuttings Transport

The effects of particle density on the cuttings distribution and fluid carrying capacity have been studied. Three more particle true densities: 1500, 3000, and 4000 kg/m³, besides the case base particle density (2300 kg/m³) have been employed. The volume fraction of cuttings in the effluent fluid outside the annular is inversely changed with a density of cuttings, as illustrated in Figure (8).

The higher density, the heavier cutting particles are released. Hence the fluid carrying capacity becomes weaker. Pronounced drag force dominates even the micro-sized particles of density above 3000 kg/m³. Furthermore, the cuttings distribution directly dominated by a density of cuttings. Where, the heavier the cuttings, the higher in-situ quantitative concentration can be obtained. In contrast, low-density particles can be carried by the fluid easier than others. Figure 8 reveals the distribution of cuttings volume fraction in the annular space with different particle densities under static condition.

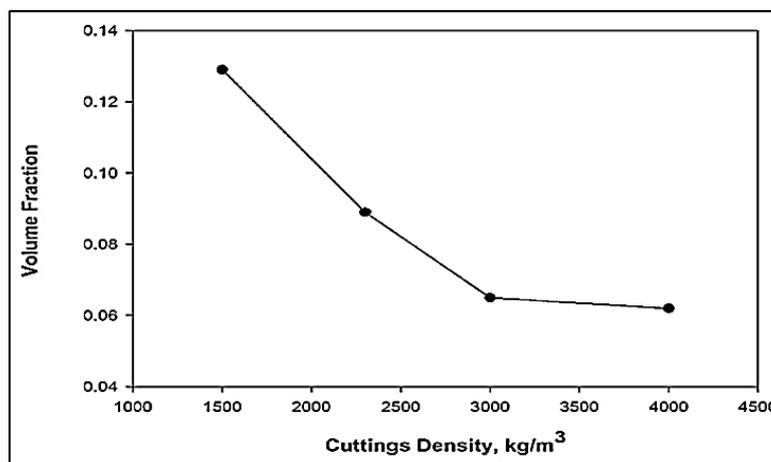


Fig. (8) The effects of cuttings right density on the volume fraction in the released drilling fluid from the wellbore.

Effect of Inlet Drilling Fluid Velocity on the Cuttings Transport

To some extent, mud flow rate in the drilling annular plays a vital role in improving a cuttings removal from a wellbore. Figure (9) displays the influence of mud inlet flow rate on various concentrations of cuttings at 100 rpm. As anticipated, the flow rate of drilling fluid improves cuttings transportation and the outlet volume fraction. As fluid flow turbulence increases, uniform drag distribution is formed to lift the cuttings to the surface. These results agree with Hussaini and Azar results; they suggested that mud annular velocity decreases in-situ cuttings concentration [18]. However, increasing the mud flow rate needs to be restricted to control the pressure drop, pumping cost, and wellbore stability.

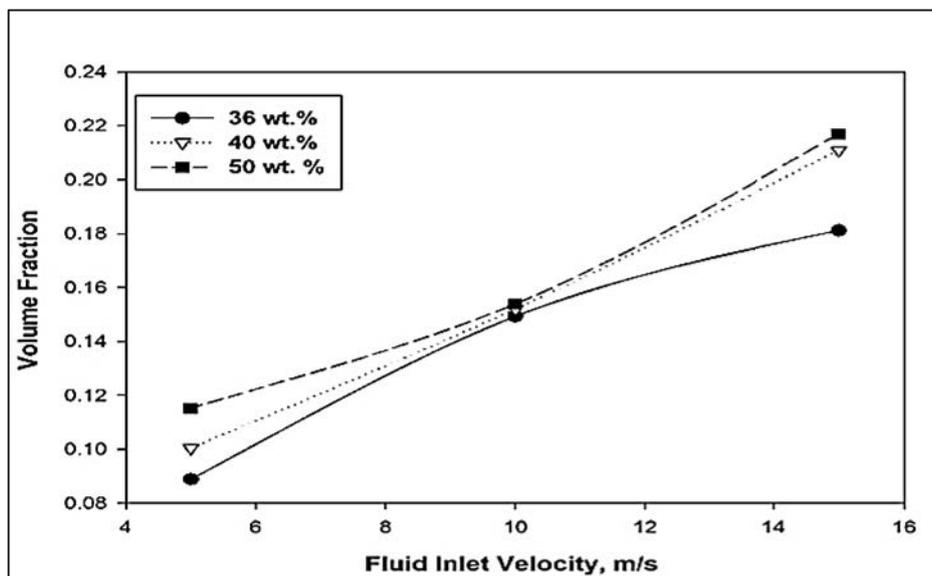


Fig. (9) Effects of cuttings concentration and mud flow rate on the cuttings transport (100 rpm)

As the cutting particle shape approaching spherical shape, the cuttings transport likely to be improved. Similarly, the cuttings outlet volume fraction increases with inlet fluid velocity, as illustrated in figure (10) Also, the figure shows that the particles with higher sphericity can be smoothly transported during the hole cleaning process.

Effect of Drilling Stem Speed on the Cuttings Transport

Cuttings distribution within a wellbore space for different rotational speeds is illustrated in figure (11). The cuttings transport with 36% in-situ concentration has been enhanced as the rotational speed increases.

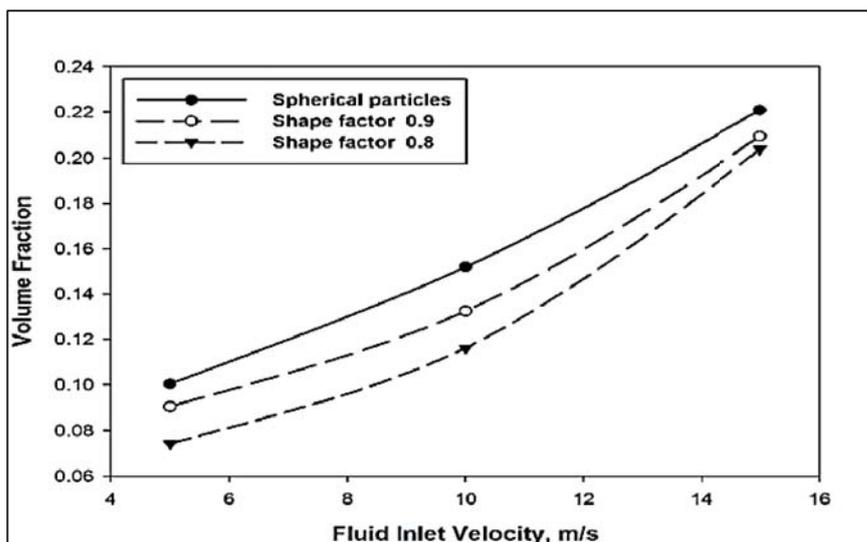


Fig. (10) Effects of cuttings shape and inlet velocity on the cuttings transport (100rpm)

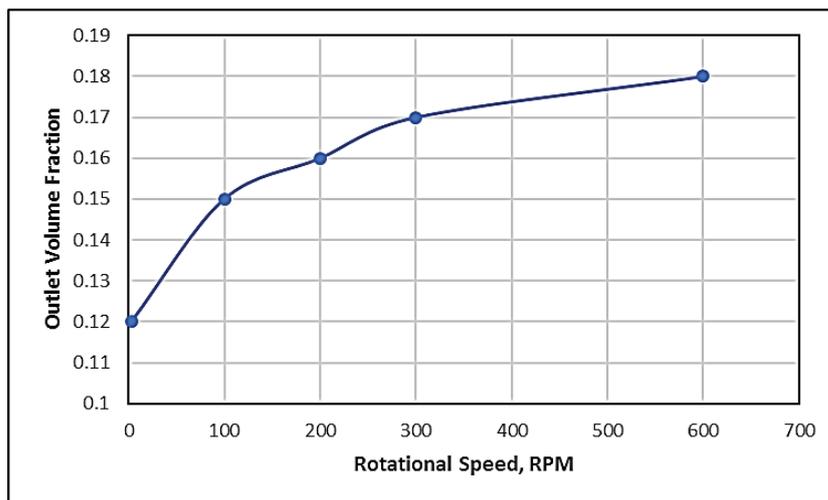


Fig. (11) Effects of drilling stem speed on the cuttings transport for 36% cuttings concentration

The analysis result reveals that as the drilling stem speed increases, spherical particles those leaving from the wellbore increases. While the drilling stem speed lowers the

transportation of the non-spherical cuttings, as shown in figure (12). As the cutting particle shape approaching spherical shape, the cuttings transport likely to be improved.

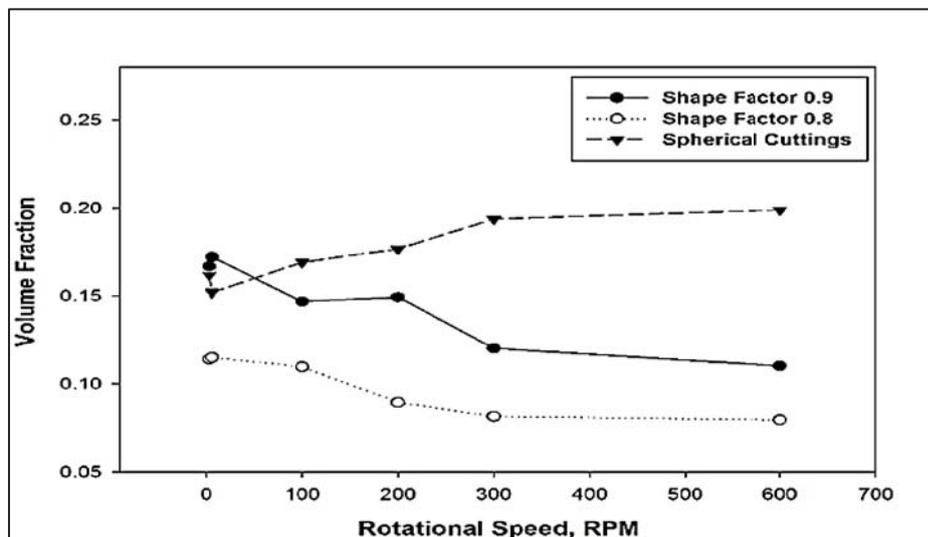


Fig. (12) Effects of rotational speed on the cuttings transport (inlet velocity= 10 m/s)

Comparison of Two-Way (DDPM) and Four-Way Coupling

Figure (13) illustrates the comparison of two-way (DDPM) and four-way coupling (DDPM+DEM) regimes, which define the relationship between continuous and discrete phase. Employing four-way coupling regime fulfils a realistic and robust interaction between the two phases. On the other hand, particle movement will be restricted due to collision with an adjacent particle or solid surface. Where, this collision can lead to a loss of particle kinetic energy, and hence reducing cuttings transport. Figure (13) content supports this argument, where pronounced variation between volume fraction from two-way and four-way coupling methods. As the cutting particle in two-way coupling (DDPM) does not affect contact dynamics, the particle accessible to transports to a surface. Therefore, the quantity of drilling cuttings leaving the wellbore in DDPM case is greater than that in DDPM+DEM case. Longitudinal sectional contours of cuttings volume fraction depict a frivolous difference in cuttings distribution between DDPM and DDPM+DEM cases, as shown in figure (14).

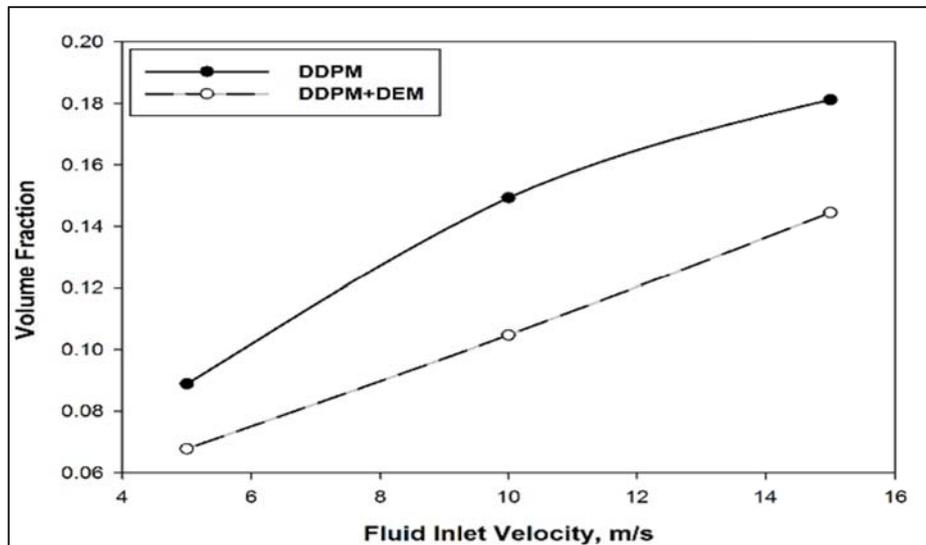


Fig. (13) Comparison of Two-Way and Four-Way coupling regimes.

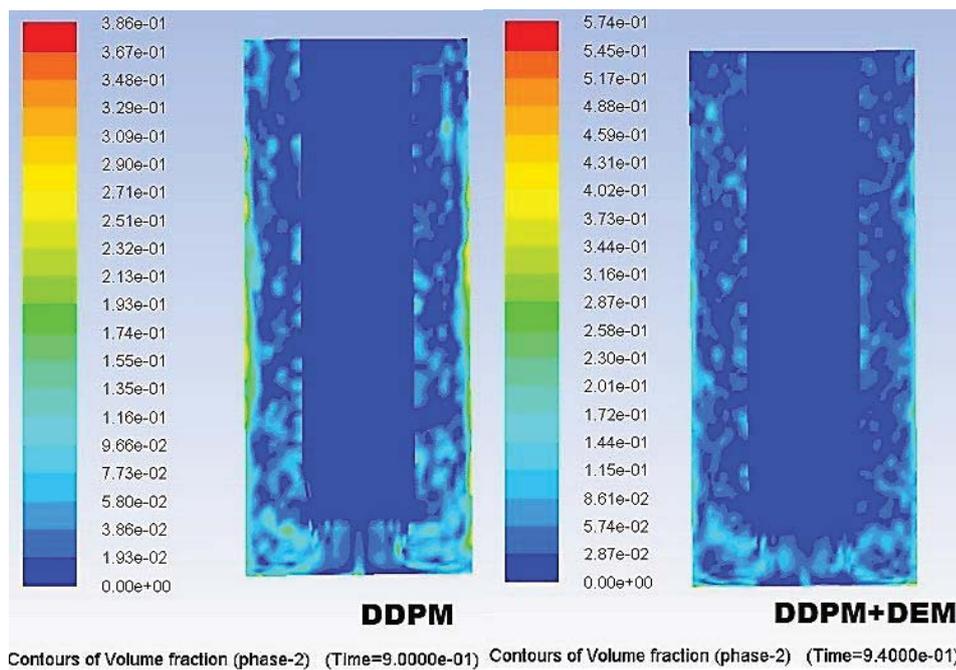


Fig. (14) Contours of cuttings volume fraction using Two-Way and Four-Way coupling regimes.

Conclusions:

Cuttings can be affected by a variety of variables such as fluid flow rate, drilling pipe rotation, cuttings shape, size, concentration, and density. The fluid flow rate and drilling pipe speed enhance cuttings transport efficiency for particular cuttings concentration, density and the spherical ones. However, as the shape factor of cutting particle falls below 1, the amount of transporting cuttings decreases with the pipe rotation. Cutting density does have an inverse influence on the cuttings transport. The coupling regime of continuous and discrete phases significantly affects the cuttings transport and fluid carrying capacity. The outlet volume fraction of cuttings using two-way coupling regime is greater than that of four-way coupling regime. The four-way coupling is much closer to reality as a result of considering the contact dynamics into the account.

Appendix

Greek symbols

α_q	Phase volume fraction
ρ_q	Phase density
τ	Stress Tensor
μ_t	Turbulent viscosity
ε	Turbulent dissipation rate
η	Dynamic viscosity
γ	Shear rate
τ_0	Yield stress threshold
γ_c	Critical shear rate.

Nomenclature

V_q	The volume of phase
q & q	Phases
v	Velocity
p	Pressure
g	Acceleration gravity constant
F_D	Drag force
K	Consistency Index or thermal conductivity
n	Power law index

Abbreviations

CFD	Computational Fluid Dynamics
DDPM	Dense Discrete Phase Model
DEM	Discrete Element Method
PL	Power Law
RANS	Reynolds-averaged Navies–Stokes
UDF	User-Define Function
WBM	Water Based Mud

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