

MAGIC SQUARE CONSTRUCTION WITH GENETIC ALGORITHM⁺

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Abstract

Genetic algorithm has wide applications in applied mathematics and computer problems. For instance, it is used in fractal image compression instead of iterated function systems algorithm , solving knapsack cryptographic problem , power dispatch and high-resolution spectra .In this paper, construction of magic squares using genetic algorithm , generating population using random number function , and selected method using reordered numbers are introduced .

المستخلص

الخوارزمية الجينية لها تطبيقات واسعة في الرياضيات التطبيقية و مسائل الحاسوب و من الأمثلة على ذلك أنها تستخدم في ضغط الصورة الكسوري بدلا من خوارزمية الأنظمة الدالية المتعاقبة و في حلول مسائل النابساك التشفيرية و مسائل القدرة و الطيف. في هذا البحث تم تكوين المربعات السحرية باستخدام الخوارزمية الجينية ، و توليد عينة باستخدام دالة الأرقام العشوائية ، و طريقة انتخابية باستخدام إعادة ترتيب الأرقام .

Introduction

One of applications of Galio fiel is the construction of magic squares. This is considered one of usage of algebraic complicated finite Galio field [1 , 2] . Mac Mahon introduced the method of Partition Analysis to construct magic squares [3] .

In this paper, genetic algorithm is suggested for the construction of magic squares. Two independent results are achieved ,the first is concerned with generating the samples of population using the random number function ,and the second studies adding new method in selection approach.

Magic square

A magic square of order n consists of the integers 1 to n^2 arranged in $n \times n$ square array so that the row sums ,column sums, and corner diagonal sums are all the same ,and equal to $\frac{1}{2} n (n^2+1)$ [1 ,2] .

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Genetic Algorithms

Applying a genetic algorithm involves the following steps [4] :

- Choosing space of " potential answers " for one's problem
- Determining an appropriate measure of "fitness " on this space
- Defining appropriate genetic operators on this space for instance crossover and mutation operator.

Generating Samples of Population

The following a procedure is presented for generating a sample of m .population , each vector (chromosome) with length equals n :

Procedure (A)

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FOR j=1 to m
FOR i=1 to n
K= int (z * rand ) +1;
C ( i ) = k ;
ENDFOR i
P ( j ) = c(1) c(2) ...c(n) ;
ENDFOR j

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where z is the number of symbols and $p (j)$ is the sample of j order. And denotes the multiplied operation .

Ordering Selection Method

To select a pair of vectors from current population there are several selection methods .The most important ones are roulette wheel ,sigma scaling , Boltzman , elitism , rank , tournament ,and steady state [5,6,7,8].

One can propose the following method for selecting two chromosomes of population :

- Calculating the sum of each chromosome.
- Reordering the chromosome with respect to its sums.
- Selecting the two first chromosomes.

To clarify the above method, one can take the following eight 0-1 chromosomes:

No.	Chromosome								sum
1	1	1	0	0	1	1	0	1	5
2	1	0	1	0	1	0	1	0	4
3	0	0	0	0	1	1	1	1	4
4	0	0	1	0	0	1	0	0	2
5	1	1	0	1	1	0	1	1	6
6	1	0	1	0	0	1	0	0	3

So we select the fifth and first chromosomes.

Construction of Magic Square Using Genetic Algorithm

In the following, we suggest the method of genetic algorithm for constructing magic square .In order to clarify proposed method one can study the problem of magic square of order three:

Step (1) Generating population using procedure A

For p₁:
 $r=0.1 \implies K = \text{Int}(9*0.1)+1 \implies K=1$
 $r=0.2 \implies K = \text{Int}(9*0.2)+1 \implies K=2$
 $r=0.9 \implies K = \text{Int}(9*0.9)+1 \implies K=9$

Then , P₁= 1 2 9

For p₂:
 $r=0.4 \implies K = \text{Int}(9*0.4)+1 \implies K=4$
 $r=0.8 \implies K = \text{Int}(9*0.8)+1 \implies K=8$
 $r=0.3 \implies K = \text{Int}(9*0.3)+1 \implies K=3$

Then , P₂= 4 8 3

For p₃:
 $r=0.5 \implies K = \text{Int}(9*0.5)+1 \implies K=5$
 $r=0.7 \implies K = \text{Int}(9*0.7)+1 \implies K=7$
 $r=0.6 \implies K = \text{Int}(9*0.6)+1 \implies K=6$

Then , P₃= 5 7 6

So the initial square is

1	2	9
4	8	3
5	7	6

When the sums of rows , columns , and diagonal corner are written, the initial square has the form

1	2	9	12
4	8	3	15
5	7	6	17
10	17	18	15

Step (2) Selection and crossover on the rows

<u>1</u>	<u>2</u>	9	12	\implies	4	2	9	15	\implies	4	2	9	15
<u>4</u>	<u>8</u>	3	15	\implies	1	8	<u>3</u>	12	\implies	1	8	6	15
5	7	6	18	\implies	5	7	<u>6</u>	18	\implies	5	7	3	15

Step (3) Selection and cross over on the columns

4	2	9	\implies	4	2	9	\implies	4	<u>2</u>	<u>9</u>	\implies	4	<u>9</u>	<u>2</u>
<u>1</u>	<u>8</u>	6	\implies	<u>8</u>	<u>1</u>	6	\implies	8	<u>1</u>	<u>6</u>	\implies	8	<u>1</u>	<u>6</u>
5	7	3	\implies	<u>5</u>	7	<u>3</u>	\implies	3	<u>7</u>	<u>5</u>	\implies	3	<u>5</u>	<u>7</u>
10	17	18	\implies	17	10	18	\implies	15	10	20	\implies	15	15	15

Step (4) permutation on row and columns

4	9	2	$\xrightarrow{r\ 2:r\ 3}$	4	9	2
8	1	6		3	5	7
3	5	7		8	1	6

Clearly the last square is magic .

The Results

One can write the new results in this paper as follows :

1. Adding a new method of samples to generate a sample of population by using the random algebraic function (rand) as shown in section (4).
2. Adding a new method of selection methods named (Ordering Selection Method) . This method used for choosing pair of vectors from current population as showed in section (5) .
3. Adding a new method of construction of magic square by using genetic algorithm as shown in section (6) . This method is added to the two famous methods of Galio and Mac Mohan .

Conclusion

In this paper the method of genetic algorithm is used for constructing magic square instead of the famous complicated methods of Galio field and Mac Mohan's partition . To make our method more compact, two methods are introduced .The first is for generation a population .This practical procedure is introduced and used in magic square population via the function of random number. The second method is named Ordering Selection Method depending on the sum of each chromosome in sample of population .

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