

How to use Temimi Transformation for solving L.O.D.E without using any initial Conditions.

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Abstract :

Our aim is to apply Temimi transformations to solve linear ordinary differential equations(L.O.D.E) with variable coefficients without using any initial conditions.

المستخلص :

هدفنا استخدام تحويلات التميمي في حل المعادلات التفاضلية الخطية الاعتيادية ذات المعاملات المتغيرة وغير الخاضعة لشروط ابتدائية.

Introduction :

The usual method of Temimi transformation (T.T) to find solutions of linear ordinary differential equations with variable coefficients that satisfy some initial conditions is summarized as taking Temimi transformation to both sides of differential equation and putting Temimi transformation of general solution which is considered as a fraction whose numerator and denominator are polynomials. Then we can put it into partial fractions whose number equals to the number of prime factors of denominator that contain constants whose number equals to the degree of the polynomial which is in the denominator, and the values of these constants depend on coefficients of numerator, that can be evaluated by comparison of both sides of last equation, and during constructing system of linear equations which can be evaluated by known algebraic methods, and after taking the inverse of Temimi transformation (T^{-1}) of Temimi transformation we obtain the solution of the differential equation.[1] We shall assume that the function that appears on the right side of the equations whose Temimi transformation can be determined. In this paper we also use the Temimi transformation to find the solution of linear ordinary differential equations but without using any initial conditions.

Basic definitions and concepts

In order to make the work is self-contained as possible, we will start by introducing some of the most important definitions and concepts that used later in the among of the search.

Definition 1:-[3]

let f is defined function at period (a,b) then the integral transformation for f whose it's symbol

$$F(s) \text{ is defined as } \int_a^b k(s,x) f(x) dx$$

where $k(s,x)$ is function of s and x , and the integral is exist.

Definition 2:-[1]

The AL-Temimi transformation for the function $f(x)(x > 1)$ is defined by the following integral:

$$T\{f(x)\} = \int_1^{\infty} x^{-s} f(x) dx = F(s),$$

Such that this integral is convergent, s is constant.

Property 1:

The Temimi transformation is characterized by the linear property, that is

$$T\{Af(x) + Bg(x)\} = AT\{f(x)\} + BT\{g(x)\}$$

where A,B are constants ,the functions $f(x), g(x)$ are defined when $x > 1$.

See [1].

Now, we'll give the table for AI- Temimi transformation for some functions

Table1:Transformation for some functions [1]

ID	FUNCTIONS, $f(x)$	$F(s) = \int_1^{\infty} x^{-s} f(x) dx = T\{f(x)\}$	Regional convergence of
1	$k ; k = \text{constant}$	$\frac{k}{s-1}$	$s > 1$
2	$x^n, n \in \mathfrak{R}$	$\frac{1}{s-(n+1)}$	$s > n+1$
3	$\ln x$	$\frac{1}{(s-1)^2}$	$s > 1$
4	$x^n \ln x, n \in \mathfrak{R}$	$\frac{1}{(s-(n+1))^2}$	$s > 1$
5	$\sin(a \ln x)$	$\frac{a}{(s-1)^2 + a^2}$	$s > 1$
6	$\cos(a \ln x)$	$\frac{s-1}{(s-1)^2 + a^2}$	$s > 1$
7	$\sinh(a \ln x)$	$\frac{a}{(s-1)^2 - a^2}$	$ s-1 > a$
8	$\cosh(a \ln x)$	$\frac{s-1}{(s-1)^2 - a^2}$	$ s-1 > a$

From the Temimi definition and the above table, we get

Theorem 1:

If $T\{f(x)\} = F(s)$ and a is constant, then $T\{x^{-a} f(x)\} = F(s+a)$. see [1]

Definition 3:

Let $f(x)$ be a function where $(x > 1)$ and $T\{f(x)\} = F(s)$, $f(x)$ is said to be an inverse for the Temimi transformation and written as: $T^{-1}\{F(s)\} = f(x)$, where T^{-1} returns the transformation to the original function.

Property 2:

If $T^{-1}\{F_1(s)\} = f_1(x), \dots, T^{-1}\{F_n(s)\} = f_n(x)$ and a_1, \dots, a_n are constants then,
 $T^{-1}\{a_1F_1(s) + a_2F_2(s) + \dots + a_nF_n(s)\} = a_1f_1(x) + a_2f_2(x) + \dots + a_nf_n(x)$.

Definition 4:[2]

The equation $a_0x^n \frac{d^n y}{dx^n} + a_1x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \dots + a_{n-1}x \frac{dy}{dx} + a_ny = f(x)$, where a_0, \dots, a_n are constants and $f(x)$ is a function of x , is called Euler's equation.

Theorem 3:

If the function $f(x)$ is defined for $x > 1$ and its derivatives $f^{(1)}(x), f^{(2)}(x), \dots, f^{(n)}(x)$ are exist then
 $T\{x^n f^{(n)}(x)\} = -f^{(n-1)}(1) - (s-n)f^{(n-2)}(1) - \dots - (s-n)(s(n-1)) \dots (s-2)f(1) + (s-n)!F(s)$ see[1].

How to use Temimi Transformation for solving (L.O.D.E) without using any initial conditions.

Suppose the (L.O.D.E) of order (n) with variable coefficients and its general form

$$a_0x^n \frac{d^n y}{dx^n} + a_1x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + \dots + a_{n-1}x \frac{dy}{dx} + a_ny = f(x) \dots(1)$$

Without using any initial conditions, i.e.

$y(1), y'(1), \dots, y^{(n-1)}(1)$ are unknown and the Temimi transformation of $f(x)$ is known. To solve equation(1), we take (T.T) of both sides, we get:

$$T\{y\} = \frac{K(s)}{(a_0s^n + a_1s^{n-1} + \dots + a_n)H(s)} \dots(2)$$

Whereas $H(s)$ is a polynomial of s represents denominator of (T.T) of the function $f(x)$ and $K(s)$ is also a polynomial of s with degree smaller than the degree of the product of $(a_0s^n + a_1s^{n-1} + \dots + a_n)$ and $H(s)$ and not necessary to know the terms of $K(s)$ we only denoted it by this symbol. Now by taking T^{-1} of both sides of equation(2), we get the following solution:

$$y = A_1g_1(x) + A_2g_2(x) + \dots + A_n g_n(x) + B_1h_1(x) + B_2h_2(x) + \dots + B_r h_r(x) \dots(3)$$

Whereas A_1, A_2, \dots, A_n and B_1, B_2, \dots, B_r are constants, the g_1, g_2, \dots, g_n and h_1, h_2, \dots, h_r are functions of x .

The number of the constants B_i and the number of the functions $h_i, i = 1, 2, \dots, r$ are equal to the degree of $H(s)$ which is supposed to be (r) .

Note that the order of equation(1) is (n), therefore its general solution contains (n) constants, but the solution in (3) contains (n+r) constants and to solve this problem we can eliminate some of these constants B_1, B_2, \dots, B_r whose values obtaining by substituting the solution (3) in equation(1), so we get a solution contains (n) constants (as unknown) as the require solution. By this method we get the general solution of equation(1) without using any initial conditions by using Temimi transformation.

Example1:

To solve the (D.E) $xy' + y = 16\sin(\ln x)$

By using (T.T) without using any initial conditions, we take (T.T) to both sides of it we get:

$$-y(1) + (s-1)T\{y\} + T\{y\} = \frac{16}{(s-1)^2 + 1}$$

$$T\{y\} = \frac{K(s)}{s((s-1)^2 + 1)}$$

Whereas $K(s)$ has a degree less than three, i.e. less than the degree of denominator and the quantity s is resulting from $xy' + y$ and $[(s-1)^2 + 1]$ represents the denominator (T.T) of the function $16\sin(\ln x)$. Now, we can write the solution y after taking $(T^{-1} \cdot T)$ to both sides as follows:

$$y = T^{-1}\left\{\frac{A}{s} + \frac{Bs + C}{(s-1)^2 + 1}\right\}$$

$$= \frac{A}{x} + B \cos \ln x + J \sin \ln x \dots(4)$$

Where $J = B + C$

The given equation is of order one, so the general solution must contain only one constant while equation(4) contains three constants, therefore, we should eliminate the constants B, J , for this we get y' from equation(4) as follows:

$$y' = \frac{-A}{x} - \frac{B}{x} \sin \ln x + \frac{J}{x} \cos \ln x \dots(5)$$

And after we substitute y, y' in (D.E) to find the values of B, J we get:

$$B = -8$$

$$J = 8$$

Therefore the general solution is:

$$y = \frac{A}{x} - 8 \cos \ln x + 8 \sin \ln x$$

This solution contains only one constant A equal to the order of (D.E).

Example2:

To solve the (D.E) $x^2 y'' + 4xy' + 2y = x \ln x$

By using (T.T) without using any initial conditions, we take (T.T) to both sides of it we can write:

$$T\{y\} = \frac{K(s)}{s(s+1)(s-2)^2}$$

Whereas $K(s)$ has a degree less than four, i.e. less than the degree of denominator and the quantity $s(s+1)$ is resulting from $x^2 y'' + 4xy' + 2y$ and $(s-2)^2$ represents the denominator (T.T) of the function $x \ln x$. Now, we can write the solution y after taking $(T^{-1} \cdot T)$ to both sides as follows:

$$y = T^{-1}\left\{\frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-2} + \frac{D}{(s-2)^2}\right\}$$

$$= \frac{A}{x} + \frac{B}{x^2} + Cx + Dx \ln x \dots(6)$$

The given equation is of order two, so the general solution must contain only two constants while equation(6) contains four constants, therefore, we should eliminate the constants C, D , for this we get y', y'' from equation(6) as follows:

$$y' = \frac{-A}{x^2} - 2\frac{B}{x^3} + C + D + D \ln x$$

$$y'' = 2\frac{A}{x} + 6\frac{B}{x^4} + \frac{D}{x}$$

And after we substitute y, y' and y'' in (D.E) to find the values of C, D we get:

$$C = \frac{-5}{36}$$

$$D = \frac{1}{6}$$

Therefore the general solution is:

$$y = \frac{A}{x} + \frac{B}{x^2} - \frac{5}{36}x + \frac{1}{6}x \ln x.$$

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