Regular Strongly Connected Sets in topology

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Abstract:

This work presents definitions of regular strongly connected sets and regular weakly disconnected sets as well as investigation for the relationship between these sets. Also a definition has been presented for the regular totally weakly disconnected topological space and to introduce a relation between this space and a regular T_1 as a separation axiom in topological space .

الخلاصة:

يقدم هذا البحث تعاريف لمجموعات الترابط المنتظم الشديد ومجموعات الانفصال المنتظم الضعيف بالإضافة إلى بحث العلاقة بين المجاميع. كذلك تم تقديم تعريف إلى الفضاء التبولوجي المنفصل الكلي المنتظم الضعيف وتقديم علاقة بين هذا الفضاء وبديهية الانفصال T_1
في الفضاء المنتظم.

1. Introduction:

A subset A of a topological space (X,T) said to be regular open (briefly R.O) iff $A=int(cl_{(A)})$ and A is called regular closed (briefly R.C) iff it is complement is R.O. The set of all regular open sets in X is denoted by R.O(X),[3], A subset A of a topological space X will be termed Strongly Connected (briefly S.C.) if $A \subset U$ or $A \subset V$ whenever $A \subset U \cup V$,V and U being open set in X[5] ,A subset A of X will be termed weakly disconnected (briefly W.d.) if it is not S.C[5].

A topological space X is totally weakly disconnected (T.W.d) if singleton sets are the only nonempty S.C sets [5] .In this work define a Regular strongly connected sets and a Regular weakly disconnected sets and investigate a relation ship between these sets ,also we shall define a Regular totally weakly disconnected topological space and to introduce a relation between this space and a Regular T_1 as a separation axiom in topological space .

2-Regular strongly connectivity .

Definition(1.2) [1,4]

A space *X* is *connected* if it cannot be decomposed as the union of two disjoint nonempty open sets.

Definition (2.2)

A subset A of a topological space (X,T) is said to be Regular strongly connected (briefly R.S.C.) iff $A \subset U$ or $A \subset V$ whenever $A \subset U \cup V$,U and V being Regular open sets in X.

Theorem (3.2):

Every R.S.C. is S.C. sets

Proof : Let A is R.S.C. set, then $A \subset U \cup V$, wherever $A \subset U$ or $A \subset V$ and U and V being Regular open sets in X ,Since every regular open set is open set so , U and V are open set in X and $A \subset U \cup V$ hence A is S.C. set.

Theorem (4.2):[2]

If A is S.C, then A is connected .

Proof : If A is disconnected , then there exist open sets U and V for which $A=(A\cap U)\cup(A\cap V)$, $A\cap U\neq\phi, A\cap V\neq\phi$ and $A\cap U\cap V=\phi$ then $A\subset U\cup V$, but $A\not\subset U$ and $A\not\subset V$ which contradicts A being S.C.

Theorem (5.2):

If A is R.S.C. , then A is connected .

Proof : Easy by using theorem (2.2),(3.2).

Remark(6.2):

The converse of theorem (4.2) is not true .As shown by following example .

Example (7.2):

Let X={1,2,3},T={φ, X,{2},{1,2},{2,3}} it is clear that X is connected but not regular strongly connected .

Definition (8.2):

A subset A of a topological space X is said to be regular weakly disconnected (briefly R.W.D.) iff it is not R.S.C.

Remark (9.2) :

If A is Regular disconnected then A is R.W.D.

Proof: Since A is regular disconnected ,then A is not regular connected and by theorem (4.2), A is not R.S.C. and hence A is R.W.D.

Theorem(10.2):

A subset A is R.W.D. iff there exist two nonempty disjoint sets M and N each regular closed in A.

Proof: Let A is R.W.d. ,then Ais not R.S.C., which is mean there is no regular open set U and V whenever $A\subset U$ or $A\subset V$ and $A\subset U\cup V$ so let $M=U^C$ and $N=V^C$ M and N are regular closed since $A\subset U\rightarrow A\not\subset U^C=M$ or $A\subset V\rightarrow A\not\subset V^C=N$ and $A\not\subset M\cup V$, since A is not R.S.C. this implies that M and N are two nonempty disjoint regular closed in A .

The other hand is easy from a definition of Regular closed sets in A.

Corollary(11.2):

A subset A is R.W.D. iff there is exist two points x and y in A such that $A\cap cl(x)\cap cl(y)=\phi$, where cl(x) and cl(y) be the closure of x and y respectively.

Theorem (12.2):

A topological space (X,T)is Regular strongly connected if the only non-empty subset of X which is both regular open and regular closed in X is X itself .

Theorem (13.2):

Suppose (X,T) is R.S.C. and let F be a regular closed subset of X then F is R.S.C.

Proof :

If F is not R.S.C. it is to say F is R.W.D. , then there exist regular open sets V and U for which $F=(F\cap U)\cup(F\cap V)$. $F\cap U\neq\phi$, $F\cap V\neq\phi$ and $F\cap U\cap V=\phi$ then $F\subset U\cup V$, but $F\not\subset U$ and $F\not\subset V$ which make F is Regular open , which contradicts with F as a regular closed .

Corollary(14.2):

A space (X,T) is R.S.C. iff every regular closed subset F is R.S.C.

Proof:

suppose (X,T) is R.S.C. then by theorem(13.2) Every regular closed subset is connected .

suppose that F is regular closed subset and F is R.S.C then $F\subset U$ or $F\subset V$ when ever $F\subset U\cup V$,U and V being regular open sets in X , but $F\subset U\cup V$ and F is regular closed , Hence the only non empty subset of X is X itself which is mean that (X,T) is R.S.C.

Definition(15.2):[2]

Let f be a mapping from topological space (X,T) in to topological space (Y,F) , then f is said to be continuous function if the inverse image of any open (closed) in Y is open (closed) in X .

Definition (16.2):

Let f be a mapping from topological space (X,T) in to topological space (Y,F) , then f is said to be regular continuous if the inverse image of any open (closed) in Y is regular open (regular closed) in X .

Theorem (17.2):

If f: (X,T)→(Y,F) is regular continuous and if A is R.S.C. in X ,then f(A)is R.S.C. in Y

Proof:

let f: X→Y be a regular continuous of a R.S.C. A in X f(A) is R.S.C .in Y

Assume that f(A)is not R.S.C. in Y →there exist U,V both regular open in Y s.t $U\cap f[A]\neq\phi,\ V\cap f[A]\neq\phi$ and $(U\cap f[A])\cap(Vf[A])=\phi$ and $(U\cap f[A])\cup(V\cap f[A])=f[A]$,

$\phi=f^{-1}(\phi)=f^{-1}\big((U\cap f[A])\cap(V\cap f[A])\big)=f^{-1}\big((U\cap V)\cap f[A]\big)=f^{-1}(U\cap V)\cap f^{-1}(f[A])$

$=f^{-1}(U)\cap f^{-1}(v)\cap A \quad =f^{-1}(U)\cap f^{-1}(V)$

$A=f^{-1}(f[A])=f^{-1}(U\cap f[A])\cup(V\cap f[A])=f^{-1}\big((U\cap V)\cap f[A]\big)$

$=f^{-1}(U)\cup f^{-1}(V)\cap f^{-1}(f[A])=f^{-1}(U)\cup f^{-1}(V)\cap A=f^{-1}(U)\cup f^{-1}(V)$

,Since f is R.S.C. and U,V are regular open in Y this impels $f^{-1}(U)$,$f^{-1}(V)$ are regular open in X so X is not R.S.C. which is contraction so f[A] must be R.S.C. in Y.

Definition (18.2):

A topological space X is totally weakly disconnected (R.T.W) if Singleton sets are the only non empty R.S.C. sets .

Theorem (19.2):

A topological space X is RT_1iff it is R.T.W.D. whereRT_1 be the regular T_1-space .

Proof: Necessity singleton sets are clearly R.S.C. .Now suppose A is a subset of X with two or more points , let $x\neq y$ in A , then x and y are nonempty disjoint regular closed subset of A and by theorem (17.2) A is R.W.S.

Sufficiency , Let $x \in X$ and suppose $[x] \neq cl[x]$, let $y \in cl[x]-[x]$ and let A=[x,y] ,then every regular open set which contains also contains x and thus A is R.S.C. which is contradiction .

Corollary(20.2):

Let a topological space (X,T) is R.T.W.D then X is RTO

Proof :

Easy by using the fact that every RT_1-Space is RT_2-Space and theorem(19.2) .

Theorem (21.2):

Every discrete space (X,D) defined by a regular open set is R.T.W.D.

Proof:

Let (X,D) be a regular discrete space and let $x \neq y$ in X , let A={x},B=X-{x}are both nonempty regular open disjoint sets whose union is X such that $x \in A$ and $y \in B$ →(X,D) is R.T.W.D.

3. Regular strong local connectivity

Definition (1.3)

A topological space (X,T) is said to be regular strongly locally connected at x (briefly as R.S.L.C.) if for $x \in O \in T$, there exists a R.S.C. open set G such that $x \in G \subset O$.

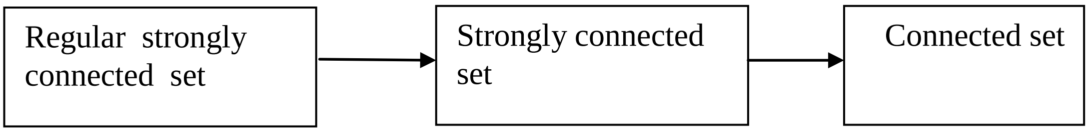
Remark (2.3) :

A topological space (X,T) is R.S.L.C. at every point of X.

Corollary (3.3):

If a topological spaces (X,T) is R.S.L.C., then X is regular locally connected .

Proof: let X is R.S.L.C. , then for $x \in O \in T$, there exists a R.S.C. ,open set G such that $x \in G \subset O$, and G is a regular neighborhood of x which is contains a connected Regular open neighborhood of x , which is mean that X is Regular locally connected .



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