# Study the Axail Dispersion Model in Ion Exchange Column

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## Abstract

A mathematical model is developed which predicates the performance of cylindrical ion exchange bed involving comparing of axial dispersion model for cation exchange column with different assumption, this model permits the performance to predicate the residence time within the bed with the variance, axial dispersion and Pecklet No. to indicated deviation from plug flow model.

Two type of systems are chosen for positive ions first with divalent ions ($Ca^{+2}$) to exchange with resin of $Na^{+1}$ form used as application in water softener units and second with monovalent ions ($Na^{+1}$) to exchange with resin of $H^{+1}$ form used as application in deionize water units with two parameters effected in the operation of the two systems different flow rate and different feed concentration by their effect on axial dispersion in each one where the system are compare from the performance which describe in term of breakthrough time other parameters (the time equivalent to usable capacity "$t_u$" , time equivalent to equilibrium capacity "$t_t$" and length of unused bed "$H_{UNB}$") are calculated in the operation condition that with axial dispersion are very important in predicated the best condition to operation the cation exchange column as a softener and deionize units.

*Keywords:* Ion exchange, axil dispersion, $Ca^{+2}$ ion, $Na^{+1}$ion, Pe No., breakthrough time, height of unused bed ($H_{UNB}$), residence time, variance

## 1. Introduction

Over the past decades the ion exchange processes has found increased applications in a range of diverse fields.

Most of these applications use cylindrical column ion exchange beds in which the electrolyte solution to be treated is injected in one end and the effluent is collected at the other such as water softner when treated a solution of divalent ion ($Ca^{+2}, Mg^{+2}$) with using cation exchange column with Na-form to exchange divalent , other application is the dionized water with using resin in H-form to treated all the divalent and monovalent positive ion where most of these application are conventional one-dimensional. Linear ion exchange column has been well investigated, and many models have been developed to predict ion exchange performance in vertical flow, however most models assume no axial dispersion model.

Important to investigated a model that account axial dispersion since the axial dispersion coefficient lumped together all mechanisms leading to axial mixing in packed bed that its account not only for molecular diffusion and conversion mixing but also for non uniformities in the fluid velocity across the packed bed .

Several workers studied dispersion with porous media, the first who deal with axial dispersion O. Levenspiel 1958 when he study the commercial application of step experiment and he design chart to ease the calculation of axial dispersion, H. Dunck 1966 study he effect of axial dispersion on the elution band of nonlinear equilibrium chromatography and the effect change in axial dispersion coffin with elution velocity , O .Levenspiel 1972 found the c-curve in closed vassal for various extend of back mixing as predicated the dispersion model , C.Y. Wen, T. Tfan l.t. 1975 point that the axial dispersion provides satisfactory description only of mixing model that not deviated significantly from plug flow, R. Shinnar 1987emphasized that the application of axial dispersion model for Pe No. value for less than 10 is not advised,

C. T. Tan1987 the value of dispersion coefficient are either assumed or calculated from molecular diffusivity and radial pore velocity prior to incorporation into the model the radial dispersion R. Y. A. Ofoli, 1991 reported that axial dispersion model is suitable for analysis of reactor up to 30% conversion , Y. Tsaur and D. Shallcross 1997 developed a model to simulate ion exchange performances with fixed annular bed which take into account the radial dispersion in effect in both solution and exchange phases, however the model assumes instantaneous equilibrium between ions in the solution and exchange phases existed throughout the exchanger, M. A. Barros and Aszpla 2002 analyze the $Cr^{+3}$ ion to exchange in NaA zeolite in presence of $K^+$,$Cu^{+3}$ ion in binary and ternary cation exchange with evaluate the dimensionless variance of Breakthrough curve which useful to estimate the axial dispersion of the bed, M. Doung and D. Shallcross 2002 developed theoretical model of fixed bed ion exchange and obtained the model for pure dispersion without ion exchange performance, I. M. Abu-Reesh and B.l F. Abu-Sharkh 2003 make a comparison of axial dispersion and tank-in-series models for simulating the performance reactors they found that tank-in-series models presents computational advantages compared to dispersion model , Doung M. and Shallcross D.2005 made a series of experiment in wedge-shape cell ion exchange for both Na-and H- form bed with different flow rate lead to indicated dispersion coefficient function of pore velocity and radial distant of the wall.

## 2. Dispersion and Axial Dispersion Model

Dispersion is a general term which is used to describe the various types of self-induced mixing processes occurs during the flow of a fluid through a pipe or vessel. Dispersion arise from the effects of molecular diffusion or as a result of the flow pattern existing within the fluid, the important consequence of dispersion is that the flow in a packed bed reactor, the truth that dispersion reflects the back mixing which accurse with flow is a trendy for both axial and radial dispersion since the bed diameter is normally greater than the particle diameter so its comments to have effects of radial dispersion but prevalence of plug and axially dispersed that it can reduce the efficiency of separation. Axial dispersion is caused by the twin effects of molecular diffusion and turbulent mixing which a rises from splitting and recombination of flow around the particle

therefore it is important to describe the behavior across axial dispersion, the model characterizes mass transfer in axial direction in term of an effective or apparent longitudinal diffusivity $D_L$ is superimposed on the plug flow velocity. In order to predicate axial dispersion many assumption are suggest the model assumes that the fluid velocity and reactant concentration are constant across the tube diameter. The magnitude of the dispersion is assumed to be independent of position within the vessel, so there will be no stagnant regions and no by passing or short-circuiting of fluid in the model column also eliminating the assumption of instantaneous equilibrium through the bed

By changing the magnitude of the dispersion parameter, one may vary the performance of the reactor from that of plug flow ($\dfrac{D_L}{uL} = 0$) to that of a single continuous stirred tank reactor ($\dfrac{D_L}{uL} = \infty$).

Since these two types of phenomena are characterized by a single parameter, and since we force the model to fit the form of Fick's law of diffusion, $D_L$ should be regarded as an effective dispersion coefficient having the units of an ordinary molecular diffusivity (length$^2$/time). However, it is significantly greater in magnitude because of turbulence effects.

Consider the binary system involving exchange between cations A and B. We assume initially that all the exchangeable sites of the resin bed are in the B-form and the solution phase within the pores of the resin bed is free of cations A. Initially, i.e., at t = 0, a solution containing cations A is injected into the top of the resin bed at a volumetric rate, $F_L$, which is constant with time and uniform across the top of the resin bed. The feed solution concentration is constant for t >0 with time. A solution is distributed at a rate uniform across the resin bed.

For some time after injection begins, A cations do not appear in the effluent. As the resin gradually converts to the A form, the concentration of A cations in the effluent increases from zero to the feed concentration. Plotting the concentration of A cations in the effluent against the produced effluent volume gives the breakthrough curve.

The ion exchange process involving A and B cations may be presented by:

$$aR - B^{b+} + bA^{a+} \iff bR\text{-}A^{a+} + aB^{b+} \qquad \ldots(1)$$

Where, R is the solid cation exchange resin, a is the valance of A and b is the valance of B and the process is considered reversible.

A thin but finite slice of the bed having the thickness $\Delta Z$ at position $Z$ is considered now. For $t > 0$, $C_A$ and $C_{RA}$ will be volumetric concentration of cations A in the solution phase and in the solid resin phase respectively.

$J_A$ is the dispersion flux of the A cations.

a material balance for A cations over the section may be written as

$$\varepsilon_b A_c \Delta Z \frac{dC_A}{dt} + (1-\varepsilon_b)A_c \Delta Z \frac{dC_{RA}}{dt} =$$

$$\varepsilon_b A_c (J_A\big|_Z - J_A\big|_{Z+\Delta Z}) + F_L(C_A\big|_Z - C_A\big|_{Z+\Delta Z})$$

$$\ldots(2)$$

In this equation the terms on the left hand side arise from the accumulation of A cations in the solution and resin phases. The first term on the right hand side arises from the dispersion terms and the second results from the convection terms then taking limits as $\Delta z. \rightarrow 0$

$$\frac{\partial C_A}{\partial t} + \frac{1-\varepsilon_b}{\varepsilon_b}\frac{\partial C_{AR}}{\partial t} = -\frac{\partial J_A}{\partial Z} - \frac{F_L}{\varepsilon_b A_c}\frac{\partial C_A}{\partial Z}$$

$$\ldots(3)$$

Fick's Law can be used to relate the dispersion flux to the concentration gradient:

$$J_A = -D_L\frac{\partial C_A}{\partial Z} \qquad \ldots(4)$$

where, the axial dispersion coefficient, $D_L$ is assumed to be independent of $C_A$ and constant with respect to Z

The substitution of the derivative of the axial dispersion flux into equation (3) yields

$$\frac{\partial C_A}{\partial t} + \frac{1-\varepsilon_b}{\varepsilon_b}\frac{\partial C_{RA}}{\partial \tau} = D_L\frac{\partial^2 C_A}{\partial Z^2} - \frac{F_L}{\varepsilon_b A_c}\frac{\partial C_A}{\partial Z}$$

$$\ldots(5)$$

Equation (5) is the basic of the theoretical model for ion exchange performance in a fixed bed.

Several assumptions are implicit in equation (5):
- The resin bed is homogeneous with uniform porosity
- The bulk phase is plug flow with axial dispersion and mass transfer between phases is accounted for by film diffusion

- Fluid flows at a rate constant with time, under isotherm condition and is uniformly distributed
- The axial dispersion is independent of chemical concentration
- The bed is chemically inert and there is no mass transfer in the solid film that

$$C_A = C_p\big|_{A \ r=Rp} \text{ that mean } \frac{\partial C_{RA}}{\partial t} = 0 \qquad \ldots(6)$$

$$\frac{\partial C_A}{\partial t} = D_L\frac{\partial^2 C_A}{\partial Z^2} - \frac{F_L}{\varepsilon_b A_c}\frac{\partial C_A}{\partial Z} \qquad \ldots(7)$$

by expressing in dimensionless form equation (7) May be more easily solved

$$u = \frac{F_L}{A_c} \quad , \quad Pe = \frac{uL}{D_L \varepsilon_b} \qquad \ldots(8)$$

$$z = (ut + x)/L \ , \quad \theta = \frac{t}{t} = tu/ \qquad \ldots(9)$$

then equation (7) become

$$\frac{\partial C}{\partial \theta} = \frac{1}{Pe}\frac{\partial^2 C}{\partial z^2} - \frac{\partial C}{\partial z} \qquad \ldots(10)$$

that axial dispersion model is characterized by the Pecklet number (Pe No.) which can be regarded as the ratio

PeNo.= rate of transport by convection /rate of transport by diffusion or dispersion.

axial dispersion model

$$\frac{\partial C}{\partial \theta} = \frac{\varepsilon_b D_L}{uL}\frac{\partial^2 C}{\partial^2 z} - \frac{\partial C}{\partial z} \qquad \ldots(11)$$

where the dimensionless group ($\frac{D_L}{uL}$) is call vessel dispersion number which is measures the extent of axial dispersion where

$$\frac{D_L}{uL} \rightarrow \quad 0 \quad \text{negligible dispersion (plug flow)}$$

$$\frac{D_L}{uL} \rightarrow \quad \infty \quad \text{large dispersion (mixed flow)}$$

This model represents quite satisfactorily flow that deviates not too greatly from plug flow.

Equation (11) may be more easily solved using numerical techniques with initial and boundary

condition, since the resin is initially in the B-form and the solution within the pores of the bed contains B cations only, the initial condition for the partial differential equation (11) may be written

$$C\ (0,z) = 0 \qquad \qquad …(12)$$

At $t = 0$, there is a step change in the composition of the solution injected into the bed, the boundary conditions that apply to this equation depend on characteristics of the system at the tracer injection and monitoring stations according to these points the bed is assumed as "open," depending on plug flow into the test section that open boundary is one at which the same dispersion parameter characterizes the flow conditions within and adjacent to the test section.

There are then three different possible sets of boundary conditions depending on whether a completely open vessel or a closed-open vessel or an open-closed vessel is assumed different solutions will be obtained for different boundary conditions. if we now consider concentration in the feed to an "open" tube that can be regarded as extending to infinity in both directions from the injection point, that the boundary condition are

$$c(\theta,0)=1, \quad \frac{\partial^2 c}{\partial z^2}(\theta,1) = 0 \qquad …(13)$$

The basic method of estimating $D_L$ relating the measured mean residence times ($\bar{t}$) variances Pecklet number or dispersion parameter for the test section where ($\bar{t}$) calculated by the equation (14)

$$\bar{t} = \int_0^\infty t\left(\frac{dF(t)}{dt}dt\right) \qquad …(14)$$

By using F =C/C0

Then determined the variance of the response to a pulse input which is measures the spread of the distribution about the mean defined as

$$\sigma_t^{\,2} = \frac{\int_0^\infty (t-\bar{t})^2\, \frac{dF(t)}{dt}\, dt}{\int_0^\infty \frac{dF(t)}{dt}\, dt} \qquad …(15)$$

Where $\sigma_t^2$ is the variance of the response to a pulse input, in order determination of the reactor dispersion parameter

$$\sigma_t^2 = (\bar{t})^2\left[\frac{2D_L}{uL} + 8\left(\frac{D_L}{uL}\right)^2\right] \qquad …(16)$$

## 3. Experimental Work

A series of experiment were conduct to determined the axial dispersion by examine the breakthrough curve for ion exchange bed using a strong acid resin first with Amberlite IR-120-styrene-DVB gel type first with $Na^{+1}$ ionic form of a total capacity of 1.9-2.15 meq/ml to treated $Ca^{+2}$ ion in $CaCl_2$ solution secondly with same resin type but with $H^{+1}$ ionic form of 1.8 eq/l total capacity to treated ion in NaCl solution.

The both kind of resin is packed in a glass column of inside diameter (2.45cm) and (1m)long, the column is packed with 44.5 cm resin bed depth which supported inside the column by a glass disk welded to the wall of the column on the bottom section.
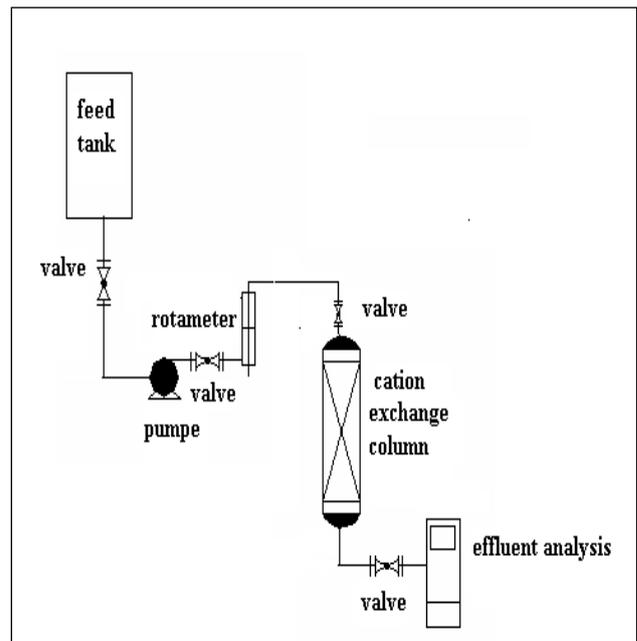


**Fig.1. Schemed Diagram for the System of Experimental Work**

The column is corporate with glass valve of shut of type located to coordinated the different concentration (500,450,350,250 ppm) first with a solution of $CaCl_2$ then with a solution NaCl of which represent the feed to be treated, a dosing pump with a rotameter is used to control the different flow injected (4, 2, 1.25, 0.9 l/hr) which is connected to the column as down flow operation condition. First with a solution of $CaCl_2$

then with a solution NaCl six experiment in each feed type were cared, the effluent was collected then analysis using Ethylen Diamine Tetracetic Acide (EDTA) titration method to determined the out put concentration for $Ca^{+2}$ ion and PH value method for determined the out put concentration for $Na^{+1}$ in both type the experiments is continuous until ($C_{out} = C_o$) figure (1) show a schemed diagram for the system of experimental work.

## 4. Result
### 4.1 The Breakthrough time

The breakthrough time of $Ca^{+2}$ -$Na^{+1}$ exhaustion and $Na^{+1}$-$H^{+1}$ runs effected by the $CaCl_2$, NaCl solution concentration and the flow rate in these curves three zone can be distinguished the first zone is almost saturated where the exist concentration is practically zero then the exchange take places that concentration is increases until the concentration limited value or where it exhausted where the zone is call mass transfer zone which its width measuring the efficiency of ion exchange column, the third zone is almost free of ionic spaces.

The result in tables (1) and (2) shows that at high $CaCl_2$, NaCl solution concentration feed input reducing the breakthrough time and lead the effluent concentration equal to initial concentration in shorter time at the same flow rate and same bed depth because the concentration of ions in the solution increases which raises the mass transfer rate through the film until it exceed the diffusion rate through the resin bed then the diffusion become controlling kinetics which is slower the process , figures (2) and (3) shows the breakthrough curve for different $CaCl_2$, NaCl solution (500,350,250ppm) at the same flow rate (1.25 l/hr)and same bed depth (44.5cm).

**Table 1,**
**Experiments Runs for CaCl$_2$ Salt at Cation Exchange with Na- Form**

| Exp no. | Salt Con. (ppm) | Flow rate l/hr | Break-through time(min) | tt | tu | $H_{UNB}$ |
|---------|-----------------|----------------|-------------------------|---------|---------|-----------|
| 1 | 450 | 4 | 582 | 453.75 | 397.42 | 12.415 |
| 2 | 450 | 2 | 1470 | 1366.5 | 1289.74 | 5.7 |
| 3 | 450 | 0.9 | 2310 | 2183.02 | 2070.7 | 5.145 |
| 4 | 500 | 1.25 | 540 | 419.7 | 361 | 13.8 |
| 5 | 350 | 1.25 | 885 | 744.74 | 629.718 | 15.45 |
| 6 | 250 | 1.25 | 972 | 892.8 | 779.054 | 12.7 |

**Table 2,**
**Experiments Runs for NaCl salt at Cation Exchange with H-Form**

| Exp no. | Salt Con. (ppm) | Flow rate l/hr | Break-through time(min) | tt | tu | $H_{UNB}$ |
|---------|-----------------|----------------|-------------------------|---------|---------|-----------|
| 1 | 450 | 4 | 400 | 232.95 | 125.988 | 45.92 |
| 2 | 450 | 2 | 1080 | 774.115 | 364.4 | 52.925 |
| 3 | 450 | 0.9 | 1761 | 1421.29 | 1162 | 18.243 |
| 4 | 500 | 1.25 | 300 | 182.4 | 122.36 | 32.92 |
| 5 | 350 | 1.25 | 700 | 504.39 | 404.65 | 19.80 |
| 6 | 250 | 1.25 | 740 | 612.4 | 322.66 | 47.312 |

The result in tables (1) and (2) shows that high $CaCl_2$, NaCl solution flow rate input reducing the breakthrough time and lead the effluent concentration equal to initial concentration also in shorter time at the same concentration and same bed depth that lower flow rate there will be sufficient contact time ions exchanges that the equilibrium is established soon, as the flow rate increases equilibrium is no longer reached and the mass transfer through the film rises until the diffusion through the resin becomes the controlling factor and the system is said to exhibit particle controlled kinetics, figures (4) and (5) shows the breakthrough curve for different $CaCl_2$ ,NaCl flow rate(4, 2, 1.25, 0.9 l/hr) at the same concentration (450 ppm)and same bed depth (44.5 cm).
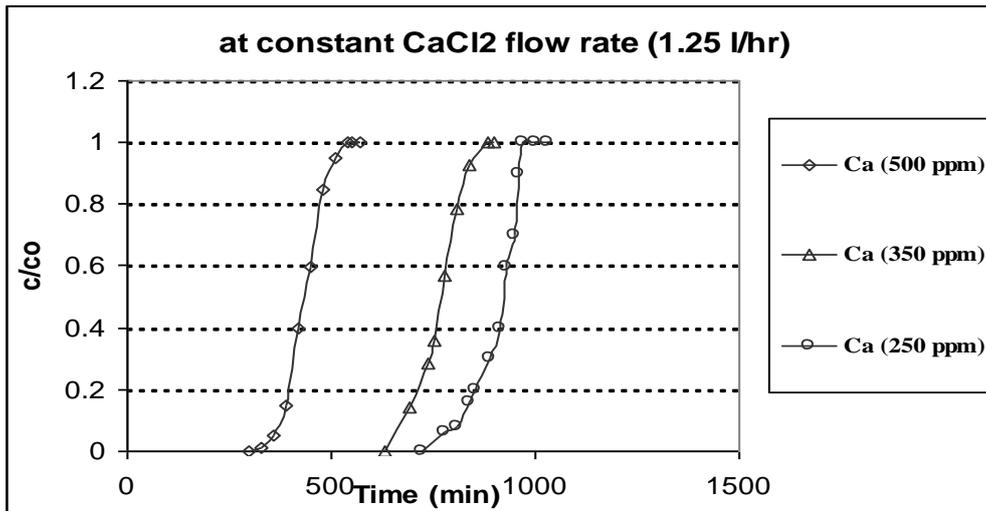
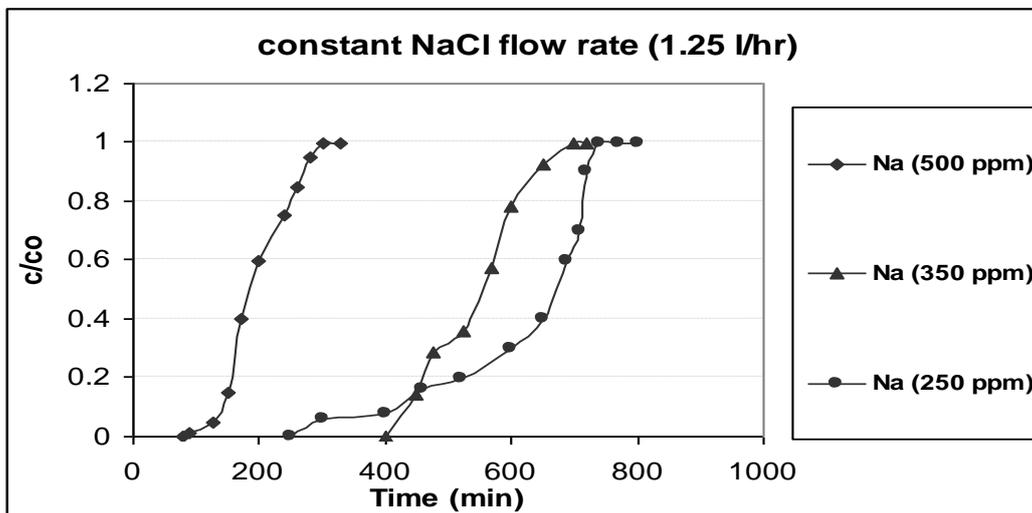**Fig.2. Breakthrough Time for CaCl₂ Solution at Constant (1.25 l/hr) Flow Rate**



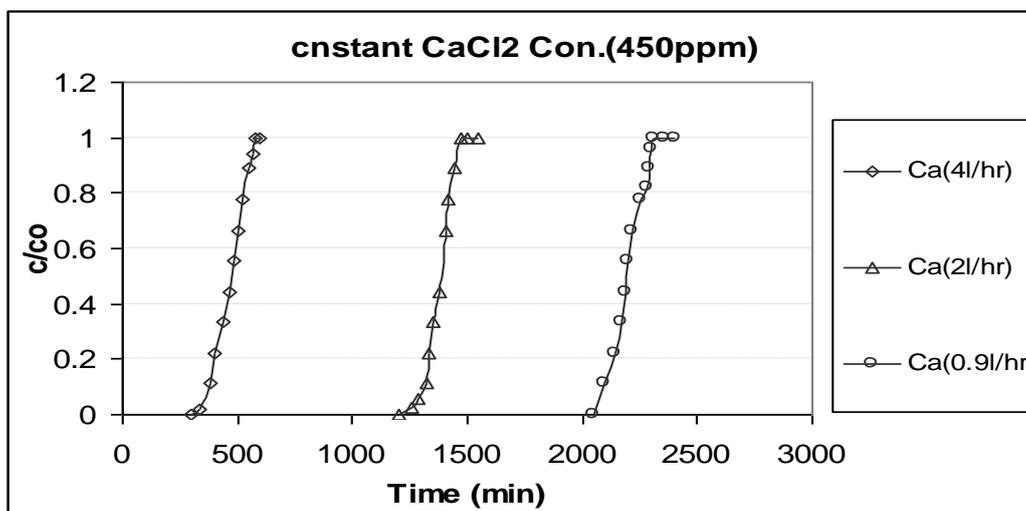**Fig.3. Breakthrough Time for NaCl Solution at Constant (1.25 l/hr) Flow Rate**



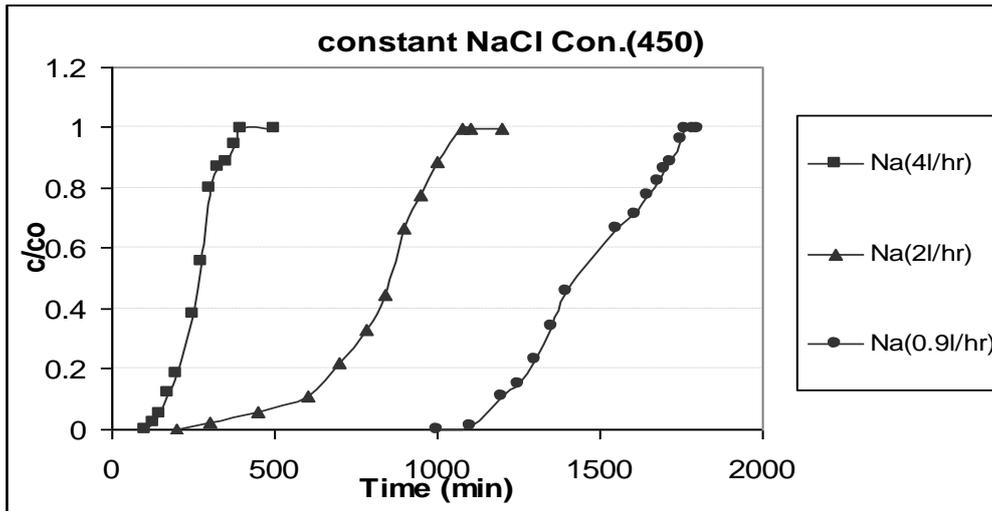**Fig.4. Breakthrough Time for CaCl₂ Solution at Constant Concentration (450ppm) Flow Rate**

**Fig. 5. Breakthrough Time for NaCl Solution at Constant Concentration (450ppm)**

It important to notice that breakthrough time of $Ca^{+2}$ -$Na^{+1}$ exhaustion for $CaCl_2$ solution is greater than $Na^{+1}$-$H^{+1}$ exhaustion for NaCl solution in the same condition of salt concentration and flow rate that the exchange of divalent with monovalent can go much completely because the concentration of the monovalent ions is squared in $Ca^{+2}$ -$Na^{+1}$ exchange that effected on the selectivity of the resin with taking in a count the decreasing of size of hydrated ions from $Na^{+1}$ to $Ca^{+2}$ which make the exchange capacity for divalent ions is grater than for monovalet ion .

The time equivalent to usable capacity ($t_u$) and the time equivalent to total stoichiometric capacity of packed bed ($t_t$) if the entire bed attains to equilibrium are easily to determined by eq. (17) where $t_b$ is the time related to $C = 0.05 C_o$

$$t_u = \int_0^{t_b} (1 - \frac{C}{C_o}) dt \qquad t_t = \int_0^{\infty} (1 - \frac{C}{C_o}) dt \qquad \dots(17)$$

The length of unused bed is determined by equation (18) that ($t_u/t_t$) is the fraction of the total bed capacity or length utilized to the breakpoint.

$$H_{UNB} = \left[ 1 - \frac{t_u}{t_t} \right] L \qquad \dots(18)$$

Where L is the total bed length

The $H_{UNB}$ values is listed in tables (1)and(2) for $CaCl_2$ and NaCl ion exchange operation solution at different feed concentration and different flow rate that the small value means that

the breakthrough curve is close to an ideal step with negligible mass transfer resistance.

It can be notice that $H_{UNB}$ depends on concentration of the ion to be exchange, ions types, the temperature and fluid velocity while it independent of column length which allows the design and scale-up from small scale laboratory to be determined that the full-scale column obtained by simply adding the $H_{UNB}$ to the length of bed needed.

It's important to taken into account the axial dispersion because differences in the extent of axial dispersion in small-scale laboratory and full-size column may caused discrepancies.

## 4.2 Axial Dispersion

Residual time axial dispersion for $Ca^{+2}$-$Na^{+1}$ and $Na^{+1}$-$H^{+1}$ ions exchange is calculated and listed in tables (3) and (4), the values of dispersion number for $Ca^{+2}$-$Na^{+1}$ and $Na^{+1}$-$H^{+1}$ exhaustion is increasing with increasing flow rate at the same salt concentration that at height flow rate the bed is consist of a series of mixing cell the dimension of each of which is of the same as the size of the bed particles forming while at low flow rate the effect of molecular diffusion predominate and the cell mixing contributes relatively little to dispersion.

For $Ca^{+2}$-$Na^{+1}$ figure(6) show small extend of dispersion ($\frac{D_L}{uL}$ <0.01) which mean small deviation from plug flow that the spreading tracer curve does not significantly change in shape as it passes( during the time it is measuring it)which gives symmetrical curves.

**Table 3,**
**Experiments Runs for CaCl₂ Solution at Cation Exchange with Na- Form**

| Exp no. | Salt Con. (ppm) | Flow rate l/hr | Residence time (min) | $\alpha\theta^2$ | $\dfrac{D_L}{uL}$ | $D_L$ | Pe NO. |
|---|---|---|---|---|---|---|---|
| 1 | 450 | 4 | 449.4 | 0.145 | 0.0588 | 77.36 | 45.98 |
| 2 | 450 | 2 | 1453.19 | 0.0306 | 0.0145 | 9.538 | 229.885 |
| 3 | 450 | 0.9 | 2291 | 0.00596 | 0.00294 | 0.87 | 1133.78 |
| 4 | 500 | 1.25 | 441 | 0.0878 | 0.00432 | 1.806 | 771.6 |
| 5 | 350 | 1.25 | 756.05 | 0.00726 | 0.00358 | 1.47 | 915.36 |
| 6 | 250 | 1.25 | 899.18 | 0.00294 | 0.00145 | 0.596 | 2298.85 |

**Table 4,**
**Experiments Runs for  NaCl Solution at Cation Exchange with H-Form**

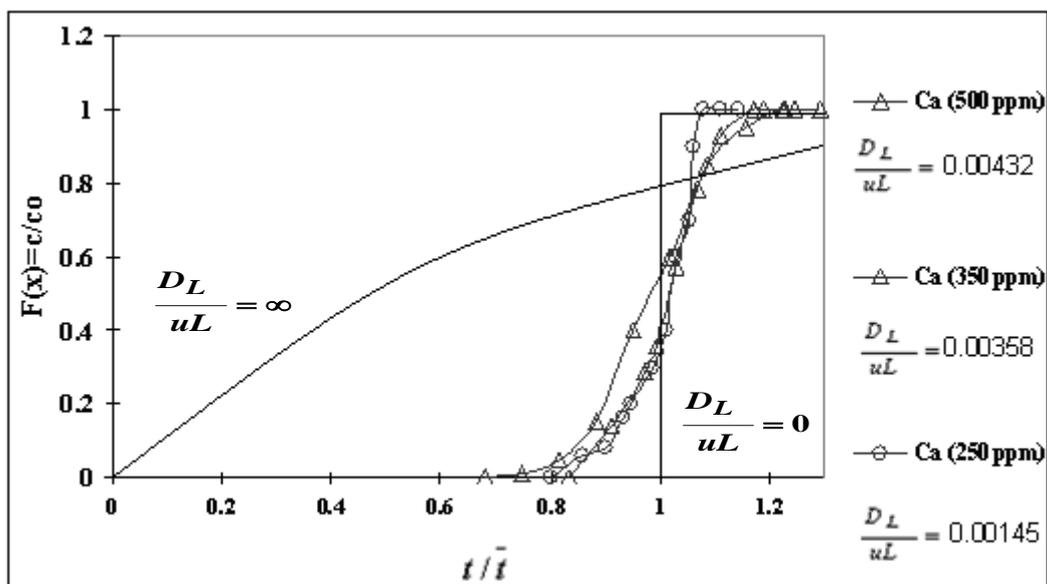| Exp no. | Salt Con. (ppm) | Flow rate l/hr | Residence time (min) | $\alpha\theta^2$ | $\dfrac{D_L}{uL}$ | $D_L$ | Pe NO. |
|---|---|---|---|---|---|---|---|
| 1 | 450 | 4 | 273.959 | 0.296 | 0.1044 | 137.359 | 31.928 |
| 2 | 450 | 2 | 841.03 | 0.1514 | 0.061 | 40.125 | 54.64 |
| 3 | 450 | 0.9 | 1684.92 | 0.0066 | 0.0295 | 8.79 | 112.765 |
| 4 | 500 | 1.25 | 195.966 | 0.0239 | 0.01145 | 4.707 | 291.12 |
| 5 | 350 | 1.25 | 516.316 | 0.017 | 0.00824 | 3.387 | 404.53 |
| 6 | 250 | 1.25 | 659.88 | 0.035 | 0.01648 | 6.7757 | 74.57 |



**Fig.6. Axial Dispersion for CaCl₂ Solution at Constant Flow Rate (1.25l/hr)**

But with $Na^{+1}$-$H^{+1}$ ion exchange figure (7) show small extend of dispersion ($\frac{D_L}{uL} > 0.01$) which mean small deviation from plug flow that resulting S-shape response curves is not symmetrical that mean the flow is undistributed as it passing the entrance and exit boundaries .
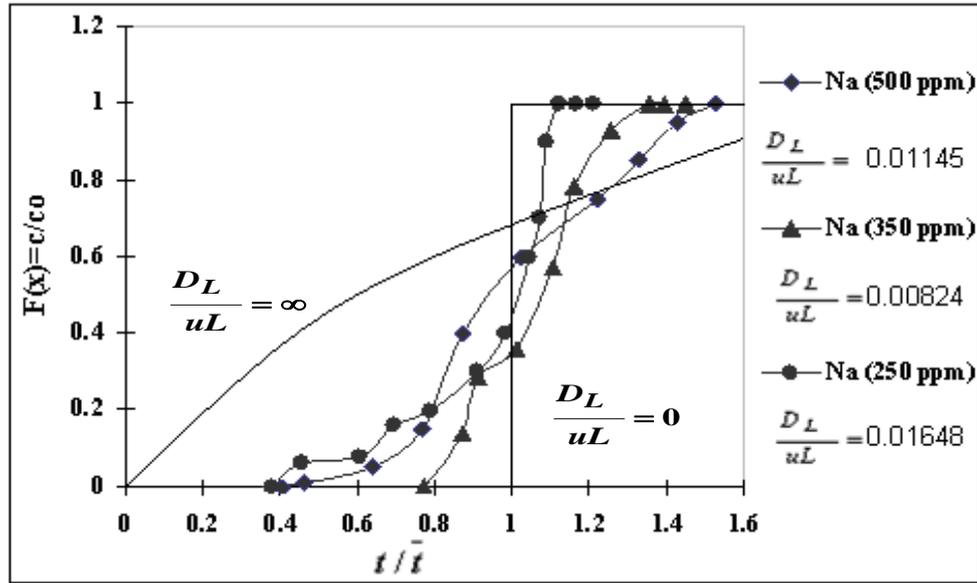
**Fig.7. Axial Dispersion for NaCl Solution at Constant Flow Rate (1.25l/hr)**

The values of vessel dispersion number for $Ca^{+2}$ -$Na^{+1}$ and $Na^{+1}$-$H^{+1}$ is increasing as the $CaCl_2$ , NaCl salt concentration increasing for the same flow rate as shown in tables (3) and (4)that when concentration differences exist the rate of transfer(exchange) is proportional to the product of the molecular diffusivity and the concentration gradient where the fluid at centre moves with twice the mean velocity this can rise the dispersion since elements of fluid will take different times to traverse the length of the column.

Figures (8) and (9) show small extend of dispersion ( $\frac{D_L}{uL}$ <0.01) which mean small deviation from plug flow for $Ca^{+2}$ -$Na^{+1}$ and $Na^{+1}$-$H^{+1}$ ion exchange.
The dimensionless variances of breakthrough curve is calculated and listed in table (3) and(4) for the both case of ion exchange($Ca^{+2}$,$Na^{+1}$) that it value of $\sigma^2 \approx 0$ means the packed bed close to an ideal flow reactor with negligible axial dispersion
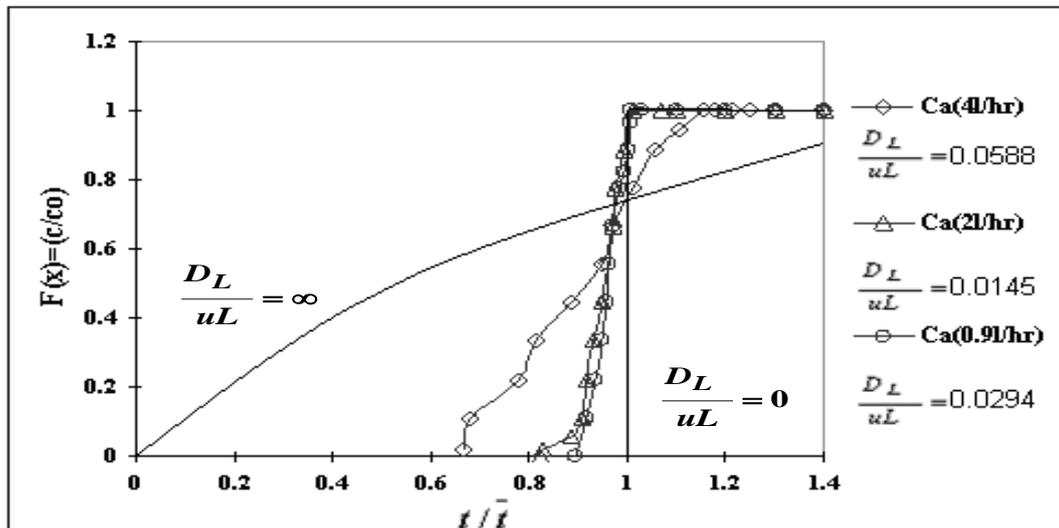


**Fig.8. Axial Dispersion for CaCl₂ Solution at Constant Concentration (450ppm)**
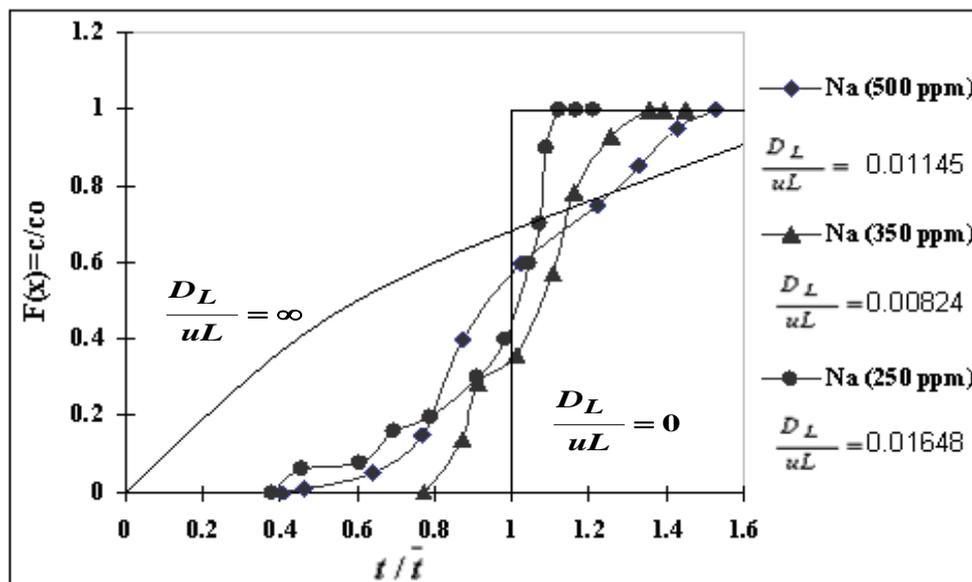
**Fig.9. Axial Dispersion for NaCl Solution at Constant Concentration (450ppm)**

Pe No. represent all of the effects that cause deviations from ideal plug flow behavior such as nonuiform velocity profiles or eddies listed in table (3) and(4) for the both type ion exchange ($Ca^{+2}$, $Na^{+1}$) its mean increasing as the flow rate increasing and decreasing with increasing ($Nacl$, $CaCl_2$) salt concentration that as Pe No. increasing from 0 to $\infty$ the flow pattern in the column changes from complete mixing (CSTR) to no mixing, with low Pe No. high conversion with low residence time accurse because molecular diffusion strongly effects the rate of dispersion in laminar flow means at low flow rate it promotes dispersion but at higher flow rate it has opposite effect .

## 5. Conclusion

A mathematical model has been developed to simulate ion exchange with fixed bed; the axial dispersion model is applied with experimental observation for both $Ca^{+2}$ and $Na^{+1}$ ion in cation exchange column.

The breakthrough time is decreasing with increasing flow rate and feed concentration and its greater for divalent ions from monovalent ions.

The axial dispersion model show small extend from plug flow for $Ca^{+2}$ ions exchange both exchange in both increasing flow rate and increasing salt concentration While its show large extend from plug flow for $Na^{+1}$ ions with increasing flow rate and salt concentration.

## Nomenclature

| | |
|---|---|
| a | valence of cation A |
| Ac | circular cross-sectional area |
| b | valence of cation B |
| CSTR | continuous stirrer tank reactor |
| C A | solution phase concentration |
| CRA | resin phase concentration |
| C0 | total normality in the solution |
| CsA | surface phase concentration in particle |
| CpA | pore phase concentration in particle |
| DL | solution phase dispersion coefficient |
| d | column inner diameter |
| $\varepsilon_b$ | bed porosity |
| F | C/C0 |
| FL | flow rate |
| $H_{UNB}$ | length of unused bed |
| JA | dispersion flux of A cations in  s o solution |
| L | bed length |
| Pe | Peclet number |
| $\theta$ | dimensionless time |
| R | solid cation exchange resin |
| Rp | resin radius |
| t | time |
| $\bar{t}$ | mean residence times |
| $t_b$ | time related to C = 0.05Co |
| $t_t$ | time equivalent to total stoichiometric |

| | capacity |
|---|---|
| $t_u$ | time equivalent to usable capacity |
| u | velocity for linear flow |
| z | dimensionless column distance |
| $\sigma^2_t$ | variance of the response to a pulse input |

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# دراسة التشتت المحوري لعمود الطبقة الثابته من المبادلات الايونية

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## الخلاصة

تم تطوير موديل رياضي لوصف اداء عمود اسطواني نوع الطبقة الثابته من المبادلات الايونية يتضمن مقارنة موديل التشتت المحوري بعدد من الفرضيات حيث ان هذا الموديل يسمح بتحيد الاداء للعمود و حساب الوقت المقيم لتحديد الانحراف عن عمود الجريان المضغوط.

تم اختيار نموذجين من انظمة التشغيل للايونات الموجبة  تم العمل اولا مع ايونات ثنائية التكافؤ ( ايون الكالسيوم الموجب) ليتبادل مع رزن بصيغة ايون الصوديوم والذي يطبق بوحدات الماء الرائق والثاني مع ايونات احادية التكافؤ (ايون الصوديوم الموجب) ليتبادل مع رزن بصيغة ايون الهيدروجين والذي يطبق في وحدات الماء اللايوني.

تم اختيار عاملين  لكل واحد من النظامين  هما اختلاف معدل جريان المغذي واختلاف تركيز الملح  عن طريق تاثيرهم  في التشتت العمودي لكل حالة بواسطة مقارنة الاداء والذي يعبر  عنه بزمن الاستنزاف حيث تم حساب محددات اخرى (الزمن المكافئ للسعة المستعملة والزمن المكافئ لسعة الاتزان وطول العمودى غير المستعمل)حسبت خلال ظروف العمل لان هذه المحددات مع التشتت المحوري مهمة جدا لتحديد افضل ظروف لتشغيل عمو د  الطبقة الثابته للمبادل الايوني الموجب كوحدات الماء الرائق ووحدات الماء اللايوني.

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