



## Mathematical Modeling for the Clarifier Units and Turbidity Parameters in AL-KARAMA Treatment Plant

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### Abstract:-

The high cost of chemical analysis of water has necessitated various researches into finding alternative method of determining portable water quality. This paper is aimed at modelling the turbidity value as a water quality parameter. Mathematical models for turbidity removal were developed based on the relationships between water turbidity and other water criteria. Results showed that the turbidity of water is the cumulative effect of the individual parameters/factors affecting the system. A model equation for the evaluation and prediction of a clarifier's performance was developed:

$$\text{Model: } T = T_0(-1.36729 + 0.037101 \cdot 10^{\text{pH}} + 0.048928t + 0.00741387 \cdot \text{alk})$$

The developed model will aid the predictive assessment of water treatment plant performance.

The limitations of the models are as a result of insufficient variable considered during the conceptualization.

**Keywords:** Performance evaluation, pH, Alkalinity, Mathematic model, Simulation

### 1. Introduction

Tebbutt (1983) describes Water as the World's most important natural resource since without it life cannot exist and industry operate. Unlike many other resources, it has no substitute in many of its uses. Water plays an essential role in community development since a reliable supply is a prerequisite for establishing a permanent settlement.

There is a vast amount of water present in the earth and its' surrounding atmosphere. About 7% of the earth's mass is made up of water. 97% of this is found as saline water in oceans, about 2.3% is in the polar caps and only 0.7% exists in fresh water lakes, rivers, aquifers and in the atmosphere.

Water that is pure is not found in nature; even water vapour condensing in air contains solids and dissolved gases

(Tebbutt, 1983). As it condenses and falls, it sweeps up other materials from the air, it becomes still more contaminated on reaching the ground, as it runs running over soil surface and percolate the soil strata.

Waste from human activities, either industrial or domestic, according to Tchobanoglous and Burton (1991), introduce even more pollutants, than any natural sources, into water bodies.

Since the available water is seldom found in conditions that meet potable specifications, a treatment plant is clearly necessary to improve the water quality. Water supply to a community goes through the following stages; the community water demand is carefully estimated with allowances for population growth; the most suitable raw water source is identified and analysed; then a

water treatment plant to effect the required changes is designed, constructed and operated along with it's distribution network.

When online, a periodic review of plant performance is undertaken to ascertain if, or otherwise, the plant works according to prediction. Hammer (1973) and Cairncross (1980) both agree that record-keeping and periodic reviews of plant performance are necessary decision tools when the plant requires expansion or when operational problems arise.

The aim of this paper is to model water turbidity as function of several other parameters that will aid in assessing the performance of a Al-Karama water treatment plant over a period of time and to suggest ways of improving plant performance

## **2. EXPERIMENTAL WORK AND PROCEDURE**

Al-Karama drinking water treatment plant located on the western-bank of Tigris river in Al-Karkh district and the water supplies to the plant by four in river intakes, the plant consists of 20 clarifiers that their functions are to reduce the solid content in the water coming from the river to pass through the sub-sequent treatment stages till pumps through the network.

The sampling procedure followed the way of sampling from the basin mentioned in the standard method of water analysis all and all the test were done on – line near the basins to reduce the physical effects of the surrounding on the results.

### **Determination of Temperature**

The temperature of the sample of water was determined with the aid of thermometer, beakers, and electrodes.

### **Determination of Turbidity**

This involves the use of turbid meter. The sample cells were washed with sample water, and was discarded and refilled with the same sample.

### **Determination of pH**

The PH of the sample was determined with aid of colormetric comparator.

### **Determination of Total Alkalinity**

The total alkalinity was determined using titration.

## **3. Modelling for Turbidity Removal**

For a constant flow rate and a fixed set of conditions the suspended solids (and hence turbidity) removal is a fixed fraction of inlet suspended solid. Assuming also, in accordance with Belan (1988), that for mildly contaminated rivers the suspended solid is linear with turbidity i.e. that the particle size and type distribution is constant over a time period.

Then Turbidity removal

$$(T_0 - T)/T_0 = k_0 \quad \dots(1)$$

where T = turbidity of clarified water,  $T_0$  = Turbidity of aerated water,  $k_0$  = constant

$$1 - (T/T_0) = k_0, \quad T/T_0 = 1 - k_0 = k', \quad T = T_0 \cdot k' \quad \dots(2)$$

In reality operating conditions do vary. Therefore  $k'$  is only one of the function by which water quality may be measured. Other essential parameters according to Nikoladze et al (1989) are temperature (t), pH and alkalinity (alk). It is well established that these parameters exhibit independent and cumulative effect on T.

### **Effect of Water Temperature**

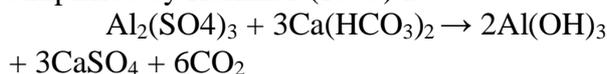
Water molecules and impurity particles are in thermal Brownian motion whose intensity is directly proportional to temperature (Nikoladze et al, 1989). It is clear that the probability of collision of individual particles with one another and their consequent aggregation depend on their relative velocities i.e. on thermal Brownian motion (and therefore on water temperature). Again, it shall be assumed that the temperature profile within the clarifier is constant and is equal to that of the clarified water samples.

$$(T_0 - T)/T_0 \sim t \dots(3)$$

Where t = temperature in °C.

### Effect of Alkalinity

Turbidity removal is preceded by and dependent on the formation of chemical flock. The equation of the overall process as simplified by Hammer (1975) is



Obviously, good coagulation is dependent on the presence of sufficient alkalinity ( $HCO_3^-$ ) and therefore

$$(T_0 - T)/T_0 \sim alk \quad (4)$$

where alk = alkalinity of water in mg/l as  $CaCO_3$ .

### Effect of pH

Suspended matters in water are surface-charged particles and it is the function of the coagulant to neutralize the charges. Different particles types have been seen to have a particular pH at which the net charge on them is zero and coagulation optimum (Nikoladze et al 1989). This pH is the isoelectric point ( $pH_{is}$ ). A large difference between pH of the water medium and  $pH_{is}$  confers greater anticoagulation properties. For clay and humus  $pH_{is} = 7.1$  and  $7.0$  respectively. This effect is confirmed by graphs obtained by Tebbutt (1983) where turbidity (color) removal is reduced as the water pH deviates from an optimum value of about 7.0.

Therefore it can be concluded that:

$$(T_0 - T)/T_0 \sim 1/(pH - pH_{is}) \quad \dots(5)$$

Assuming  $pH_{is}$  as 7 then  $pH - 7 = \Delta pH$  has a positive value.

$$(T_0 - T)/T_0 \sim 1/(-\Delta pH) \quad \dots(6)$$

$pH = -\log[H^+]$ , therefore a more appropriate form of the equation would be:

$$(T_0 - T)/T_0 \sim 1/10^{-\Delta pH} \quad \dots (7)$$

An expression similar to this was used by Belan (1988) to express the lime dosage required to bring about a desired change in the pH of water to  $pH_{is}$  of its contaminants.

Introducing constants into equations 3, 4 and 7:

$$(T_0 - T)/T_0 = k_1/10^{-\Delta pH} \quad \dots(8)$$

$$(T_0 - T)/T_0 = k_2t \quad \dots (9)$$

$$(T_0 - T)/T_0 = k_3alk \quad \dots(10)$$

Other factors, such as organic content do exert influence on turbidity removal; their contribution can be accounted for by the introduction of corrective coefficient.

Assuming that hydrodynamic conditions are approximately constant, the influences of temperature, pH, and alkalinity on the turbidity of water in a clarifier basin may be additive or multiplicative (Lucey, 2000). However a change in one will cause disequilibrium in the overall turbidity. Such changes are accounted for by the various constants. Using the additive method:

$$(T_0 - T)/T_0 = k_0 + k_1/10^{-\Delta pH} + k_2t + k_3alk \quad \dots(11)$$

from which:

$$T = T_0(1 - k_0 + k_1/10^{-\Delta pH} + k_2t + k_3alk) \quad \dots(12)$$

Because  $1 - k_0$  is a constant, let us say  $k_0$  in place of:

$$T = T_0(k_0 + k_1/10^{-\Delta pH} + k_2t + k_3alk) \quad \dots(13)$$

The values of these constants maybe determined using the least – square method for multiple regression as outlined below adapted from Stroud (1995).

The various effects of pH, temperature and alkalinity on turbidity removal may also be combined in a multiplicative or exponential manner since no evidence exists

to suggest the relationship used. Following from the same premise as before, a possible second model equation such as 61 could be developed.

$$T = k_0(T_0 \cdot 10^{k_1 \Delta pH_i} \cdot t^{k_2} \cdot alk^{k_3}) \quad ..(26)$$

Comparison of the models results indicates their extent of validity. The constants  $k_0, k_1, k_2, k_3$  could be obtained from the solution of the generated 4x4 matrix.

The alternatively the developed models can be solved software packages such as Polymath3 (Himmelblau, 1996). The package was used to find the coefficients in both models. The developed models are presented below:

$$T = T_0(-1.36729 + 0.037101 \cdot 10^{\lambda pH} + 0.048928 \cdot t + 0.00741387 \cdot alk) \quad (\text{Model 1})$$

$$T = 2.20673 \cdot 10^{11} \cdot T_0 \cdot 10^{0.045606 \Delta pH} \cdot t^{5.35007} \cdot alk^{1.55059} \quad (\text{Model 2})$$

The graphical error analysis technique was then used as the criteria for selecting the best-fit model.

#### 4. Results

The results of the various experimental methods are as presented in Table 1.

**Table 1 weekly water quality parameters values**

Weeks	Quality parameters				
	T <sub>0</sub>	T	T	ΔHπ	alk
1	1.50	1.25	27.75	0.10	23.80
2	2.00	1.25	26.74	0.20	21.80
3	1.80	1.00	29.81	0.10	21.00
4	2.00	1.00	28.52	0.10	23.00
5	2.25	1.50	29.00	0.00	21.84
6	2.13	1.64	30.81	0.10	23.29
7	2.00	0.50	28.75	0.40	23.75
8	2.00	1.00	29.50	0.30	29.50
9	3.50	1.00	28.00	0.50	30.00
10	6.00	1.00	29.00	0.30	27.50
11	4.50	1.25	29.33	0.40	30.00
12	4.25	1.08	29.05	0.40	29.66
13	5.00	1.00	30.00	0.40	29.50
14	4.80	2.31	29.47	0.10	25.50
15	4.25	2.50	28.88	0.30	28.25
16	6.50	5.75	28.43	0.20	26.15

The simulated results from the models are presented in figure 1.

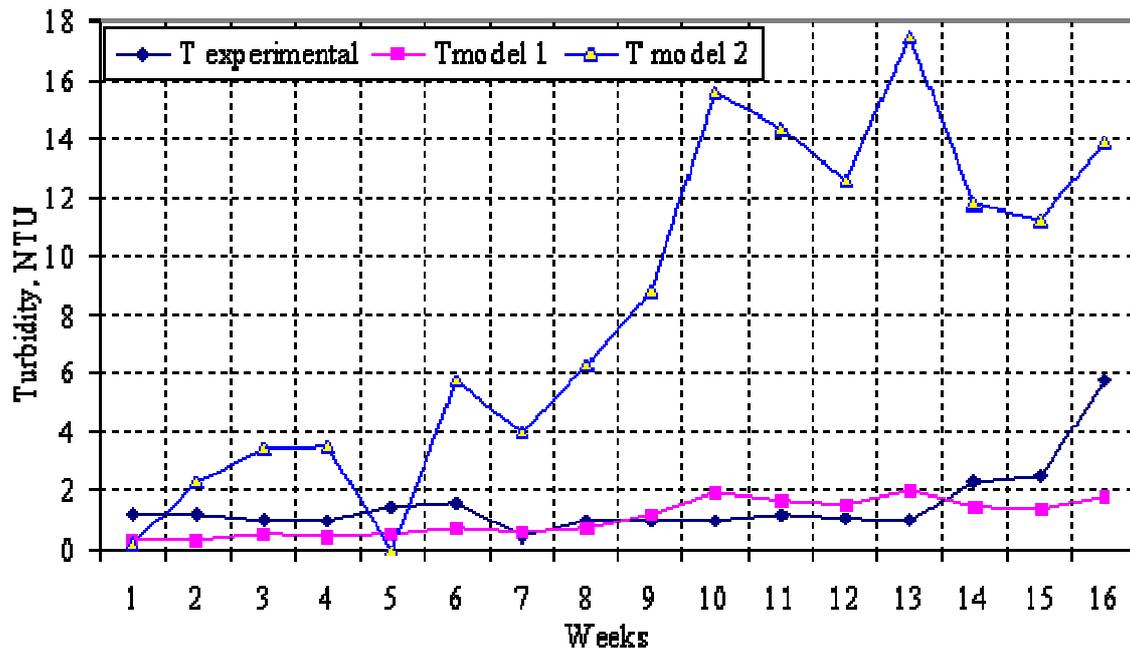


Fig. 1 Comparative analyses of the experimental and simulated values from models 1 and 2

### 5. Discussion

Results from table 1 showed that turbidity in the treated water fell short of standards (5NTU max.) on one occasion (5.75NTU) but the mean value of 1.56NTU is well within the guide line value (WHO,1985). The cause of this is most likely the high  $T_0$  and the consequent clarifier's overload. This may also suggest that the clarifier was overdue for desludging.

Mathematical modeling of the clarifier's performance based on additive and multiplicative models using the pre-determined parameters gave models 1 and 2 respectively.

$$\text{Model 1: } T = T_0(-1.36729 + 0.037101x 10^{\lambda \text{pH}} + 0.048928t + 0.00741387. \text{alk})$$

$$\text{Model 2: } T = 2.20673 \cdot 10^{-11} T_0 \cdot 10^{0.045606 \text{ pH}} \cdot t^{5.35007} \cdot \text{alk}^{1.55059}$$

Simulation results of the models showed that model 1 to a large extent will give a better turbidity prediction. It showed also that factors affecting turbidity values are mainly independent in operation. The turbidity of water is the cumulative effect of the

individual parameters/factors affecting the system.

From model 1 the change in turbidity caused by one of the independent variables is the cumulative effects of individual contributions of  $T_0$ ,  $t$ , and  $\text{alk}$ . In model 2 the relationship of the turbidity parameters is as the synergy or interdependence of the variables. The nature of the model 1 equation gives turbidity change a dimensionless significance. Therefore  $k_2$  the coefficient of temperature  $t$  will have a unit of  $1/^\circ\text{C}$ . It could be defined as the change in ratio of inlet and outlet turbidity caused by changing the temperature of the clarifier by  $1^\circ\text{C}$ .

The discrepancies between the experimental and simulated values could be attributed to the assumptions made during the formulating of the model. The numbers of variables considered were quite small. The influence of human activities, impact of environmental pollution/contamination etc. on the coagulation and settling processes and much other need to be considered to enhance the reliability of the model.

## 6. Conclusion

Developed a model equation for the evaluation and prediction of a clarifier's performance as is presented below:

$$\text{(Model 1) } T = T_0(-1.36729 + 0.037101 \cdot 10^{\lambda_{\text{pH}}} + 0.048928 \cdot t + 0.00741387 \cdot \text{alk})$$

The limitations of the models are as a result of insufficient variables considered during the conceptualization process.

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## التمثيل الرياضي للعوامل المؤثرة على عكورة المياه في احواض الترويق في مشروع ماء الكرامة

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قسم الهندسة البيئية

### الخلاصة:

يهدف البحث الى ايجاد تمثيل رياضي للعوامل المؤثرة على العكورة المزالة في احواض الترويق, حيث اعتمد التمثيل الرياضي على العلاقة بين عكورة الماء وباقي العوامل الفيزيائية الاخرى وقد وجد من العلاقة الرياضية ان عكورة الماء تتاثر بصورة كبيرة وتراكمية لبعض المؤثرات الفيزيائية وقد تم ايجاد معادلة رياضية لتقييم اداء احواض الترويق اعتمادا على العكورة المزالة .