



## Free Vibration Analysis for Dynamic Stiffness Degradation of Cracked Cantilever Plate

**Dr: Hussain A. Dawood**

*Mechanical Engineering Department  
College of Engineering/ University of Baghdad*

**Oday. I. Abdullah**

*Nuclear Engineering. Department.  
College of Engineering/ University of Baghdad*

(Received 29 August 2005; accepted 27 October 2005)

---

### Abstract:-

In the present work a dynamic analysis technique have been developed to investigate and characterize the quantity of elastic module degradation of cracked cantilever plates due to presence of a defect such as surface of internal crack under free vibration. A new generalized technique represents the first step in developing a health monitoring system, the effects of such defects on the modal frequencies has been the main key quantifying the elasticity modulii due to presence any type of un-visible defect. In this paper the finite element method has been used to determine the free vibration characteristics for cracked cantilever plate (internal flaws), this present work achieved by different position of crack. Stiffness reduction in term of elastic material properties is analyzed through a parametric study of crack density factor. Results are given for Young's modulus and shear modulus variation with respects the vibrational characteristics.

**Key words:** Free vibration, cantilever plate, stiffness degradation.

### 1. Introduction

In many branches of modern engineering the vibration of cantilever plate, plays an important role in the design and operation of the final engineering systems. Whether one is concerned with propellers, turbine cantilever plates, or satellite booms, where the vibrational behaviors of such system is perpendicular to the plane of rotation, this behavior is one of the most serious problems that should be studied and considered. Basically in the most

general terms damage can be defined as changes introduced into system that adversely affect the future performance of the plate structure. Thus, the study of damage identification in structures will be limited to changes of material and for/or geometric properties, but still the need for the ability to monitor and detect damage at earliest possible stages is objected engineering aim. Such investigations of damaged plates are determined whether the damage

recognized by measuring the vibration characteristics so that, severe damage can be prevented, the need for quantitative global damage detection techniques that can be applied to complex structures. This vibration based damaged deflection is deals with the significantly alter the stiffness, mass or energy dissipation properties of plates.

Such structural element in rotor craft requires assessment of their dynamic tolerance, and then accurately predicting technique tool for the behavior of damaged structure [1]. The dynamic analysis under present of internal (invisible) flaws is an important issue of the structural investigation and design, and determine the free vibration characteristics often appears to be the fundamental task in dynamic analysis [2]. Carnaage studied the vibration of cantilever plate by using the energy method [3]. The increment of strain energy due to rotation was first investigated. Further studies of the vibration characteristics of thin cantilever plates using the finite difference method [4] and the extended Holzer's method [5] were presented by Carnaage and co-workers. Krupka and Baumanis [6] studied the bending-bending mode of a cantilever plate including rotary inertia and shear deflection by the Myklestad method. Stafford and Giurgiutiu [7] used a semi-analytic method based on transfer matrix method to study a Timoshenko beam. Abbas [8] and Thomas and Abbas [9] developed a finite element model which can satisfy all the geometric and natural boundary conditions of a thick non-cantilever plate. Abbas [10] further used the finite element model for a thick cantilever plate with flexible root. The effect of the local flexibility of a cracked column upon its buckling load was studied by liebowitz et al. [11,12] and

okamura et al. [13]. Rice and Levy [14] recognized the coupling between bending and extensional compliance of a cracked column in compression. Dimargonas [15], Dimargonas and Paipetis [16] and Chondros and Dimargonas [17] studied the effect of cracks upon the dynamic behavior of cracked beams. The effect of peripheral cracks upon the torsional vibration of a rod of circular cross-section was studied by Dimargonas and Paipetis [16].

Recently, design for durability can be important reason for the long time material performance, thus for a rotational design it is necessary to quantify the damage tolerance of the cantilever plate. The assessment of this tolerance requires a capability to simulate the progressive damage and cracks characters of structures and loading. A different type of defects and typical notable in cantilever plates and the presence of these defects changes the dynamic characteristics of structure [18].

These changes can be used to identify the existence location and magnitude of effect damage, before they can grow to their critical size [17]. The present work will be concerned with the main macroscopic effects of cracks on cantilever plates, where such defects effects is reduction of stiffness. Where it is more deterministic than strength, such reduction can be related to physical damage, In present work the characteristics features of the stiffness reduction cantilever plate will be considered, and investigated under unique boundary condition, While the material properties presented by elastic module take as variable, It must be realized that the changes in deformations response are given by the total changes in stiffness matrix if the crack is presence and characterized.

## 2. Theory

In the present work, the finite element formulation for a superparametric shell element is introduced followed by the required procedure to evaluate the Eigen- Value problem, for calculating natural frequencies, the present proposed technique, Proceed for investigate the influence of cracks.

The present element is a modified form of shell element described by Ahmed [20] is selected. The super parametric shell element with eight nodes and five degree of freedom at each node is investigated. It is presented by Zienkiewicz [21]. Which the geometric shape functions are higher order than the displacement shape functions. The element is called a superparametric element. If the reverse the element is called subparametric.

$$[M]\{\ddot{U}\}+[K]\{U\}=0 \quad \dots(1)$$

$$U_i = \Phi_i \sin(\omega_i t + \theta_i) \quad i=1,2,\dots,\text{DOF} \quad \dots(2)$$

In this harmonic expression,  $\Phi_i$  is a vector of nodal amplitudes (mode shape) for the  $i$ th mode of vibration. The symbol  $\omega_i$  represents the angular frequency of mode  $i$ , and  $\theta_i$  denotes the phase angle. By differentiating Eq. (2) twice with respect to time  $t$  to get:

$$\ddot{U}_i = -\omega_i^2 \Phi_i \sin(\omega_i t + \theta_i) \quad \dots(3)$$

In which  $[A]$  is a symmetric matrix (dynamic matrix) and  $[I]$  is an identity matrix. The symbol  $\lambda_i$  denotes the  $i$ th eigenvalue. And  $XX_i$  is the corresponding eigenvector for a new system of homogeneous equations. Putting Eq. (4) into form of Eq. (5) by factoring either matrix  $[K]$  or matrix  $[M]$ , using the Cholesky square root

The superparametric shell elements are derived from the original three-dimensional isoparametric elements. The development and application of isoparametric family of elements is presented by Zienkiewicz [21].

### 2.1. Free Vibration Analysis

If any elastic structures are disturb in an appropriate manner initially at  $t=0$ , the structure can be made to oscillate harmonically. This oscillatory motion is a characteristic property of the structure and it depends on the distribution of mass and stiffness in the structure Rao [19].

The oscillatory motion occurs at certain frequencies known as natural frequencies or characteristic values, and it follows well define deformation patterns known as mode shapes or characteristic modes. The equation of motion by assuming the external force vector  $\{R\}$  to be zero; i.e. homogenous equation; and the displacements to be harmonic [22] so:

Substitution of Eq. (3) and Eq. (2) into Eq. (1) allows cancellation of the term  $\sin(\omega_i t + \theta_i)$ , which leaves,

$$([K] - \omega_i^2 [M])\Phi_i = 0 \quad \dots(4)$$

Eq. (4) has the form of the algebraic eigenvalue problem.

The most efficient type of Eq. (4) for structural vibrations accepts the eigenvalue problem only in the following standard, symmetric form:

$$([A] - \lambda_i [I])XX_i = 0 \quad \dots (5)$$

method (which is a direct method for solving a linear system which makes use of the fact that any square matrix  $[A]$  can be expressed as the product of an upper and lower triangular matrix, weaver [22] ).

The step solution to solve Eq. (4) by using this method (if the stiffness matrix is positive definite) was:

a- Choosing of factor [K] for an important reason that will soon be

$$[K]=[Q]^T [Q] \quad \dots(6)$$

Where the factor [Q] is an upper triangular matrix,

b- Substituting Eq. (6) into Eq. (4) to obtain:

$$([Q]^T [Q] - w_i^2 [M]) \Phi_i = 0 \quad \dots (7)$$

c- Pre-multiply this equation by  $[Q]^{-T}$  and insert  $I = [Q]^{-T} [Q]$  after matrix [M], which yields:

$$[Q]^{-T} ([Q]^T [Q] - w_i^2 [M]) \Phi_i = 0 \quad \dots (8)$$

d- Rearranging terms in reverse order, this found that:

apparent. Thus:

$$([M_A] - \lambda_i [I]) \Phi_{Ai} = 0 \quad \dots (9)$$

Where,

$$[M_A] = [Q]^{-T} [M] [Q]^{-1}$$

$$\text{and } \lambda_i = \frac{1}{\omega_i^2}; \quad \Phi_{Ai} = [Q] \Phi_i$$

...(10)

e- Determination the angular frequencies and mode shapes (in original coordinate):

$$w_i = \frac{1}{\sqrt{I_i}}; \quad \Phi_i = [Q]^{-1} \Phi_{Ai} \quad \dots (11)$$

Because the matrix  $M_A$  in the new coordinates is symmetric, all of its eigenvectors are linearly independent.

## 2.2. Crack Characteristics Modeling

Following the suggested procedure for the crack characterization of the damage mode and free vibration consideration, for simplicity in one off-exist plane, and the thickness of the cracking ( $t_1$ ), and ( $t$ ) is the total thickness of plate. Let ( $b$ ) is the width of a typical crack, which for a uniformly vibrated cantilever plate.

The study of cracked cantilever plate is based on the assumption that cracks can be modeled as a statically uniform array localized in a specified position at surface, in transverse direction. Usually these cracks almost instantaneously propagate through the thickness of the plate. Fig (1) describes schematically the geometrical dimension  $a$  of cracked damaged cantilever plate.

The damage vector in cantilever plate can be defined as a different mode for presenting the states of defects. In the present work the focusing study on the most common type, which and surface cracks, then the point of concentration subjected to the failure tolerance in working performance of such cantilever plates, so the attempts of the stiffness degrading quantification based on condition of unchanged geometry and boundary condition, and loading where the remain effective parameters still the elastic material properties (i. e.  $E, G, n$  ).

Firstly, it is important to focus the light on the stiffness-damage relation, the connected to dynamic load conditions.

Consider now load-time history, illustrated in Fig (2-a), which induced damage in given cantilever plate [17].

If the loading is interrupted at times ( $t_1, t_2, t_3, \dots etc$ ) and the cantilever plate is subjected to small, monotonically increasing stress with rotational speed increasing stresses at these times such that no additional

damage is induced under these stresses then we might obtain the stress- strain behavior illustrate in Fig. (2-b). Thus the changes in stiffness with time ( $t$ ) would reflect the development of damage under the applied load-time history.

Based on this fact the present study aims for investigation the reduction in modules of elasticity (both of axial, torsional modulus, at the Same time it is important to quantify this changes effects in material properties to decide the critical damage tolerances, then Therese assessment the cost-effective performance).

### 3. Results and Discussion

Several numerical investigations are conducted to characterize the dynamic detection of cracked cantilever plates under free vibration consideration, comparisons are made between the response of healthy cantilever plate and other cracked ones. Then a parametric study is the main purpose of using the developed computer program. Firstly results showed this numerical methodology technique a accurately predicts the undamaged reduction in natural frequencies, for comparison examples of detailed validation studies, then a various parameters effects being taken into consideration with a new developed dynamic stiffness degradation of cracked cantilever plate.

The present works were compared with experimental and theoretical results in Ref. [23] to find the natural frequencies. Table (1) explains the current results with experimental and theoretical results in Ref. [23], and the values of percentage error with experimental and theoretical results. Table (1) shows that the percentage errors between the current results with experimental results are less than the percentage error between the

experimental results and theoretical results in Ref. [23].

The calculating frequencies of cracked plated are proceed the proposed block diagram in Fig. (3), [24].

The following case will be used as verification case to the equations derived in this work. The results to be presented here obtained by using the following data [21]:

$$E=200 \text{ G N/m}^2, \quad r=7850 \text{ Kg/m}^3, \\ u = 0.3$$

Width of cantilever plate ( $b$ ) =0.025 m,  
thickness of the cantilever plate ( $t$ ) =0.011m

The numerical values of the fundamental natural frequencies for different length are shown in Table (1). The numerical results reported in Table (1) Show good agreement with the experimental and theoretical results in Ref. [23]. For this table, it can be observed that when length decreased, the percentage error increased especially with experimental result.

The second set of results will initiate the tendency of changes in the elastic parameters with the variation of crack location and relatively length. In Figs.(4),(5),(6),(7) the percentage loss in elastic modules with different location of crack ( $L1/L$ )at certain relatively crack length( $t1/t$ ), are seems logically changes, in dramatic form , where it is maximum near the fixed end , then decreases as far as the crack tends to be closer from the free end, these values starts with 5% approximatly closer to clamped edge , and tends to 1.5% at ( $L1/L$ ) =0.4, the same trend can be noticed from all mentioned figures but with different grades.

In the next demonstrated figures, are shows the inversely relatively crack length variation at different location. The Figs. (8), (9), (10) and (11), shows the percentage losses in elastic

modulus with quantification values. The distinguished changes from these figures indicates the increasing of these quantified losses values with increasing relatively crack depth it can be noticed clearly, and such behavior refer to the clear degradation in stiffness. The normalized elastic modulus changes with different crack length and location is shown clearly .The degree for stiffening the cracked cantilever plate can be related by recognized the normalized reduction degree for each location and relative length under one crack only.

Where the resonant frequencies are enforced in iterative process for determining the percent variation in elastic stiffness parameters, where the geometry and boundary conditions are considered fixed parameters under free vibration analysis. .The results from this mode analysis for cracked cantilever plate can be input initially for the iterative during natural frequency calculation where it is used as a designated point for reaching, when some elastic modules are changed.

Where the recalculation the normalized elastic modules ( $E_c/E_h$ ) ratios, the influence of stiffness losses degree on the axial and shear stiffness, are well documented to an accurate variation in elastic modules Figures.(12),through (15), shows the percent variation in axial and shear stiffness modules, ratios ( $E_c/E_h$ ) ,and ( $G_c/G_h$ ) for cracked cantilever plates having such dynamic characteristics , the clear percentage changes are clear starts at 5% for this case study as an example , tends to 2% nearly, free edges .The same note it can be distinguished from the reduction in axial/shear ratio variation with different cracks length or location.

Finally, the correlation between the present reduction, and stiffening in elastic modules ratio with

different crack length or location on dynamic characteristics detection signifies the importance of plate stiffness .And more detailed observations can be useful in the development of damage indices for durability detection based on vibration techniques

#### 4. Conclusion

The present work gives a simplified design methodology for determining the dynamics stiffness characteristics degradation based on the crack location and thickness variety. This design methodology is can be provide a design charts for any types of surface damages. The conclusions obtained from the present analysis can be

1. The crack position is very important and effective on the frequency parameters, the maximum effect of crack occur at the fixed end of cantilever plate and the minimum of crack occur at near of the free end.

2. The maximum value of relative frequency reduction is (4.85%) at  $[(t_1/t) = 0.3]$  and  $[(L_1/L) = 0]$ .

3. The maximum value of relative elastic modulus reduction and relative shear modulus reduction is (9.44%) at  $[(t_1/t) = 0.3]$  and  $[(L_1/L) = 0]$ .

summarized as follows:

**Table 1.** Values of the fundamental natural frequencies for different length of cantilever plate.

Length (m)	Present	Exp. Ref.[23]	Theo. Ref.[23]	Error %	
				Exp.	Theo.
0.0635	2268.35	1968.3	2282.20	15.44	0.60
0.0529	3268.49	2736.8	3286.30	19.42	0.54
0.0454	4437.58	3594.7	4473.10	23.44	0.79
0.0397	5803.33	4550.0	5842.40	27.54	0.66
0.0353	7340.22	5513.5	7394.30	33.13	0.73
0.03175	8546.00	6731.80	8358.70	25.61	1.16

**Table 2.** Relative elastic modulus reduction variation with varies ( $t_1/t$ ) and ( $L_1/L$ ).

		$L_1/L$				
		<b>0</b>	<b>0.1</b>	<b>0.2</b>	<b>0.3</b>	<b>0.4</b>
$t_1/t$	<b>0.02</b>	0.47	0.42	0.33	0.17	0.13
	<b>0.04</b>	0.83	0.67	0.53	0.30	0.20
	<b>0.06</b>	1.22	0.97	0.74	0.44	0.28
	<b>0.08</b>	1.65	1.28	0.94	0.60	0.36
	<b>0.10</b>	2.11	1.62	1.17	0.75	0.45
	<b>0.12</b>	2.59	1.98	1.45	0.93	0.56
	<b>0.14</b>	3.14	2.38	1.72	1.12	0.67
	<b>0.16</b>	3.72	2.80	2.08	1.31	0.81
	<b>0.18</b>	4.35	3.27	2.38	1.54	0.92
	<b>0.20</b>	5.02	3.78	2.75	1.78	1.07
	<b>0.22</b>	5.77	4.34	3.15	2.05	1.22
	<b>0.24</b>	6.77	4.95	3.58	2.34	1.42
	<b>0.26</b>	7.45	5.60	4.08	2.67	1.61
	<b>0.28</b>	8.40	6.33	4.60	3.04	1.81
<b>0.30</b>	9.44	7.15	5.17	3.44	2.07	

**Table 3.** Relative frequency reduction variation with varies ( $t_1/t$ ) and ( $L1/L$ ).

		$L1/L$				
		<b>0</b>	<b>0.1</b>	<b>0.2</b>	<b>0.3</b>	<b>0.4</b>
$t_1/t$	<b>0.02</b>	0.25	0.22	0.18	0.10	0.08
	<b>0.04</b>	0.43	0.35	0.27	0.17	0.11
	<b>0.06</b>	0.63	0.50	0.38	0.24	0.15
	<b>0.08</b>	0.84	0.66	0.49	0.31	0.20
	<b>0.10</b>	1.07	0.83	0.61	0.39	0.24
	<b>0.12</b>	1.32	1.01	0.74	0.48	0.29
	<b>0.14</b>	1.59	1.21	0.88	0.57	0.35
	<b>0.16</b>	1.89	1.43	1.04	0.67	0.41
	<b>0.18</b>	2.21	1.67	1.21	0.79	0.48
	<b>0.20</b>	2.56	1.92	1.39	0.91	0.55
	<b>0.22</b>	2.94	2.21	1.60	1.05	0.63
	<b>0.24</b>	3.35	2.52	1.82	1.19	0.72
	<b>0.26</b>	3.81	2.86	2.06	1.36	0.82
	<b>0.28</b>	4.31	3.23	2.33	1.54	0.93
	<b>0.30</b>	4.85	3.65	2.63	1.74	1.05

**Table 4.** Relative shear modulus reduction variation with varies ( $t_1/t$ ) and ( $L1/L$ ).

		$L1/L$				
		<b>0</b>	<b>0.1</b>	<b>0.2</b>	<b>0.3</b>	<b>0.4</b>
$t_1/t$	<b>0.02</b>	0.47	0.42	0.34	0.17	0.13
	<b>0.04</b>	0.83	0.68	0.52	0.30	0.20
	<b>0.06</b>	1.22	0.96	0.74	0.44	0.29
	<b>0.08</b>	1.65	1.27	0.94	0.60	0.36
	<b>0.10</b>	2.11	1.63	1.17	0.75	0.46
	<b>0.12</b>	2.59	1.99	1.46	0.94	0.56
	<b>0.14</b>	3.15	2.38	1.73	1.12	0.68
	<b>0.16</b>	3.73	2.80	2.08	1.31	0.83
	<b>0.18</b>	4.36	3.28	2.38	1.55	0.91
	<b>0.20</b>	5.02	3.77	2.76	1.78	1.07
	<b>0.22</b>	5.77	4.34	3.15	2.05	1.22
	<b>0.24</b>	6.57	4.95	3.58	2.34	1.42
	<b>0.26</b>	7.45	5.60	4.08	2.68	1.61
	<b>0.28</b>	8.40	6.33	4.60	3.04	1.82
	<b>0.30</b>	9.45	7.15	5.17	3.45	2.07

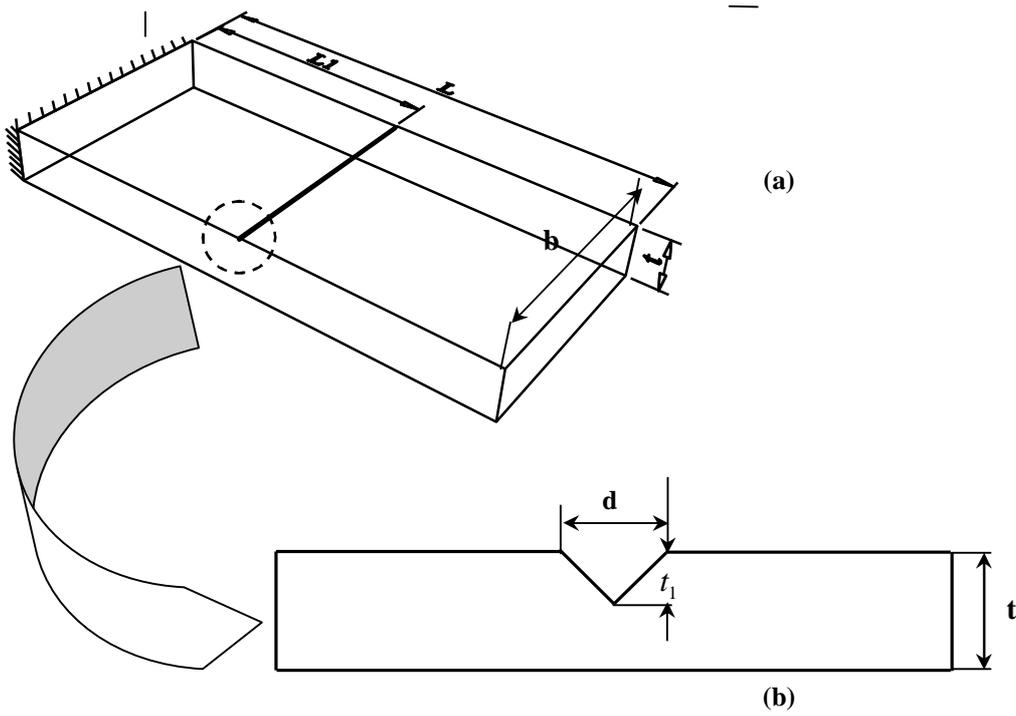


Fig. 1

- (a) The cracked cantilever plate.
- (b) The section of side view of crack.

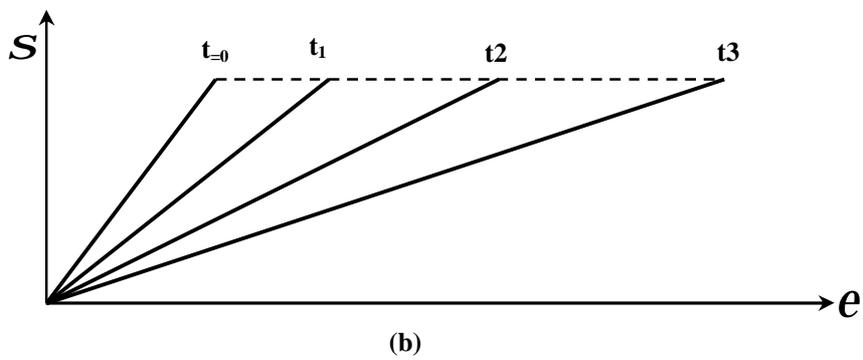
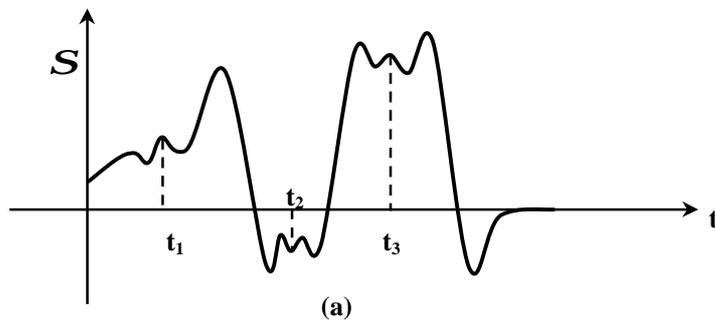


Fig. 2

- (a) A load-time history
- (b) Stress-strain response at various times in load-time history

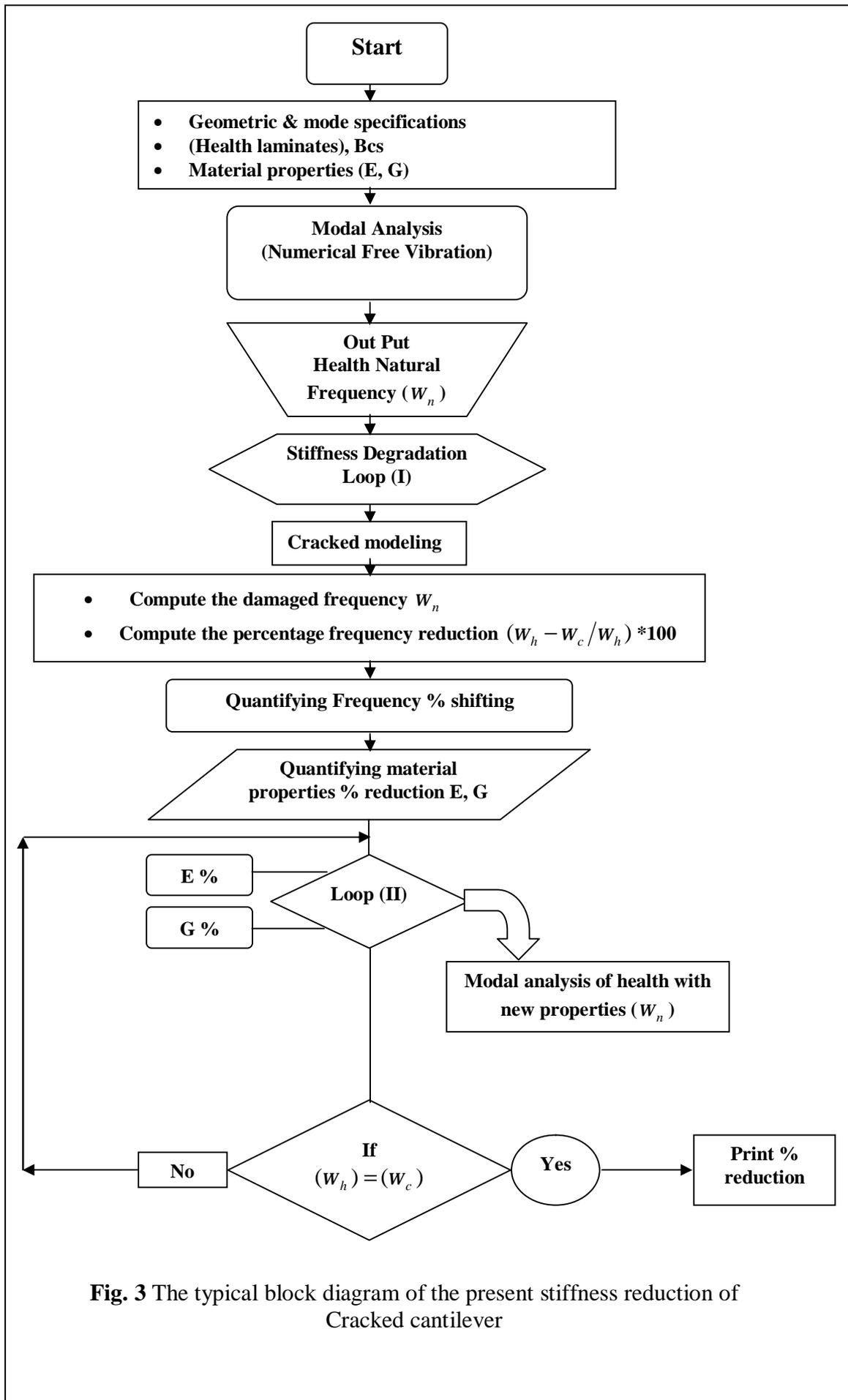


Fig. 3 The typical block diagram of the present stiffness reduction of Cracked cantilever

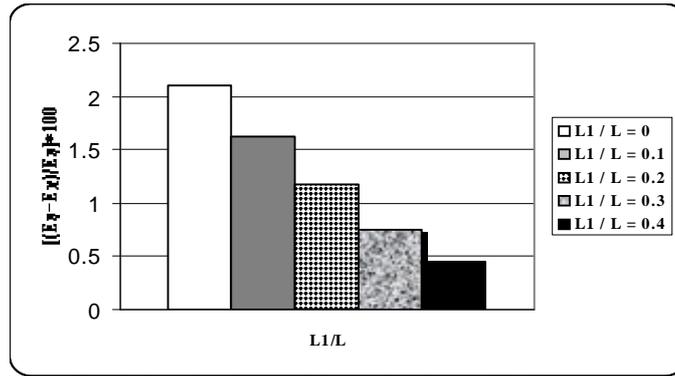


Fig. 4 Variation of Relative elastic modulus reduction with different ( $L1/L$ ) at ( $t_1/t=0.1$ )

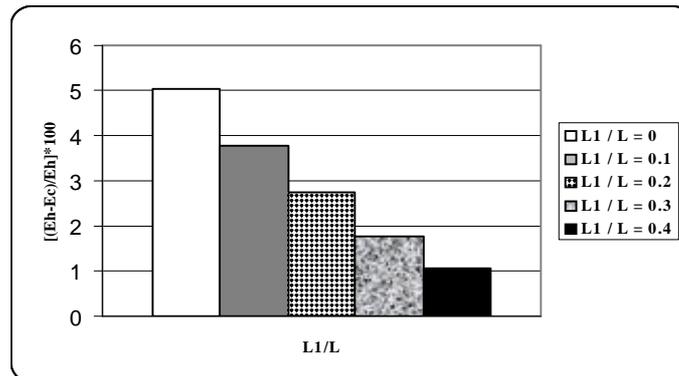


Fig. 5 Variation of Relative elastic modulus reduction with different ( $L1/L$ ) at ( $t_1/t=0.2$ )

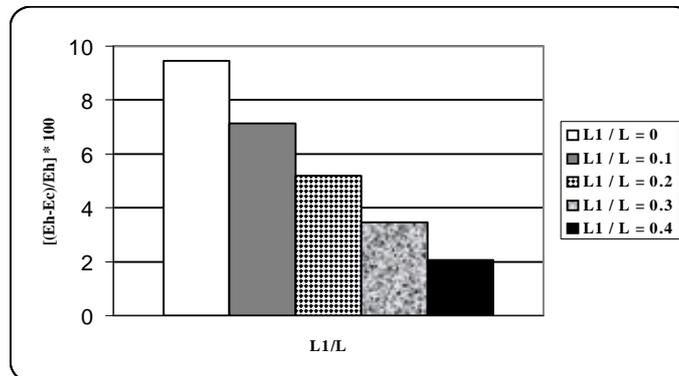


Fig. 6 Variation of Relative elastic modulus reduction with different ( $L1/L$ ) at ( $t_1/t=0.3$ )

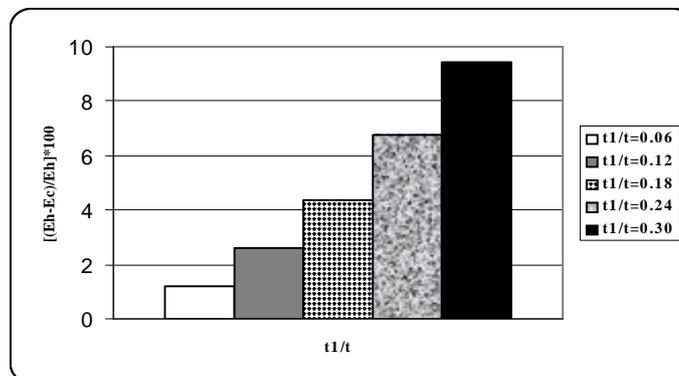


Fig. 7 Variation of Relative elastic modulus reduction with different ( $t_1/t$ ) at ( $L1/L=0$ )

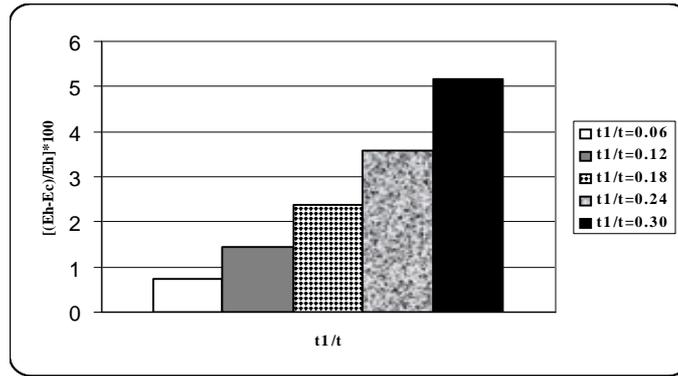


Fig. 8 Variation of Relative elastic modulus reduction with different ( $t1/t$ ) at ( $L1/L=0.2$ )

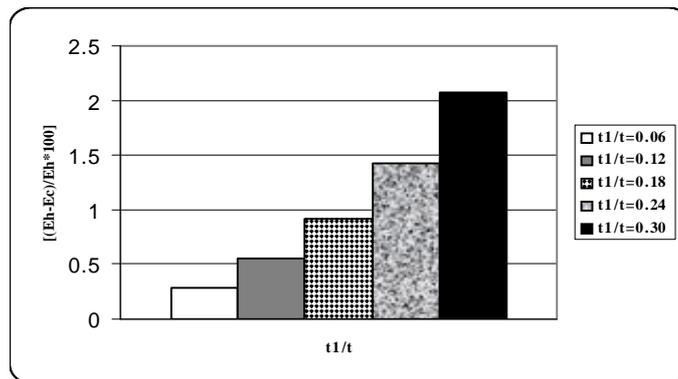


Fig. 9 Variation of Relative elastic modulus reduction with different ( $t1/t$ ) at ( $L1/L=0.4$ )

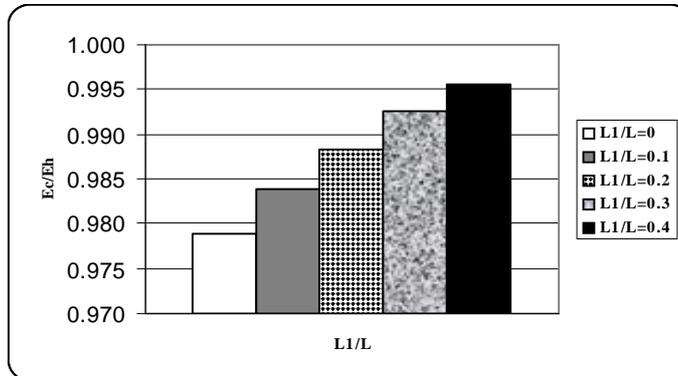


Fig. 10 Variation of Relative elastic modulus ratio with different ( $L1/L$ ) at ( $t1/t=0.1$ )

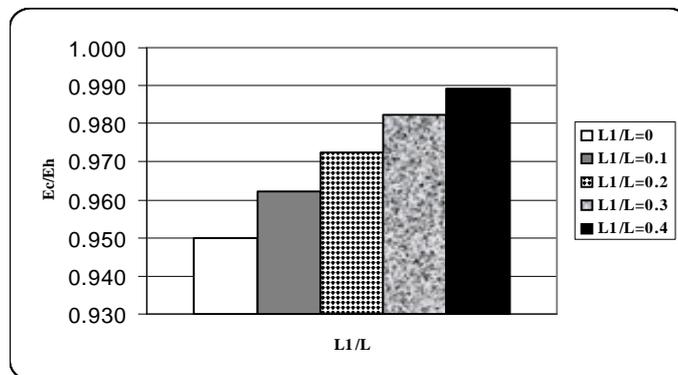


Fig. 11 Variation of Relative elastic modulus ratio with different ( $L1/L$ ) at ( $t1/t=0.2$ )

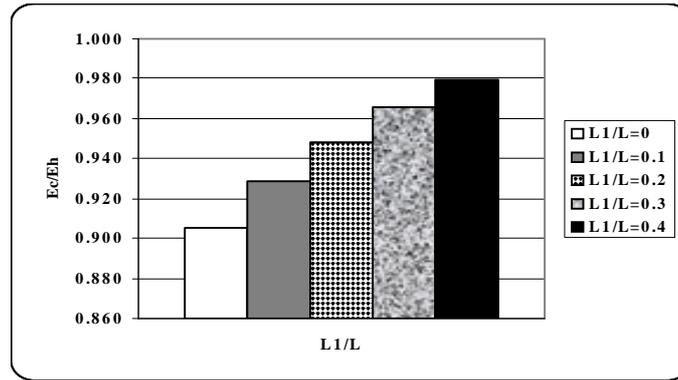


Fig. 12 Variation of Relative elastic modulus ratio with different (L1/L) at (t1/t=0.3)

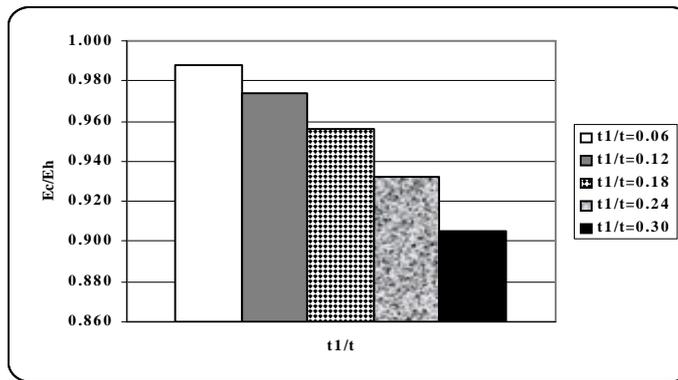


Fig. 13 Variation of Relative elastic modulus ratio with different (t1/t) at (L1/L=0)

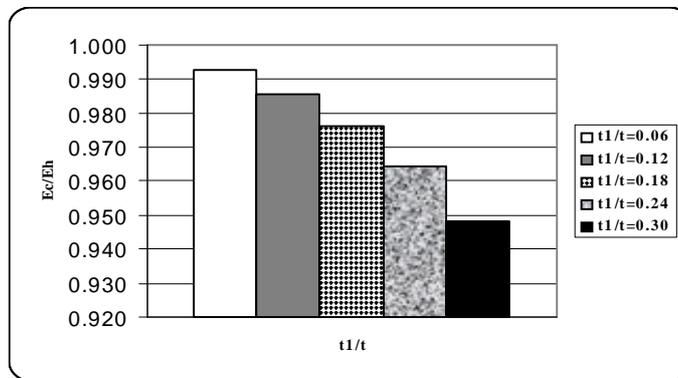


Fig. 14 Variation of Relative elastic modulus ratio with different (t1/t) at (L1/L=0.2)

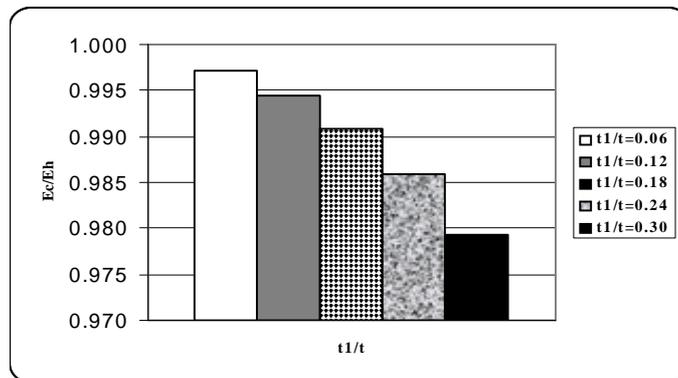


Fig. 15 Variation of Relative elastic modulus ratio with different (t1/t) at (L1/L=0.4)

## 5. References

1. Andrew .M. Erian . A, " influence of damage on the elastic behavior of beams", Inter. Journal Americal helicopter sol. 55 th, Amual Forum ,May 1999.
2. Int. Jouml Structunal engineering and Mechanics Vol. (14), No.3(2002)245-202
- 3.W. Carnegie, "Vibrations of rotating cantilever blading" , theoretical approaches to the frequency problem based on energy methods. J. Mech Sci. 1, 235-240 (1959).
- 4.W. Carnegie, C. stirling and J. Fleming, "Vibration characteristics of turbine blading under rotation results of an initial investigation and details of high speed test installation. Proc. Inst. Mech. Engrs 180, 1-9 (1966)..
- 5.W. Carnegie and J. S. Rao, "Effect of pretwist and rotation on flexural vibrations of cantilever beams treated by the extended holzer method" Bull. Mech. Engng Education 10, 1971.
- 6.R. M. Krupka and A. M. Baumanis, " Bending-bending mode of a rotating tapered twisted turbomachine blade including rotary inertia and shear deflection" , J. Engng. Ind., ASME 91, 1017 (1969).
- 7.R. O. Stfford and V. Giurgiutiu, " Semi-analytic methods for rotating Timoshenko beams". Int. J. Mech. Sci. 17, 719-727 (1975).
- 8.B. A. H Abbas, " Simple finite elements for dynamic analysis of thick pre-twisted blades ". Aeronaut. Jnl, 450-453 (1979).
- 9.J. Thomas and B. A. H. Abbas," Finite element model for dynamic analysis of Timosheno beam". J. Sound Vibr. 41, 291-299 (1975).
- 10.B. A. H. Abbas, " Dynamic analysis of thick rotating blade with flexible roots". Aeronaut. Jnl, 10-16 (1985).
- 11.H. Liebowiz, H. Venderveldt and D. W. Harris, "Carrying capacity of notched column". Int. J. Solids Struct. 3, 489-500 (1967).
- 12.H. Liebowiz and W. D. S. Claus, Jr, "Failure of notched column", Engng Fract. Mech. 1, 379-383 (1969).
- 13.H. Okamura et al. , "Acracked column under compression", Engng Fract. Mech. 1, 547 (1969).
- 14.J. R. Rice and N. Levy, "The part-through surface crack in an elastic plate", J. appl. Mech. 185 (1975).
- 15.A. D. Dimargonas, "Vibrations Engineering", West, St. Paul (1976).
- 16.A. D. Dimargonasand S. A. Paipetis, "Analytical Methods in Rotor Dynamics". Applied Science, Barking, U. K. (1983).
- 17.T. G. Chondros and A. D. Dimargonasand, "Identification of cracks in welded joints of complex structures", J. Sound Vibr. 69, 531 (1980).
- 18.Ramesh T, "Stiffness properties of composites", J. Int. J. of Fracture Mechanics, Vol(25) 12, pp (751), (1986).
- 19.Rao S. S. "Finite Element Method in Engineering", Pergamon Press, Oxford, 1982.
- 20.Ahmad S, Irons B., Zienkiewicz O. C. "Analysis of Thick and Thin Shell Structures by Curved Elements", I. J. For Numerical Methods in Eng., Vol. 2, PP. 419-451, 1989.
- 21.O. C. Zienkiewicz, "The finite element Method", 3rd Ed., Mc Graw-Hill Book Company (UK) Limited, London, 1977.
- 22.William Weaver and Paul. R. Johnston, "Structural Dynamic by Finite Elements", Standford University, Pretice-Hall Inc., N. J., 1987.
- 23.B. A. H. Abbas and H. Irretiev, "Experimental and theoretical investigation of the effect of root flexibility on the vibration characteristics beam", J. of sound and Vibration, Vol.130, No. 3, PP. 353-362, (1989).
24. Adnan. N. J. Muhsin J. Jweeg, Hussain A. D,"determination of

dynamic characteristics, and inter laminar shear stress of composite plate under various loading", Phd. Theses

Uni. Of Baghdad, College of Engg. Mech. Dept, 2004.

## 6. Nomenclature:

$t$ : Thickness of the cantilever plate (m)

$t_1$ : Thickness of the crack (m)

$L_1$ : The location of the crack (m)

$L$ : The length of the cantilever plate (m)

$b$ : The width of the cantilever plate (m)

$E_h$ : Health modulus of elasticity  $N/m^2$

$E_c$ : Cracked modulus of elasticity  $N/m^2$

$G_h$ : Health modulus of rigidity  $N/m^2$

$G_c$ : Cracked modulus of rigidity

$[(E - E_c)/E_h]*100$ : Relative elastic modulus reduction

$(E_c/E_h)$ : Relative elastic modulus ratio

$(G_c/G_h)$ : Relative shear modulus ratio

$[(G_h - G_c)/G_h]*100$ : Relative shear modulus reduction

$[(w_h - w_c)/w_h]*100$ : Relative frequency reduction

$d=0.05$  mm

## تحليل الاهتزاز الحر لتكميم الجساءة لعتبة كابولية

عدي ابراهيم عبد الله

جامعة بغداد / كلية الهندسة

قسم الهندسة النووية

د.حسين علي داود

جامعة بغداد / كلية الهندسة

قسم الهندسة الميكانيكية

### الخلاصة:

تم في هذا العمل تقديم تقنية عديدة لتحليل والتوصيف الديناميكي للانحدار في قيم الجساءة المرنة في عتبة كابولية تعاني عيبا "دخليا" مثل التشقق. التقنية العامة المقدمة تمثل الخطوة الأولى في منظومة عمل سلامة الهياكل وتشخيصها. تأثير وجود هذه العيوب والتشوهات على قيم الترددات الطبيعية مثل نقطة تقويم و تكميم مستويات الانحدار في قيم ثابت المرونة الهندسي لهذه الهياكل بوجود عيوب غير ظاهرة. في هذا البحث تم استخدام تقنية العناصر المحددة لإيجاد المواصفات الاهتزازية الحرة لعتبة كابولية تعاني عيبا "دخليا" مثل التشقق وبوجود شقوق متغيرة الموقع. أن قيم الانحدار في الجساءة بدلالة الثوابت الهندسية المرنة ثم تحليل وفق النتائج التي تم الحصول عليها وكذلك تقديم دراسة معلمية بدلالة معاملات تصميمية أساسية. وبالاعتماد على قيم معامل المرونة و الجساءة الهندسية نسبة إلى المواصفات الاهتزازية.