



Time Prediction of Dynamic Behavior of Glass Fiber Reinforced Polyester Composites Subjected to Fluctuating Varied Temperatures

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Abstract

The reduction of vibration properties for composite material (woven roving E-glass fiber plies in thermosetting polyester matrix) is investigated at the prediction time under varied combined temperatures (60°C to -15°C) using three types of boundary conditions like (CFCF, CCCF, and CFCC). The vibration properties are the amplitude, natural frequency, dynamic elastic moduli (young modulus in x, y directions and shear modulus in 1, 2 plane) and damping factor. The natural frequency of a system is a function of its elastic properties, dimensions, and mass. The woven roving glass fiber has been especially engineered for polymer reinforcement; but the unsaturated thermosetting polyester is widely used, offering a good balance of vibration properties at moderate or ambient temperatures, and also at relatively low cost. The mismatch between matrix and fiber yarns gives a predominant role for the fiber's mechanics where the matrix is the area where most damage mechanisms develop. The free vibration test was carried out for (5, 10, 15, 20, 25, 30) minutes. The composite plate was exposed to (75°C) of thermal gradient for ten times in various times at different stages. The results were classified into experimental and finite element using software ANSYS Ver. 9.

Keywords: Time prediction, composite material, varied temperatures

1. Introduction

Composites are used almost in all engineering fields especially in aerospace and defense industries these days and their applications are growing very rapidly. The simulation of a reduction in the fundamental frequency and of damping with an increase in the percentage of uniform moisture and temperature (hygro-thermal effect) at (20°C and 80°C) are accomplished for material properties (extensional and flexural stiffness) of graphite/ epoxy composite beams, plates, and shells with respect to ply angles, fiber volume ratio, and temperature using finite element discretization, [1, 2, 3]. Random response in using shape memory alloy hybrid composite can be reduced greatly when the plates are subjected to a steady state constant temperature and uniform sound pressure with the aiding of nonlinear finite element model, [4]. The dynamic load factor of simply supported glass/epoxy composite beams

increase due to moving loads (forced vibration) and hygro-thermal effects because of degrading the material stiffness properties of the matrix by varying the fiber volume fraction and the fiber orientation of the angle plies in the laminate, [5]. A damping monitoring method is used to measure, calculate, and investigate the natural frequency, elastic modulus in (x, y) directions and the damping factor for polyethylene fiber composite beam under varied temperature (-10°C to 60°C) for free-free boundary conditions, but both natural frequency and elastic modulus decreased with the increase of temperature adversely for damping factor, [6]. A shooting method is applied numerically to calculate the first three frequencies of orthotropic circular plate due to uniform temperature rise and found that the three lowest frequencies of the buckled plate decrease with an increase in the temperature using clamped or simply supported boundary conditions moving in the radial direction, [7]. On the other

hand, ultrasonic oscillator technique was used to generate ultrasonic stress waves at 80 and 150 kHz (forced frequency) to measure the mechanical damping (in longitudinal vibration mode) and temperature dependence of dynamic modulus in the longitudinal, transverse fiber direction. Ramadan J. Mustafa [8] suggested that the dynamic modulus decreases with the increase of temperature from room temperature to 450 °C. The present investigation aim is to predict the perfect time that participates in the reduction of natural frequency, dynamic elastic moduli and in the increase of the damping factor under varied temperatures.

2. Experimental Setup and Procedure

2.1 Composite Laminate Fabrication

Fabrication of composite laminates was conducted in a mold consisting of (24 cm * 24 cm) aluminum with two X-ray photo sheets to avoid abrasive and to insure the flattening of the specimen surface. The X-ray photo sheet, which was first placed on the bottom of the aluminum mold, was wet with the catalyzed polyester resin before the first ply was placed on it. More catalyzed resin was applied to the first ply with brush until it was thoroughly wet. Following this, the remaining plies were placed in the mold following the same sequence. The mold left for one day with sufficient pressure ($2604.1667 \frac{N}{m^2}$) to get rid of the excess resin and entrapped air bubbles and to remove the composite plate from the mold. The assembly was heated to (70 °C) in an oven at (3 hours) curing time to complete cross-linking. The fiber volume fraction was determined for the glass fiber composites from the following relationship, [9]:

$$v_f = \frac{1}{1 + \frac{1 - \varphi}{\varphi} * \frac{\rho_f}{\rho_m}} \quad \dots(1)$$

$$\varphi = \frac{m_f}{m_c} \quad \dots(2)$$

2.2 Vibration Measurement

The fundamental natural frequency for the first mode can be estimated by using the rubber tip hammer test; but it can be calculated before and after exposure to combined temperatures to show

how they have affect on the composite material free vibration property reduction. There are many parts that are needed in this test are as follows:

- (1) Frame Fixture.
- (2) The rubber tip hammer.
- (3) WINSCOPE Programming.
- (4) Wave Transporter.

The plate has the dimensions (a = 0.3 m, b = 0.24 m, h = 0.0016 m, $\nu = 25.076 \text{ Vol.}\%$). The specimen was treated in a thermal box environment at (60 °C) temperature for (5 minutes) and then was immediately exposed at (-15 °C) temperature of the same period of time for one cycle of ten cycles [10]. The heating temperature (60 °C) was obtained using a heater (400W) made from ceramic material. The heater was in contact with electrical contactor and mesh with thermal controller, adversely the cooling temperature (-15 °C) obtained from use the refrigerant ferion gas (R-134a) flows from solenoid valve. Solenoid valve contacts with gas bottle and when temperature reaches above (-15 °C) the solenoid gate valve must be opened to compensate gas flow to the plate; but if the plate reaches the steady state setting temperature (-15 °C) the solenoid gate valve closes.

After applying fluctuating temperatures, the elastic moduli have been changed due to these loadings causing a change in bending stiffness material and that affects the values of vibration properties for different conditioning time intervals. A free vibration test has been achieved with inconstant impact load which is applied to induce vibration using a spherical rubber ball hammer. To measure the amplitude's magnitudes, wave transporters (one kind of many accelerometers kinds) are placed on the mid-point of the specimen to convert the vibration response to the computer using linking wire. A WINSCOPE programming with a bandwidth limitation between (20 Hz to 20 KHz) that picks up or records the frequency wave of the composite plate and gives all the vibration properties was needed. The operation sequence of the present paper can be illustrated in Fig. (1a, b).

After measuring the amplitudes of two successive sinusoidal waves and damped natural frequency from WINSCOPE Programming, the value of natural frequency can be found from the following relationships, [11]:

$$\delta = \ln \frac{x_1}{x_2} = \frac{2 * \pi * \xi}{\sqrt{1 - \xi^2}} \quad \dots(3)$$

$$\omega_d = \omega_n * \sqrt{1 - \xi^2} \quad \dots(4)$$

$$p.o. = 100 * e^{\frac{-\xi * \pi}{\sqrt{1 - \xi^2}}} \quad \dots (5)$$

Fig.2 shows the sequence of the edged boundary conditions and can be named as:
 CFCF which means: edge (3) fixed, edge (2) free, edge (1) fixed, edge (4) free.
 CCCF which means: edge (3) fixed, edge (2) fixed, edge (1) fixed, edge (4) free.
 CFCC which means: edge (3) fixed, edge (2) free, edge (1) fixed, edge (4) fixed.

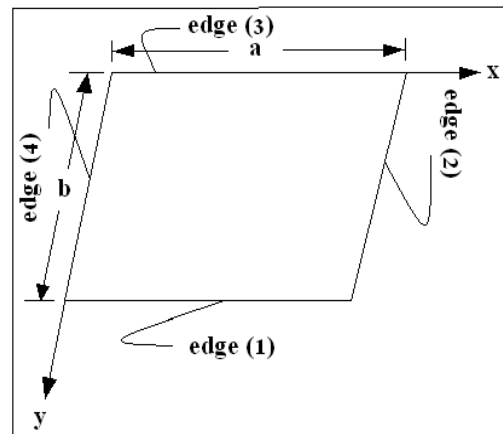


Fig.2. Sequence Edges of Boundary Conditions.

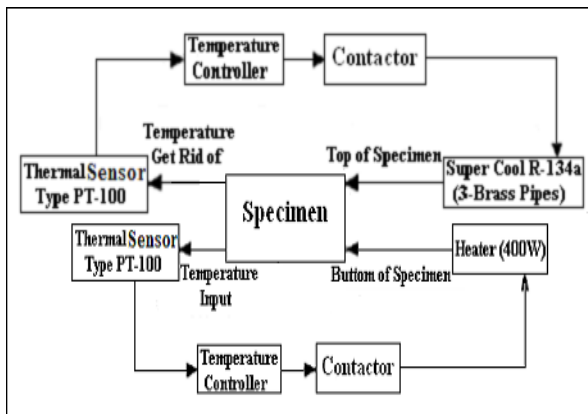


Fig.1a. Block Diagram of Combined Temperature Loading Sequence Operation.

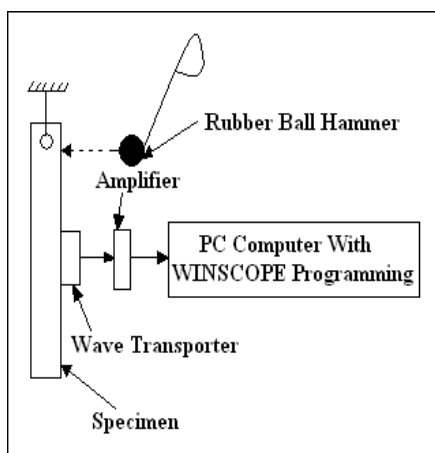


Fig.1b. Measurement of Natural Frequency Due to Free Vibration Test.

2.3 Dynamic Elastic Moduli Measurement

The tension moduli, for the E-glass and polyester matrix were determined by taking the average Young's Modulus of three specimens with loading (10 KN). The machine that made a tensile and compression testing device was of type INSTRON, model 1195, at a speed of (1mm/min). The test specimens of the composites were prepared according to the (ASTM D 3039) standard. The INSTRON'S plotter draw the load-deflection curves. The stress can be calculated from the equation ($\sigma = \frac{P}{A}$), where (P) and (A) represent the load taken from the curve, and the instantaneous cross sectional area respectively. Before and after applying combined loading temperatures, the elastic moduli have changed. As an example, when the composite specimen is exposed to heating (60 °C) for (5 minutes) and they immediately to cooling (-15 °C) for (5 minutes) with ten times repeated of cycle number one alone then the plate can be cut into tensile ASTM dimensions and begin the tension test with INSTRON plotter device. This operation is repeated each time for (10, 15, 20, 25, and 30) minutes to obtain the dynamic elastic moduli. The shear modulus for composite plate can be found from the following formula, [12]:

$$G_{12} = \frac{1}{\left(\frac{4}{E_x} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2 * \nu_{12}}{E_1}\right)} \quad \dots(6)$$

The specimen geometry used for both tension and shear moduli test can be shown in Fig. 3.

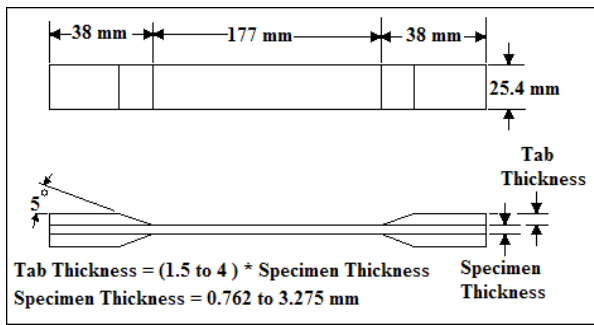


Fig. 3. Tensile Test Specimen.

3. Numerical Procedure

Table 1,
 Mechanical Property of Composite Laminate before Combined Temperatures Effect.

E_x^c (GPa.)	19.933
E_y^c (GPa.)	19.933
E_z^c (GPa.)	3.0896
ν_{12}	0.3835
ν_{23}	0.32
ν_{13}	0.32
G_{12} (GPa.)	1.07675
G_{23} (GPa.)	0.681
G_{13} (GPa.)	0.681
ρ_c ($\frac{kg}{m^3}$)	1464.18

Modal analysis is carried out with ANSYS Ver. 9 finite element software. The linear elastic orthotropic model is used to investigate the cloth composite laminated plate. For this problem, the (SOLID45) element is used. This element is used for the three-dimensional modeling of solid structure and is defined by eight nodes. It has three degrees of freedom at each node: translations in the nodal x, y, and z directions. Mechanical properties of the composite plate are taken from Table 1. To determine the natural frequencies, modal analysis is carried out using the subspace method with ANSYS in CFCF, CCCF, and CFCC boundary conditions. After load applying, the mechanical properties varied

Table 3,
 Experimental and Numerical Comparison before Temperatures Effect.

Boundary Conditions	Fundamental Natural Frequency (Hz)		
	Experimental Work	SYSNA	Percentage Error
CFCF	64.9917	69.914	7.0405%
CCCF	65.4616	71.63	8.6114%
CFCC	64.9918	71.543	9.15701%

with each time with intervals as mentioned above. From the experimental work that can be taken the change in elastic mechanical properties due to temperature loadings effect might be entered instead of the mechanical properties before this applying the loadings listed in Table 1 for the same ANSYS program, and that gives different values for natural frequency according to its conditioning time. The same ANSYS program is repeated each time for (5, 10, 15, 20, 25, and 30) minutes because the elastic moduli have been changed.

4. Results and Discussion

In general the logarithmic decrement, damping factor, and damped forced frequency are given in Table 2 using CFCF, CCCF, and CFCC boundary conditions. In Table 2 the damped forced frequency is the same for all boundary conditions because the damping factor is nearly constant. The constant damping factor refers to the stiffness and mass properties of composite plate before applying temperature loadings and that does not change with the variation of boundary conditions because the inverse of critical damping coefficient is still constant.

Table 2,
 Vibration Parameters before Combined Temperatures Effects.

Boundary Conditions	δ	ξ	ω_d (Hz)
CFCF	- 0.6931	0.10963	64.6
CCCF	1.0296	0.1617	64.6
CFCC	0.6931	0.1096	64.6

Table 3 gives the comparison between modal experimental and numerical analysis before temperatures effect using different boundary conditions. It can be noticed that the clamed boundary conditions from all plate edges are not take in our account because there is something wrong in the nature of fixture. One-third of the plate length must be fixed with bolts from every directions.

Table 4 shows the variation of dynamic mechanical properties as a result of temperatures effect with different conditioning time. All properties varied sinusoid ally. It can be noticed that the Young modulus in (x, y) directions is the same because the distance between fiber yarn is the same in (x, y) directions, but the perpendicular

effect gives a different Young modulus in z-direction. The shear modulus in 23, 13 planes is the same because the thickness along z-direction for all points of the specimen is the same; adversely, the shear modulus in 12 plane is different since the dimension in (x, y) directions is larger than the dimension in z-direction.

**Table 4,
Dynamic Mechanical Property of Composite Laminate After Combined Temperatures Effect.**

Time (min)	5	10	15	20	25	30
E_x^c (GPa.)	19.363	21.41	17.434	17.76	20.102	23.548
E_y^c (GPa.)	19.363	21.41	17.434	17.76	20.102	23.548
E_z^c (GPa.)	3.001	3.31	2.702	2.752	3.115	3.649
ν_{12}	0.372	0.411	0.335	0.341	0.386	0.453
ν_{23}	0.31	0.343	0.279	0.285	0.322	0.378
ν_{13}	0.31	0.343	0.279	0.285	0.322	0.378
G_{12} (GPa.)	1.045	1.156	0.941	0.959	1.085	1.272
G_{23} (GPa.)	0.661	0.731	0.595	0.606	0.686	0.804
G_{13} (GPa.)	0.661	0.731	0.595	0.606	0.686	0.804

**Table 5,
Experimental and Numerical Comparison after Temperatures Effect.**

Fundamental Natural Frequency (Hz)						
Boundary Conditions	CFCF			CCCC		
Time (min)	Experimental	ANSYS	Error	Experimental	ANSYS	Error
5	67.914	73.386	7.4%	69.581	74.885	7.1%
10	75.094	77.557	3.1%	76.937	79.131	2.7%
15	61.148	69.336	11.8%	62.649	70.756	11%
20	62.292	70.03	11%	63.821	71.464	10%
25	70.506	74.904	5.8%	72.237	76.432	5.4%
30	82.593	81.809	0.94%	84.62	83.451	1.3%

Table 5, Continue

Fundamental Natural Frequency (Hz)			
Boundary Conditions	CFCC		
Time (min)	Experimental	ANSYS	Error
5	69.497	74.885	7.1%
10	76.844	79.131	2.7%
15	62.573	70.756	11%
20	63.743	71.464	10%
25	72.149	76.432	5.4%
30	84.517	83.451	1.3%

Table 5 gives the experimental and numerical comparison of fundamental natural frequency after temperatures effect using different boundary conditions. At time of (t = 15min) both experimental and numerical procedures give the

lowest value of frequency because this affords a lower value of amplitude. In ANSYS program the CCCC and CFCC boundary conditions give the same value of frequency because the shear coupling effect boundary conditions is symmetric

about y-axis. All ANSYS values are different because ANSYS is an approximate solution. The error between experimental and numerical value of fundamental natural frequency is too high at time of (t = 15 min) because at this time the elastic moduli are small and the frequency is a function of elastic moduli. The denominator of percentage error law for fundamental natural frequency is quite little; but the difference between experimental and numerical values is high and this gives a large error value of percentage error. Also in Table 5, the frequency is

not strange because these values vary with time harmonically and with sinusoid ally according to a fluctuating varied temperature.

Table 6 shows the comparison of maximum percent overshoot of composite plate before and after temperature effect varies with different boundary condition. The plate has minimum percent overshoot which reaches the steady state faster than the other plates because it has minimum values of both rise time and settling time. The increasing of clamped edges gives decreasing in maximum percent overshoot.

Table 6,
Maximum Percent Overshoot Before and After Temperatures Effect.

Boundary Conditions	Before Temperatures Effect	After Temperatures Effect
CFCF	70.715%	27.2922%
CCCF	51.476%	22.205%
CFCC	70.722%	22.49%

A structural dynamic problem differs from its static loading counter part in two aspects: first, the magnification factor (dynamic vibration resistance amplitude factor) is applied as a function of time or frequency or both of them. Second, these time or frequency varying load applications induced time or frequency. As mentioned previously the Magnification factor varies sinusoid ally with time for CFCF boundary condition as given in Table 7 and the reduction in magnification factor makes the system more stable with small overshoot peak. Time (t = 15 min) chosen for this operation is because this time gives reduction in vibration resistance amplitude factor while it decrease the fundamental natural frequency.

temperatures effects vary with time for CFCF boundary condition. The two successive amplitudes to obtain the logarithmic decrement can be taken. Time chosen for this comparison is 15 min because this time gives a large reduction in both amplitude and natural frequency values than the others.

Table 7,
Magnification Factor Varies With Time After Temperatures Effect.

Time (min)	Magnification Factor
5	0.971
10	1.074
15	0.874
20	0.891
25	1.008
30	1.181

The relationship between amplitude and impact time before and after temperatures effect for (5, 10, 15, 20, 25 and 30) can be found to choose the best response time. Fig. 4 shows a comparison of the amplitudes before and after

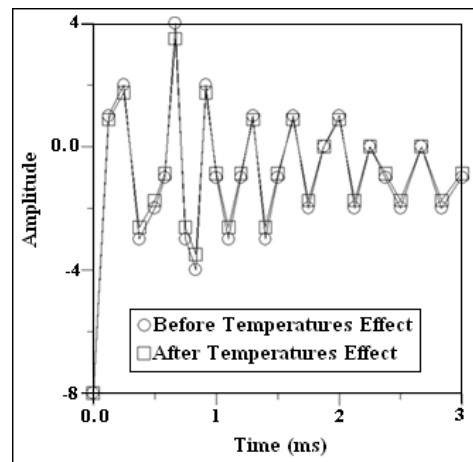


Fig.4. Amplitude vs. Time before and after Temperatures Effect.

The dangerous natural frequency at first mode has occurred and lead to resonance. Fig. 5 illustrates the comparison of the amplitudes before and after temperatures effects vary with damped forced frequency for CFCF boundary condition. The time chosen for this comparison is 15 min because this time gives a large reduction in both amplitude and damped forced frequency

values than the others time. After temperature effect the peak's amplitude is reduced with the same values above and below the vibration centerline until approaches the steady-state case of both time and damped forced frequency as shown in Figs. 4,5.

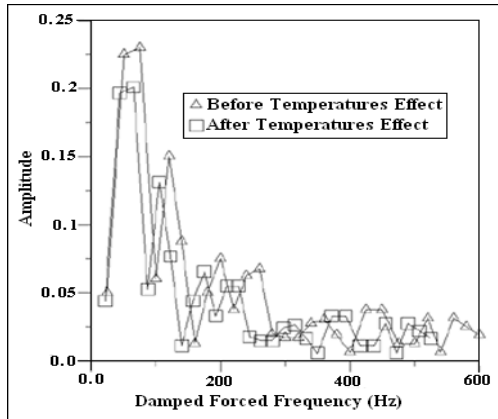


Fig.5. Amplitude vs. Damped Forced Frequency before and after Temperatures Effect.

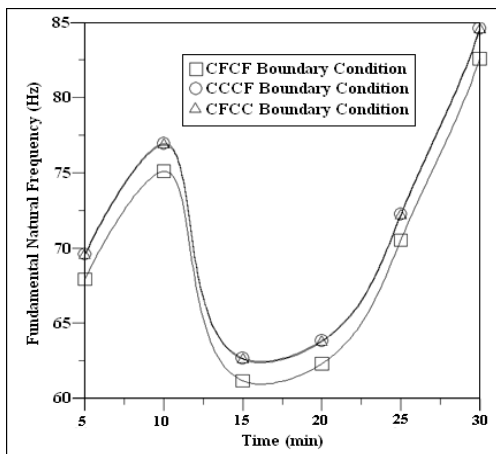


Fig. 6. Fundamental Natural Frequency vs. time after Combined Temperatures Effect.

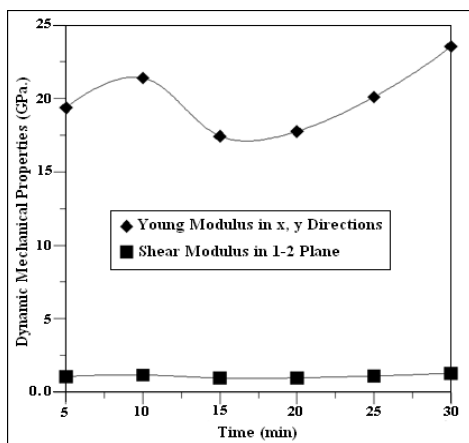


Fig. 7. Dynamic Mechanical Property vs. Time after Combined Temperatures Effect.

Figs. 6, 7 illustrate how the fundamental natural frequency and dynamic elastic moduli vary sinusoidal with conditioning holding time after combined temperatures (60°C and -15°C) affect using different boundary conditions. At time ($t = 15$ min) a large reduction in frequency and elastic moduli values occurred due to the reduction bending stiffness of the laminate.

Fig. 8 gives the logarithmic decrement after combined temperatures effect varies with time using different boundary conditions. CCCF is the best boundary condition at time ($t = 15$ min) because it affords a large value of logarithmic decrement and gives a reduction in both natural frequency and amplitude values with high damping factor.

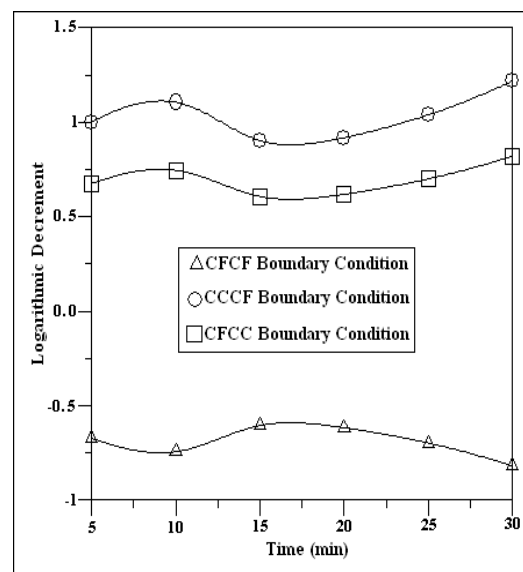


Fig.8. Logarithmic Decrement vs. Time after Combined Temperatures Effect.

5. Conclusion

- At holding time ($t = 15$ min) the increasing of free edges of laminate plate leads to decreasing the value of fundamental natural frequency because of the absence of the shear coupling effect from clamped edges.
- The best prediction holding time is at ($t = 15$ min) because this time gives a minimum of natural frequency, dynamic elastic moduli and magnification factor values to avoid a large response of composite laminate plate.
- The damping factor increases when using combined temperatures effect such (0.382, 0.432, and 0.429) at (CFCF, CCCF, and CFCC) respectively because the polymer matrix composites have temperature-dependent mechanical properties.

- CCCF is the better boundary condition in the field of maximum percent overshoot than CFCC and CFCC boundary conditions because this boundary condition affords minimum value of maximum percent overshoot.

Notation

a	large plate length
b	small plate breadth
E_x	modulus of elasticity at 45°
E_1	modulus of elasticity along x-direction
E_2	modulus of elasticity along y-direction
E_x^c	modulus of elasticity of composite plate along x-direction
E_y^c	modulus of elasticity of composite plate along y-direction
E_z^c	modulus of elasticity of composite plate along z-direction
G_{12}	modulus of rigidity in 1-2 plane
G_{23}	modulus of rigidity in 2-3 plane
G_{13}	modulus of rigidity in 1-3 plane
h	plate thickness
m_f	mass density for fiber
m_c	mass density for composite
p.o.	maximum percent overshoot
X_i	maximum alternative amplitudes (i = 1, 2)

Greek letters

v_f	fiber volume fraction
ρ_f	fiber density
ρ_m	fiber matrix
φ	fiber weight fraction
δ	logarithmic decrement
ξ	damping ratio
ω_n	natural frequency
ω_d	damped forced frequency
ν_{12}	poisson's ratio in 1-2 plane
ν_{23}	poisson's ratio in 2-3 plane
ν_{13}	poisson's ratio in 1-3 plane

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تنبؤ الوقت للتصرف الديناميكي لألياف الزجاج المقواة مع مادة البوليمر المركبة الخاضعة لتقلبات درجات الحرارة المختلفة

لؤي صباح يوسف

قسم الهندسة الميكانيكية/ كلية الهندسة/ جامعة بغداد

الخلاصة

تقليل خواص الاهتزاز للمادة المركبة (الياف زجاجية محاكاة موضوعة في مادة الترموسيت بوليستر الاساس) قد تحققت عند تنبؤ الزمن تحت درجات حرارة مشتركة ومتغيرة (60 درجة مئوية الى -15 درجة مئوية) باستخدام ثلاث انواع من شروط الدعم مثل (مثبت-حر-مثبت-حر، مثبت-مثبت-مثبت-حر، مثبت-حر-مثبت-مثبت). خواص الاهتزاز هي التردد الطبيعي، كلا من معامل المرونة بالاتجاهين الطولين ومعامل القص بالمقطع 1-2 الديناميكيين، ومعامل التخميد. التردد الطبيعي للنظام هو دالة لخواص المرونة، الأبعاد، والكتلة. الألياف الزجاجية المحاكاة هندست خصيصا لتقوية البوليمر ولكن الترموسيت بوليستر الغير مشبع الشائع الاستعمال يعرض توازنا جيدا لخواص الاهتزاز عند درجات الحرارة المعتدلة او الباردة، وايضا رخيص الثمن. ان التفاوت ما بين المادة الاساس والالياف المحاكاة تعطي الدور السائد لميكانيكا الالياف حيث ان المادة الاساس هي تطور مساحات اليات الضرر. ان اختبار الاهتزاز الحر نفذ لـ (5، 10، 15، 20، 25، 30) دقيقة من حصة الوقت حيث ان المادة المركبة معرضة الى (75 درجة مئوية) من الانحدار الحراري لعشر مرات لمختلف اوقات مراحل التشغيل. النتائج صنفتم الى نتائج عملية ونتائج بواسطة العناصر المحددة باستخدام برنامج ANSYS.