A DEVELOPED MODIFIED OSAP CONTROLLER WITH REPETITIVE CONTROL ACTION FOR UPS

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ABSTRACT

In this work, a developed modified one sampling ahead preview (OSAP) controller with repetitive control action for single–phase voltage source PWM inverters used in (UPS) systems is proposed. The proposed technique minimizes largely the plant modeling errors resulted from simplification made to obtain a linear discrete-time plant model. In addition, due to the repetitive control action, this digital control scheme can minimize the steady-state error and periodic distortions caused by nonlinear cyclic loads. Hence, the Total Harmonic Distortion (THD) will be reduced. This technique utilizes a switching frequency greater than the sampling frequency, yielding additional minimization in the plant modeling errors. As the sampling frequency is less than the switching frequency, it is possible to implement this controller on a low speed microcontroller. Plant model and theoretical analysis of the control scheme are discussed. Simulation results are presented to verify the performance of the proposed approach under different load conditions.

KEYWORDS
Uninterruptible power supply (UPS), PWM inverters, repetitive controller, one sampling ahead preview (OSAP) controller, digital control, and microcontroller.
قد بُنيت النتائج أن وجود المسيطر التكراري يخفض إلى الحد الأدنى من مقدار الخطأ الموجود في حالة الاستقرار، كما يظل من الشؤون المكررة الناتجة من الأحمال النشطة المترابطة مما يسبب اكتتافاً في نسبة مجموعة المركبات المشوهة (THD).

تم تحليل المنظومة بشكل نظري (رياضي)، كما تم تمثيل النتائج على الحاسوب لتقييم أداء التقنية المقترحة تحت عدة حالات مختلفة من الأحمال.

**INTRODUCTION**

The uninterruptible power supply (UPS) systems have been widely used as backup for critical load applications such as computers and life support systems in hospitals.

The widespread availability of low cost microcontrollers and DSP processors has yielded possible use of digital control techniques to improve the performance of UPS systems. Digital control techniques should generate the pulse width modulated (PWM) signal to produce the sinusoidal output voltage of the UPS system with low total harmonic distortion (THD) for linear and nonlinear loads. The microprocessor – based deadbeat control scheme had been used in [1, 2]. The PWM signal is determined at each sampling instant by the microprocessor, based on output measurements and the reference signal. This approach can result in a low (THD) sinusoidal output with fast transient response. The drawback of this scheme is the detection of both output voltage and output capacitor current required at each sampling instant. Thus, a deadbeat control algorithm using only a voltage sensor, which may be called one sampling ahead preview (OSAP) controller, was proposed to reduce the cost of the overall system and the computation time of the controller [3]. Although this technique demonstrates quick response for load disturbances and nonlinear loads, the output voltage waveform normally presents high (THD) for nonlinear cyclic loads such as AC phase–controlled loads and rectifier loads in computer systems. Thus, a repetitive controller was added to OSAP controller to minimize the steady–state error and periodic distortions caused by nonlinear cyclic loads [4]. In spite of this, the pulsewidth in these schemes is limited by the computation time of the microprocessor.

A modified OSAP controller [5] was proposed to increase the maximum available pulsewidth. In this approach, the pulsewidth in the (k-th) sampling interval is computed by using the output voltage sampled at the previous sampling instants. Hence, the pulsewidth is determined during the previous interval in order to extend the pulsewidth to the entire sampling interval (T). However, this digital control technique is very sensitive to parameter variations and plant modeling errors.

An improved modified OSAP controller [6] was implemented to reduce the effects of the plant modeling errors resulting from the simplifications made to obtain a linear discrete–time plant model. This digital control scheme employs a switching frequency greater than the sampling frequency, minimizing the plant modeling errors.

In this work, a developed modified OSAP controller with a repetitive control action is proposed for voltage source PWM inverter used in (UPS) systems. The proposed scheme differs from the aforementioned previous scheme, presented by reference [6], by the positions of the switching pulses within the sampling interval. Applying this scheme minimizes the effects of the plant modeling errors produced by linearization of the discrete–time plant model more than that obtained with the previous scheme. The switching frequency used in the proposed scheme is also greater than the sampling frequency, which minimizes the undesirable effects of the plant modeling errors too. As the sampling frequency is smaller than the switching frequency, it is possible to implement this controller on a low cost microcontroller.

Plant modeling and stability analysis for the proposed system are presented. Simulation results (for 110 Vrms, 60 Hz, 1 KVA system) are carried out to demonstrate the performance of the proposed
control approach due to sudden changes at load, linear and nonlinear cyclic loads, and unmodeled dynamics. Finally, conclusions and suggestions for future work are reported.

PLANT MODEL

Figure (1) shows the single–phase PWM inverter circuit of the UPS, where the full–bridge inverter, \((LC)\) filter and pure resistive load \((R)\) are considered as the plant to be controlled. The state and the output equations of such a second–order system, with state vector \([v_c(t) \ v_e(t)]^T\), are:

\[
\dot{x}(t) = Ax(t) + Bu(t)
\]

\[
y(t) = Cx(t)
\]

where

\[
x(t) = \begin{bmatrix} v_c(t) \\ \dot{v}_e(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ -\omega_p^2 - 2\zeta_p \omega_p \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \omega_p^2 \end{bmatrix}
\]

\[
C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \omega_p = \frac{1}{\sqrt{L_f C_f}}, \quad \zeta_p = \frac{1}{2R} \sqrt{\frac{L_f}{C_f}}
\]

![Figure (1)](image)

**Figure (1)** [7]

Digitally controlled PWM inverter

The proposed control scheme is very similar to the control scheme presented by [6]. The only difference between them is that *the train of input pulses in the sampling period for the proposed*
A.A. Marouf
S.A. Abdel Razzak

A Developed Modified OSAP Controller
with Repetitive Control Action for UP

system is positioned in the center of these sections instead of being in the beginning. Figures (2) and (3) demonstrate the train of pulses for the two schemes.

For the proposed controller the sampled–data state equation can be written as:

\[
x(k + 1) = e^{AT}x(k) + \int_{T - \Delta T(k)}^{T + \Delta T(k)} e^{A(T - \tau)}BV_\theta d\tau + \int_{3T - \Delta T(k)}^{3T + \Delta T(k)} e^{A(T - \tau)}BV_\theta d\tau + \cdots + \int_{(2n_p - 1)T - \Delta T(k)}^{(2n_p - 1)T + \Delta T(k)} e^{A(T - \tau)}BV_\theta d\tau
\]

\[\tag{3}\]

**Figure (2) [7]**
Multiple–Pulse Width Modulation (MPWM) pattern for the improved modified OSAP controller.
Each pulse placed in the beginning of its section

**Figure (3)**
MPWM pattern for the proposed (developed modified OSAP) controller each pulse placed in the center of its section
Integrating Eq.(3) and taking
\[ e^{\frac{-A\Delta T(k)}{2n_p}} - e^{\frac{-A\Delta T(k)}{2n_p}} \]
as a common term, the nonlinear discrete–time state equation can be obtained:

\[ x(k + 1) = e^{AT}x(k) + \left[h_{d_{max}}(k)\right] \]

where:

\[ \left[h_{d_{max}}(k)\right] = \left(\sum_{i=1,2,\ldots}^{n_p} e^{\frac{(i-1)\Delta T}{2n_p}}\right) \left(e^{\frac{-A\Delta T(k)}{2n_p}} - e^{\frac{-A\Delta T(k)}{2n_p}}\right)A^{-1}BV \]

(5)

It is possible to obtain a linear discrete–time model by computing the term \( e^{\frac{-A\Delta T(k)}{2n_p}} - e^{\frac{-A\Delta T(k)}{2n_p}} \) using Taylor series and neglecting terms of higher than \( (\Delta T)^2 \), yields:

\[ x(k + 1) = G_d x(k) + H_d \Delta T(k) \]

(6)

where:

\[ G_d = \begin{bmatrix} g_{d11} & g_{d12} \\ g_{d21} & g_{d22} \end{bmatrix} = e^{AT}, \quad H_d = \begin{bmatrix} h_{d1} \\ h_{d2} \end{bmatrix} = \frac{1}{n_p} \left(\sum_{i=1,2,\ldots}^{n_p} e^{\frac{(i-1)\Delta T}{2n_p}}\right)BV \]

(7)

Figure (4) shows the normalized errors between the parameters of the input matrix \( H_d (h_{d1} \text{ and } h_{d2}) \) and respective parameters of the nonlinear model for different values of \( (n_p) \). The normalized errors can be calculated using the following Equations:

Normalized error of \( (h_{d1}) \) = \[ \frac{h_{d1} - h_{d_{max}}(k)/\Delta T(k)}{h_{d_{max}}(k)/\Delta T(k)} \]

(8)

and

Normalized error of \( (h_{d2}) \) = \[ \frac{h_{d2} - h_{d_{max}}(k)/\Delta T(k)}{h_{d_{max}}(k)/\Delta T(k)} \]

(9)

Figure (4) demonstrates the superiority of the proposed approach upon the previous one (improved modified OSAP controller presented by reference [6]) in modeling errors amount. Moreover, it is very clear from the Figure that modeling error decreased significantly as the switching frequency of the PMW inverter increased.

Using Eq. (6), the input–output equation in the \( (z–\text{domain}) \) can be obtained:

\[ y(z) = \frac{h_{d1} + (h_{d2}g_{d12} - h_{d1}g_{d22})z^{-1}}{z - (g_{d11} + g_{d22}) + (g_{d11}g_{d22} - g_{d12}g_{d21})z^{-1}} \Delta T(z) \]

(10)

3958
Figure (4)

Modeling error for different values of $n_p (\omega_p = 11550 \text{rad/s}, \zeta_p = 0.25)$

(b) Normalized error of $h_{d1}$. (a) Normalized error of $h_{d2}$. 
Thus, the input–output difference equation can be written as:

\[ y(k + 1) + a_{d1}y(k) + a_{d2}y(k - 1) = b_{d1}u(k) + b_{d2}u(k - 1) \]  
(11)

where

\[
\begin{align*}
    a_{d1} &= -(g_{d11} + g_{d22}), \quad a_{d2} = g_{d11}g_{d22} - g_{d12}g_{d21}, \quad b_{d1} = h_{d1}T/V_B, \\
    b_{d2} &= (h_{d2}g_{d12} - h_{d1}g_{d22})T/V_B, \quad u(k) = \frac{\Delta T(k)}{T}V_B
\end{align*}
\]  
(12)

**CONTROL LAW**

The OSAP control law can be obtained from Eq.(11) considering that the output is equal to the reference signal at the next sampling instant, \( y(k+1) \) is substituted by \( r(k+1) \). Then, the following control law can be obtained:

\[
u_{\text{OSAP}}(k) = \frac{r(k + 1) + p_1y(k) + p_2y(k - 1) - q_1u(k - 1)}{q_1}
\]  
(13)

If the OSAP controller gains \( p_1, p_2, q_1 \) and \( q_2 \) are equal to the plant parameters \( a_1, a_2, b_1 \) and \( b_2 \), respectively, it becomes a deadbeat control law which forces the output voltage to be equal to the reference signal at the next sampling interval. However, if the plant parameters change after the controller gains in Eq.(13) have been determined; the deadbeat response is no longer obtained [3].

Moreover, the maximum available pulsewidth is limited by the delay time caused by the output voltage (A/D) conversion and control law computation. To solve this problem a modified OSAP controller is presented[5]. The modified OSAP controller equation becomes:

\[
u_{\text{dMOSAP}}(k) = \frac{r(k + 1) + P_{d1}y(k - 1) + P_{d2}y(k - 2) - Q_{d1}u(k - 1) - Q_{d2}u(k - 2)}{Q_{d1}}
\]  
(14)

where the developed controller gains are:

\[
\begin{align*}
    P_{d1} &= -(g_{d11}^2 + g_{d11}g_{d22} + g_{d12}g_{d21} + g_{d22}^2), \\
    P_{d2} &= -(g_{d11}g_{d12}g_{d21} - g_{d11}g_{d22}^2 + g_{d12}g_{d21}g_{d22} - g_{d11}g_{d21}g_{d22}^2), \\
    Q_{d1} &= h_{d1}T/V_B, \\
    Q_{d2} &= (h_{d1}g_{d11} + h_{d2}g_{d12})T/V_B, \\
    Q_{d3} &= \left(-h_{d1}(g_{d11}g_{d22} + g_{d22}^2) + h_{d2}(g_{d11}g_{d12} + g_{d12}g_{d22})\right)T/V_B
\end{align*}
\]  
(15)

Equation (14) shows that the pulse width determination can be completed during the previous interval, and the pulse width can be extended to the theoretically maximum limit, that is, the sampling interval \( T \).
To minimize the steady-state error and periodic distortions, a repetitive controller is added to this developed modified OSAP controller, as shown in Figure (5), such that the control law becomes:

\[ u(k) = u_{dMOSAP}(k) + u_{RP}(k) \]  

(16)

The repetitive control law can be written as [4]:

\[ u_{RP}(k) = c_{r1}e(k + N - n) + c_{r2}\sum_{i=1}^{\infty} e(k + N - iN) \]  

(17)

where \( e(k) \) is the tracking error, \( c_{r1} \) and \( c_{r2} \) are the gains of the repetitive controller, \( N \) is the time advance step size and \( n \) is the number of samples in a period of reference voltage which equal to the ratio of the sampling frequency \( f_s \) to that of the reference sinusoidal waveform \( f_r \). In this control system, each pulse width \( u_{RP}(k) \), and as a result \( u(k) \), is determined by referring the output voltage in the previous cycle. The repetitive controller gains are designed to guarantee a good steady-state response for any resistive load and fast convergence of the output error for nonlinear cyclic loads [6].

![Figure (5)](image)

Control system block diagram using the developed modified OSAP controller with repetitive control action

**STABILITY ANALYSIS**

Figure (6) shows the developed control system before adding the repetitive controller. The transfer function \( G_{dMOSAP}(z) = y(z)/r(z) \) can be obtained by taking the z-transformation of Eqs. (11) and (14), and elimination the plant input \( u_{dMOSAP}(z) \), yields:
\[
\frac{y(z)}{r(z)} = \frac{(b_dz + b_d^2)z^3}{(z^2 + a_dz + a_d^2)(Q_dz^2 + Q_dz + Q_d^2) - (P_dz + P_d^2)(b_dz + b_d^2)}
\]  
(18)

Figure (6)
Block diagram of the developed control system without the repetitive controller.

Figure (7) shows the stability region of the developed modified OSAP controller in the \(|\omega_p - \zeta_p|\) plane for two nominal values of filter parameters (case (a) \(L_f = 0.5mH, C_f = 15\mu F\), case (b) \(L_f = 0.35mH, C_f = 10\mu F\)). This is performed from the localization of the closed-loop poles (the denominator of Eq.(18)) with the variation of plant parameters (\(R, L_f,\) and \(C_f\)). Figure (7) also shows the trajectories of the plant parameters in this plane. The trajectory of one parameter is done maintaining the other two parameters constant at their nominal values. From the Figure, it is clear that stability is expected for load changes from no load to rated load. In addition, an accepted stability region with a good range of output filter parameters is obtained.

On another hand the choices of the filter parameters must be governed by the sampling frequency choice

If a repetitive controller is added to the system, the \(z\)–transform of the repetitive controller Eq. (17) can be written as:

\[
u_{RP}(z) = e(z)z^{-n} \left(c_{r1} + c_{r2} - c_{r1}z^{-n} \right)
\]  
(19)

The transfer function \((e(z)/r(z))\) of the system shown in Figure (5) can be obtained, using Eq.(19) and the \(z\)–transform of Eqs. (11), (14), and (16). Then:

\[
e(z) \quad r(z) = \frac{(1 - G_{dMOSAP}(z)(1 - z^{-n})}{1 - z^{-n}H_{dMOSAP}(z)}
\]  
(20)

where \(e(z), r(z)\) are the \(z\)–transform of the output error and the reference input respectively, and:
A.A. Marouf

A Developed Modified OSAP Controller
with Repetitive Control Action for UP!

\[ H_{dMOSAP}(z) = 1 - Q_d z^{-1} (c_r_1 + c_r_2 - c_r_1 z^{-n}) G_{dMOSAP}(z) \]  

(21)

Since the stability of Eq.(18) is established, the repetitive controller determines the overall system stability. From Eq.(20), a sufficient condition for stability [4] is:

\[ |H_{dMOSAP}(j\omega)| \leq 1 \]  

(22)

where \( \omega \) is the angular frequency of the reference input: \( \omega = 2\pi m f \), (m=0, 1, 2 ... n/2).

\[ \begin{align*}
\text{(a) } & \quad f_{sw} = f_s, \text{ controller parameters: } L_f = 0.5 \text{ mH}, C_f = 15 \mu F, \text{ and } R = 12\Omega. \\
\text{(b) } & \quad f_{sw} = 3f_s, \text{ controller parameters: } L_f = 0.5 \text{ mH}, C_f = 15 \mu F, \text{ and } R = 12\Omega. \\
\text{(c) } & \quad f_{sw} = f_s, \text{ controller parameters: } L_f = 0.35 \text{ mH}, C_f = 10 \mu F, \text{ and } R = 12\Omega. \\
\text{(d) } & \quad f_{sw} = 3f_s, \text{ controller parameters: } L_f = 0.35 \text{ mH}, C_f = 10 \mu F, \text{ and } R = 12\Omega.
\end{align*} \]

5- Simulation Results

<table>
<thead>
<tr>
<th>( \omega ) (rad/s) \times 10^4</th>
<th>( \zeta_p )</th>
</tr>
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<tbody>
<tr>
<td>0.5</td>
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<td>1</td>
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<td>3</td>
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<tr>
<td>3.5</td>
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</table>

Figure (7)

Stability region of the modified OSAP controller in \( |\omega_p - \zeta_p| \) plane with:

a) \( f_{sw} = f_s \), controller parameters: \( L_f = 0.5 \text{ mH}, C_f = 15 \mu F, \text{and } R = 12\Omega. \)
b) \( f_{sw} = 3f_s \), controller parameters: \( L_f = 0.5 \text{ mH}, C_f = 15 \mu F, \text{and } R = 12\Omega. \)
c) \( f_{sw} = f_s \), controller parameters: \( L_f = 0.35 \text{ mH}, C_f = 10 \mu F, \text{and } R = 12\Omega. \)
d) \( f_{sw} = 3f_s \), controller parameters: \( L_f = 0.35 \text{ mH}, C_f = 10 \mu F, \text{and } R = 12\Omega. \)

3963
The performance of the developed modified OSAP controller with repetitive control action is evaluated using the digital computer simulation by writing some programs (with MATLAB®, version (7.4)). Table (1) summarizes the parameters of the single-phase PWM inverter system used in simulation. The simulation results for three different cases (1), (2), and (3) of filter parameters and switching frequency values are presented according to the load conditions.

### Table (1)

**Parameters of the single-phase PWM inverter used in simulation**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
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<td>DC input voltage</td>
<td>$V_B = 200$ V</td>
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<tr>
<td>Reference voltage &amp; frequency</td>
<td>$V_{ref} = 110$ V, $f_r = 60$Hz</td>
</tr>
<tr>
<td>Nominal resistive load</td>
<td>$R = 12\Omega$</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>$f_s = 10.8$KHz</td>
</tr>
<tr>
<td>Sampling period</td>
<td>$T = 92.6$ $\mu$s</td>
</tr>
<tr>
<td>Repetitive controller gains</td>
<td>$c_{r_1} = 0$, $c_{r_2} = 0.3$</td>
</tr>
</tbody>
</table>

**Output Filter Parameters**

| Case (1)                           | $L_f = 0.5mH$, $C_f = 15\mu F$            |
|                                    | Switching frequency ($f_{sw} = 10.8$KHz)  |
|                                    | PWM (one pulse in the center)             |
| Case (2)                           | $L_f = 0.5mH$, $C_f = 15\mu F$            |
|                                    | Switching frequency ($f_{sw} = 32.4$KHz)  |
|                                    | MPWM (three pulses, each one is in the    |
|                                    | center of its section of the sampling interval) |
| Case (3)                           | $L_f = 0.35mH$, $C_f = 10\mu F$           |
|                                    | Switching frequency ($f_{sw} = 32.4$KHz)  |
|                                    | MPWM (three pulses, each one is in the    |
|                                    | center of its section of the sampling interval) |

**Nominal Load Condition**

The developed modified OSAP controller with and without repetitive control action is simulated using nominal resistive load ($R=12\Omega$). The waveforms of the output voltage $v_c(t)$, reference voltage $v_{ref}(t)$, and load current $i_L(t)$ for the three different cases (1), (2), and (3) are shown in Figure (8).
The action of the repetitive controller, THD for nominal load condition of all cases approaches zero in the latest cycles. Moreover, increasing the switching frequency minimizes the THD in cases (2) and (3) more than that for case (1). However, due to the integral action of the repetitive controller, THD for nominal load condition of all cases approaches zero in the latest cycles.

**Figure (8)**

Response of the developed modified OSAP controller for nominal resistive load condition \((R = 12\Omega)\) in:

(a) case(1), without repetitive controller. (b) case(1), with repetitive controller. (c) case(2), without repetitive controller. (d) case(2), with repetitive controller. (e) case(3), without repetitive controller. (f) case(3), with repetitive controller.

**No – Load Condition**

The simulation for the same three cases (1), (2), and (3) are shown in Figure (9). Again, better response is achieved by increasing the switching frequency in cases (2) and (3).

Figure (10) shows the total harmonic distortion (THD) to the nominal load and no-load conditions for the three cases. The influence of the repetitive integral action is clear in the consecutive cycles. Moreover, increasing the switching frequency minimizes the THD in cases (2) and (3) more than that for case (1). However, due to the integral action of the repetitive controller, THD for nominal load condition of all cases approaches zero in the latest cycles.
Figure (9)
Response of the developed modified OSAP controller with repetitive controller for no-load condition in: a) case(1). b) case(2). c) case(3).

Figure (10)
Influence of repetitive integral action on the total harmonic distortion factor (THD) with the consecutive cycles for: a) Nominal load condition. b) No – load condition.

Nonlinear Load Condition
The nonlinear loads, such as AC phase – controlled loads (Triac) and rectifier loads, are considered as an output periodic disturbance on the system. Such nonlinear loads represent a challenge to evaluate the system performance. Some of nonlinear loads will be discussed here.
Triac Connected in Series with Nominal Resistive Load

Figure (11) presents the response of the proposed controller with nominal resistive load in series with a Triac commuting at $72^\circ / 252^\circ$ for the three cases (1), (2), and (3).

![Graph](image_url)

**Figure (11)**

Response of the developed modified OSAP controller with repetitive controller for nominal resistive load in series with a Triac commuting at $72^\circ / 252^\circ$ in:

- a) case(1).
- b) case(2).
- c) case(3).

It is clear that the steady state error, caused by this nonlinear cyclic load, is minimized by adding the repetitive controller especially for case (3) in comparison with cases (1) and (2). This is due to that: the influence of reducing the filter inductance ($L_f$) minimizes the reactance affecting current changes directly, so small inductance value is quickly compensate the drawn load current in comparison with larger values, hence less input supply voltage is needed to compensate the drop in voltage.

A comparison of (THD) for nominal resistive load with a Triac commuting at angles ($36^\circ / 216^\circ$, $72^\circ / 252^\circ$, $90^\circ / 270^\circ$, and $120^\circ / 300^\circ$) each for the three different cases (1, 2, and 3) is shown in Figure (12). It is clear that the (THD) for case (1) and case (2) are very similar, while that for case (3) shows a better response. The (THD) approaches zero for Triac angles ($36^\circ / 216^\circ$) and case (3) of angles ($120^\circ / 300^\circ$) unlike for other states. This is due to the drop in the output voltage at the commuting angle that requires a low or high value of the control law ($u(k)$). High values of $u(k)$ demanded may exceed the maximum permitted value ($V_B$), i.e. the pulse width ($\Delta T$) exceeds the sampling period ($T$), according to Eq. (5), which is not possible ($\Delta T = T$ at saturation). In this case, the control law ($u(k)$) will be limited to the DC supply voltage ($V_B$), and the THD may not approaches zero.
Triac Connected in Series with Nominal Resistive Load Including Unmodeled Dynamics

In this section a nominal resistive load in series with a Triac commuting at \((72^\circ /252^\circ)\) connected to the developed modified OSAP controller including an unmodeled dynamics will be tested. An equivalent resistor of \((0.5 \, \Omega)\) in series with the filter capacitor may achieve an example for this state. From control point view, this resistor value caused an unmodeled \((\text{stable zero})\) at \((-8000 \, \text{rad/sec})\) to be added to the closed–loop transfer function of the system.

**Figure (12)**
Influence of repetitive integral action on the (THD) for nominal resistive load in series with a Triac commutes at:

- a) \((36^\circ /216^\circ)\).
- b) \((72^\circ /252^\circ)\).
- c) \((90^\circ /270^\circ)\).
- d) \((120^\circ /300^\circ)\).
Figure (13) shows the output voltage and load current waveforms for the three cases defined earlier. It is clear that the proposed controller shows a good performance with the presence of this unmodeled dynamic.

Figure (14) shows the (THD) of the output voltage for the three cases. Also the (THD) for case (3), with reduced filter parameters, is better than that for cases (1) and (2).

**Figure (13)**
Response of the developed modified OSAP controller with repetitive control action for nominal resistive load in series with a Triac commuting at $72^\circ/252^\circ$ including unmodeled stable Zero at (-8000 rad/sec): a) case (1). b) case (2). c) case (3).

**Figure (14)**
THD for nominal resistive load in series with a Triac commuting at $72^\circ/252^\circ$ including unmodeled stable Zero at (-8000 rad/sec).
Step Change – Load Condition

A step load change from no – load to full load is illustrated in Figure (15) for the three cases (1), (2) and (3). The three cases show a good transient response for a step – load change.

![Figure (15)](image)

Transient response for the proposed controller: a) case (1). b) case (2). c) case (3).

CONCLUSIONS

Results show that this controller with the proposed control technique, using input pulses in the center, minimizes largely the plant modeling errors, resulted from simplification made to obtain a linear discrete - time plant model, more than that when the pulses are placed in the beginning. Moreover, increasing the switching frequency minimizes the effects of the plant modeling errors too.

Stability analysis proves that the closed-loop system for the proposed controller is stable from no-load to full load variation. In addition, an accepted stability region with a good range of output filter parameters is obtained without distinct difference between one or three pulses per sampling period states.

Simulation results assure that adding a repetitive controller to the developed modified OSAP controller minimizes the steady-state error and hence gives the advantage of low total harmonic distortion (THD).

Results show that the proposed control scheme with three pulses, each one in the center of its section from the sampling period, (MPWM pattern) improves system transient response more than the modified OSAP control system with one pulse in the center (PWM pattern) for no-load and nominal load conditions. However, there is no distinct difference in the THD of both patterns for nonlinear load conditions.

Nevertheless reducing filter parameters, which mainly reduces the cost, (especially the inductance \(L_f\)) satisfies better performance with lower (THD) for nonlinear load conditions, even in the presence of an unmodeled dynamics, appreciating that lower stability range for the repetitive controller gains is obtained.
REFERENCES


LIST OF SYMBOLS

The symbols repeatedly used are listed below. Other symbols are explained in the text.

\( c_{r1}, c_{r2} \)  Repetitive controller gains
\( f_p \)  Resonance frequency
\( f_r \)  Reference frequency
\( f_s \)  Sampling frequency
\( f_{sw} \)  Switching frequency
\[ G_d = \begin{bmatrix} g_{d11} & g_{d12} \\ g_{d21} & g_{d22} \end{bmatrix} \]

State space matrix for the new linear discrete-time plant model

\[ G_{dMOSAP}(z) \]

Transfer function of the developed modified OSAP controller system

\[ H_d = \begin{bmatrix} h_{d1} \\ h_{d2} \end{bmatrix} \]

Input matrix for the linear discrete-time plant model

\[
\begin{bmatrix}
h_{d_{non1}}(k) \\
h_{d_{non2}}(k)
\end{bmatrix}
\]

Nonlinear input matrix for the discrete-time plant model

- \( k \): Counter of samples
- \( n \): Number of samples in a period of reference voltage
- \( n_p \): Number of pulses per sampling period
- \( N \): Time advance step size
- \( p_1, p_2, q_1, q_2 \): OSAP controller parameters
- \( P_{d1}, P_{d2}, Q_{d1}, Q_{d2}, Q_{d3} \): Developed modified OSAP controller parameters
- \( r(k), r(z) \): Reference voltage (in control notation)
- \( u(k) \): Discrete system input voltage (denoted by the control law too)
- \( u_{dMOSAP}(k) \): Control law of developed modified OSAP controller
- \( u_{OSAP}(k) \): Control law of OSAP controller
- \( u_{RP}(k) \): Repetitive control law
- \( v_{in}(t) \): Inverter input voltage
- \( v_{ref}(t) \): Reference voltage signal
- \( y(t), y(s), y(k), y(z) \): System output voltage
- \( \omega \): Angular frequency of the reference input
- \( \omega_p \): Resonance angular frequency
- \( \zeta_p \): Resonance damping ratio
- \( \Delta T(k) \): k-th pulse width
- \( MPWM \): Multiple Pulse Width Modulation
- \( PWM \): Pulse Width Modulation
- \( THD \): Total Harmonic Distortion
- \( UPS \): Uninterruptible Power Supply