Damage Detection and Assessment of Stiffness and Mass Matrices in Curved Simply Supported Beam Using Genetic Algorithm

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ABSTRACT

In this study, a genetic algorithm (GA) is used to detect damage in curved beam model, stiffness as well as mass matrices of the curved beam elements is formulated using Hamilton's principle. Each node of the curved beam element possesses seven degrees of freedom including the warping degree of freedom. The curved beam element had been derived based on the Kang and Yoo’s thin-walled curved beam theory. The identification of damage is formulated as an optimization problem, binary and continuous genetic algorithms (BGA, CGA) are used to detect and locate the damage using two objective functions (change in natural frequencies, Modal Assurance Criterion MAC).

The results show the objective function based on change in natural frequency is the best objective and no error was recorded in prediction of location and small error in detecting damage value. Also the result show that the genetic algorithm method are efficient indicating and quantifying single and multiple damage with high precision, and the prediction error for the CGA are less than corresponding value for the BGA.

Keywords: curved beam; Genetic Algorithm; damage detection

لاصفة:

في هذا البحث تم استخدام الخوارزمية الجينية لتقييم الضرر في العتبة الموؤدة، تم صياغة مصفوفة الجسماة والكتلة لعنصر العتبة الموؤدة (warping) باستخدام مبدأ هاملتون. كل عدد في العتبة الموؤدة تحوي على سبع درجات من الحرية مع الخذل بنظر الإعتبار الإعوجاج (warping). تم اشتقاق عنصر العتبة الموؤدة بالاعتماد على نظرية كايل ويو لنظرية العتبة الموؤدة ذات الة جدران الرقيقة. استخدمت الاعداد الحقيقي ونظام التشغيل (objective function) الثاني لخوارزمية الجينية لتحديد كميات الضرر وموقعه باستخدام دالتين من دوال الهند (Modal Assurance Criterion MAC) و (change in natural frequencies).

النتائج اظهرت بأن الفرق في التردديات هي أفضل دالة موضوعية و ليس هناك خطأ مسجل في التنبؤ بموقع الضرر وخطأ صغير في تحديد كميات الضرر. وكذلك أثبتت الخوارزمية الجينية كفائتها في ايجاد كمية و موقع الضرر الموؤد والمتعدد بدقة عالية وان نسبة الخطأ باستخدام الاعداد الحقيقي اقل نسبة عند استخدام نظام التشغيل الثاني.

Keywords: curved beam; Genetic Algorithm; damage detection
INTRODUCTION

At the recent years, genetic algorithms have been recognized as promising intelligent search techniques for difficult optimization problems. Genetic algorithm method is very attractive in comparison with classical methods because it does not require a solution search within the whole solution space. Instead the algorithm starts from a small initial population of approximated solutions and converges rapidly from thereon. M. I. Friswell et al. 1998 developed a technique, which is based on combined use of eigensensitivity and genetic algorithms to identify the location and magnitude of damage from measured vibration data. They employ a genetic algorithm to minimize a square-value of the frequency error. Structural damage is modeled by a reduction in Young’s modulus, while the element number in the finite element model gives damage location. The objective is to identify the position of one or more damage sites in a structure, and to estimate the extent of the damage at these sites. The GA is used to optimize the discrete damage location variables. For a given damage location site or sites, a standard eigensensitivity method is used to optimize the damage extent. This two-level approach incorporates the advantage of both the GA and the eigensensitivity methods. Damage at one and two sites have been successfully located in the simulated example of a cantilever beam, also successfully location in an experimental cantilever plate.

J.H. Chou and J. Ghaboussi 2001 used a GA to solve an optimization problem formulated for detection and identification of structural damage. The “output error” indicating the difference between the measured and computed responses under static loading and the equation error indicating the residual force in the system of equilibrium equations are used to formulate the objective function to be optimized. The method proposed is capable of successfully detecting the location and magnitude of the damage as well as correctly determining the unmeasured nodal displacement, while avoiding the complete finite element analyses. E. S. Sazonov et al. 2002 used the GA to produce a sufficiently optimized amplitude characteristic filter to extract damage information from the strain energy mode shapes. A finite element model was used to generate training data set with the known location. The filter amplitude characteristic was encoded as a GA string where the pass coefficient for each harmonic of the Discrete Fourier Transform representation was a number between 0 and 1 in an 8 bit. The genetic optimization was performed based on the minimization of the signal- to- distortion ratio. The results obtained from the GA has confirmed the theoretical predictions and allowed improvements in the method’s sensitivity to damages of lower magnitude.

In this study, it had been used a binary and continuous genetic algorithm for damage detection and location in (in and out-of-plane) curved beam by minimizing or maximizing the objective function which is based on frequency difference and modal assurance criterion MAC.

I. MODELING THE DAMAGED BEAM.

In this study the equation of motion for simply curved beam acquired from Kang and Yoo’s theory of thin- walled curved beams to drive the element stiffness and mass matrices respectively. The curved beam element is shown in Fig.1 in curvilinear coordinate system. Each node of the curved beam element possesses seven degrees of freedom including the warping degree of freedom. Using Hamilton’s principle, the dynamic equilibrium can
be expressed in the variation form as following K. Young Yoon et al. 2006.

\[ \int_{t_1}^{t_2} (\delta T + \delta U + \delta V) \, dt = 0 \]  

(1)

Where \( \delta T \) is the variation kinetic energy, \( \delta U \) is the variation strain energy, and \( \delta V \) is the variation potential energy loss due to applied loads. The symbol \( \langle \rangle \) means the first variation. For the linear elastic body, the variation of strain energy stored in the body is

\[ \delta U = \int_{V} \tau_{ij} \delta \varepsilon_{ij} \, dV \]  

(2)

Where \( \tau_{ij} \) refers to the components of the stress tensor and \( \varepsilon_{ij} \) to those of the strain tensor. The variation in kinetic energy of a thin-walled curved beam is

\[ \delta T = \int_{V} \rho \frac{d^2u_i}{dz^2} \delta u_i \, dV \]  

(3)

Where \( \rho \) is the mass density, \( u_i \) is the displacement components of the curved beam, and \( t \) is time. The variation potential energy loss due to applied loads with body forces neglected is

\[ \delta V = - \int_{V} q_i \delta u_i \, dz \]  

(4)

Where \( q_i \) stands for distributed loads applied on the line of shear center and \( l \) is the length of the element.

A linear stiffness matrix and a consistent mass matrix are developed so that various analyses such as linear and free vibration analyses can be performed. Using shape functions, the dynamic equilibrium given in eq. (1) yields a set of simultaneous equations

\[ \delta T + \delta U + \delta V = \delta d^T [Md + Kd - f] = 0 \]  

(5)

From which one obtains.

\[ Md + Kd - f = 0 \]  

(6)

Where \( K \), \( M \), \( d \), and \( f \) are the linear stiffness matrix, the consistent mass matrix, the nodal displacement vector, and the applied force vector of a global structural system, respectively. The nodal forces and the corresponding nodal displacements are shown in Fig.1 in the positive senses. The nodal forces are seven components \( (F_x, M_x, M_y, B, T_z, U_i, \text{and } V_j) \). The corresponding nodal displacements are \( (w_0, u_0, -v_0, -\tau, \beta, \alpha_0, \text{and } v_0) \) where \( \gamma \) and \( \tau \) are defined as

\[ \gamma = u_0 + \frac{w_0}{E} \]  

(7a)

\[ \tau = \beta + \frac{v_0}{E} \]  

(7b)

Where \( w_0, u_0, \) and \( \gamma \) describe the in-plane displacements whereas \( v_0, \alpha_0, \beta, \) and \( -\tau \) are the out-of-plane displacements. These two parts of displacement fields are not coupled with each other and can be formulated separately. Then, the displacement fields can be expressed in terms of nodal displacements as following K. Young Yoon et al. 2005.

\[ \begin{pmatrix} u_0 \\ v_0 \\ w_0 \\ \beta \end{pmatrix} = \begin{pmatrix} N_8 & 0 & 0 & 0 \\ 0 & N_8 & 0 & 0 \\ 0 & 0 & N_8 & 0 \\ 0 & 0 & 0 & N_8 \end{pmatrix} \begin{pmatrix} d^u \\ d^v \\ d^w \\ d^\beta \end{pmatrix} \]  

(8)

Where the shapes function, \( N \) is defined as

\[ N_8 = \left[ (1-\xi^2)(1+\xi^2) \right] \]  

(9a)

\[ N_8 = \left[ (1-\xi^2)(1+\xi^2) \right] \]  

(9b)

\[ N_0 = \left[ (1-\xi^2) \right] \]  

(9c)

Where \( \xi = z/l \)

Where the nodal displacement, \( d \) is represented

\[ d^u = [u_{\alpha}, \gamma, u_{\alpha}, \gamma]^T \]  

(10a)

\[ d^v = [v_{\alpha}, -v_{\alpha}, v_{\alpha}, -v_{\alpha}]^T \]  

(10b)

\[ d^w = [w_{\alpha}, w_{\alpha}]^T \]  

(10c)

\[ d^\beta = [\beta, -\tau, \beta, -\tau]^T \]  

(10d)
From the variation of strain energy presented in eq. (2) and the shape function in equations (9a), (9b), and (9c) the element stiffness matrix for curved beam is derived as shown in eq. (3) and following the similar procedure as used for the element stiffness matrix for curved beam formulation, the mass matrix is derived.

\[
[k] = \begin{bmatrix}
    E_k K_a & 0 & 0 \\
    0 & E_k K_b & 0 \\
    0 & 0 & E_k K_d + G K_e
\end{bmatrix}
\]  
(11)

Where:

\[
K_a = \int_N N_a^T N_a \, dz = \begin{bmatrix}
    12 & -12 & 12 \\
    -12 & 4t^2 & -6t \\
    12 & -6t & 4t^2
\end{bmatrix}
\]

\[
K_b = \int_N N_b^T N_b \, dz = \begin{bmatrix}
    12 & 6t & -12 \\
    6t & 4t^2 & -6t \\
    -12 & -6t & 4t^2
\end{bmatrix}
\]

\[
K_c = \int_N N_c^T N_c \, dz = \frac{1}{8} \begin{bmatrix}
    1 & 1 & -1 & 1 \\
    1 & -1 & 1 & -1 \\
    -1 & 1 & 1 & -1 \\
    1 & -1 & 1 & 1
\end{bmatrix}
\]

\[
K_d = \int_N N_d^T N_d \, dz = \begin{bmatrix}
    12 & -6t & 12 \\
    -6t & 4t^2 & 6t \\
    12 & 6t & 4t^2
\end{bmatrix}
\]

\[
K_e = \int_N N_e^T N_e \, dz = \begin{bmatrix}
    36 & -3t & -3t \\
    -3t & 4t^2 & 6t \\
    -3t & 6t & 4t^2
\end{bmatrix}
\]

\[
[m] = \begin{bmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & (l + \frac{1}{2}) M_0 + \frac{1}{2} M_0
\end{bmatrix}
\]  
(12)

Where:

\[
M_a = \int_N N_a^T N_a \, dz = \begin{bmatrix}
    156 & 22t & 54 & -13\, t \\
    22t & 4t^2 & 13\, t & -5\, t^2 \\
    54 & 13\, t & 156 & -22t \\
    -13\, t & -5\, t^2 & -22t & 4t^2
\end{bmatrix}
\]

\[
M_b = \int_N N_b^T N_b \, dz = \begin{bmatrix}
    156 & -22t & 54 & 13\, t \\
    -22t & 4t^2 & -13\, t & -5\, t^2 \\
    54 & 13\, t & 156 & 22t \\
    13\, t & -5\, t^2 & 22t & 4t^2
\end{bmatrix}
\]

\[
M_c = \int_N N_c^T N_c \, dz = \frac{1}{8} \begin{bmatrix}
    2 & 1 \\
    1 & 2
\end{bmatrix}
\]

\[
M_d = \int_N N_d^T N_d \, dz = \begin{bmatrix}
    36 & 3t & -36 & -3t \\
    3t & 4t^2 & -3t & -t^2 \\
    -36 & -3t & 36 & -3t \\
    -3t & -t^2 & -3t & 4t^2
\end{bmatrix}
\]

\[
M_e = \int_N N_e^T N_e \, dz = \begin{bmatrix}
    36 & 2t & 36 & 3t \\
    2t & 4t^2 & 3t & t^2 \\
    36 & 3t & 36 & -3t \\
    3t & t^2 & 3t & 4t^2
\end{bmatrix}
\]

\[
M_f = \int_N N_f^T N_f \, dz = \begin{bmatrix}
    36 & -2t & -36 & -3t \\
    -2t & 4t^2 & 3t & t^2 \\
    -36 & 3t & 36 & 3t \\
    -3t & t^2 & 3t & 4t^2
\end{bmatrix}
\]

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II. APPLICATION OF A GENETIC ALGORITHM.

GA is a global probabilistic search algorithm inspired by Darwin's survival-of-the fittest theory. In this optimization method, information about a problem, such as variable parameters, is coded into a genetic string known as an individual (chromosome). Each of these individuals has an associated fitness value, which is usually determined by the objective function to be maximized or minimized. Genetic algorithms have been shown to be able to solve the optimization problem through mutation, crossover and selection operation applied to individuals in the population.

II.I Population

The initial population are created randomly by generating the required number of individuals but a new population developed from this initial population and to do this must apply the genetic operator. The initial populations are generated by the following equation L. Randy Haupt, S. Ellen Haupt 2004:

\[ P = X_{LB} + \text{rand}(N_{pop}, N_{var})(X_{UB} - X_{LB}) \]  

(13)

Where:

\( (X_{UB}, X_{LB}) \) means the range of maximum and minimum values allowed for each variable respectively.

\( N_{pop} \) = The number of population.

\( N_{var} \) = The number of variable.

In this population, there are several individuals carrying different “genetic information“ in their string or coding. When working with binary coded genetic algorithms each of the real parameters to be optimized is translated to binary codes.

• To transform the real values \( b_i \) to binary codes the following equation is used H. M. Gomes and N. R. S. Silva (2007)

\[ s = \text{bin}_n \left\{ \text{round}(2^{n \text{bit}} - 1) \left[ b_i (k) - X_{LB} \right] \right. \\
\left. [X_{UB} - X_{LB}] \right\} \]

(14)

Where \( \text{bin}_n \) indicates a binary translation to a string \( s \), and \( n \text{bit} \) means the number of bit.

• To transform the binary codes to real values (decoding) the following equation is used.

\[ b_i (k) = X_{LB} + \text{bin}^{-1} (s) \left[ \frac{X_{UB} - X_{LB}}{2^{n \text{bit}} - 1} \right] \]

(15)

Where \( \text{bin}^{-1}(s) \) is the nonnegative integer decoded from the base 2 binary representation. From this equation it is obvious that the precision by the binary coding is \( (X_{UB} - X_{LB}) / (2^{n \text{bit}} - 1) \)

II.II Fitness Function

In order to determine the ability of an individual to search better solution, a fitness function is used to quantify how good the solution represented by a chromosome is. Depending on the problem characteristic, the fitness function can be any form of mathematical formulation, can be either a maximized or minimized function. This function generates an output from the set of input variables of a chromosome. The goal is to modify the output in some desirable fashion by finding the appropriate values of input variables.

In this work the two objective functions are used to assess the presence of damage in beam.

• Changes in Natural Frequencies.

• Modal Assurance Criterion.

Changes in Natural Frequencies

The natural frequency used as a diagnostic parameter in structural assessment procedures using vibration monitoring. One great advantage of using only eigenvalue in the damage assessment of structures is that they are cheaply acquired and the approach can
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give an inexpensive structural assessment technique. The objective function to be minimized is defined as follows M. T. V. Baghmisheh et al 2008:

\[ \Delta \omega = \sum_{i=1}^{n} (\omega_i^{T} - \omega_i^{S})^2 \]  

(16)

Where:

- \( \omega_i \) = Mode Number (i=1,2,3,....,n)
- \( \omega_i^{T} \) = Test natural frequencies
- \( \omega_i^{S} \) = Calculated natural frequencies.

The \( \omega_i^{T} \) are the natural frequencies which are applied to our damage detection system as inputs. An objective value of zero indicates an exact match between the values of frequencies.

**Modal Assurance Criterion.**

The Modal Assurance Criterion MAC value indicates the degree of correlation between two modes and varies from 0 to 1, with 1 for full correlation, and 0 for no-correlation. The deviation from 1 can be interpreted as a damage indicator in structures. This index is based on comparisons between the changes in the mode shapes obtained both from tests and from calculations, the MAC is defined by W. M. Ostachowicz et al. 1996:

\[
MAC(O_i, O_j) = \frac{\Theta_i^T \Theta_j}{\Theta_i^T \Theta_i \Theta_j^T \Theta_j} 
\]  

(17)

- \( \Theta_i \) = Test mode shape vector.
- \( \Theta_j \) = calculate mode shape vector.

**II.III Selection (reproduction)**

Reproduction is the first operator applied on a population. The first step in the reproduction is fitness assignment. Each individual receives a reproduction probability depending on the own objective (fitness) value and the objective value of all other individuals in the population. The evaluation of this objective function indicates which individuals will have more chances to procreate and to generate a large offspring.

There are various selection processes that are utilized in genetic algorithms such as roulette wheel selection, rank selection and tournament selection. A common process and used in this work are the roulette wheel selection. This selection method was used to copy individuals according to their fitness values, individuals with higher fitness have a higher probability of contributing one or more offspring in the next generation. For each population individual a probability of being selected for copying is given by the following equation D. E. Goldberg 1989:

\[
P_i = \frac{f_i}{\sum_{j=1}^{P_{size}} f_j} 
\]  

(18)

Where \( f_j \) is the fitness of individual \( j \), the sum is taken over all population members (P_size), and \( P_i \) is the probability of individual \( i \) with fitness \( f_j \) receiving an additional copy.

**II.IV Recombination (Crossover)**

Crossover is one of the recombination operators that is used for information exchange between any two individuals to create two offspring. Each pair of parents have a probability, \( P_c \), of producing offspring. Usually, a high crossover probability is used.

- **Real value Recombination:** The variable values of the offspring are chosen somewhere around and between the variable values of the parents. Offspring are produced according to the rule H. Pohlheim 2007:

\[
\text{Var}_i^O = \text{Var}_i^P_1 \cdot \alpha_i + \text{Var}_i^P_2 \cdot (1 - \alpha_i) 
\]  

(19)
Where $\alpha$ is a scaling factor chosen uniformly a random over an interval $[-0.25, 1.25]$ for each a new.

- Binary valued Recombination: The some of the crossover operators available in GA are single point crossover, two-point crossover and uniform crossover. In this work a single point crossover is applied, where one crossover position (n) along the string is selected randomly between 1 and the string length less one. Two new strings are created by swapping all characters between the individuals about this point.

II.V Mutation

Mutation means a random change in the information of a chromosome, to add diversity to the genetic characteristics of the population. It is applied at a certain probability, $P_m$, to each gene of the offspring, the mutation probability also called mutation rate, is usually a small value, to ensure that good solutions are not distorted too much. Mutation of real variables means, that randomly created values are added to the variables selected. The mutation rule is:

$$C = P + \text{rand} \ (X_{\text{UB}} - X_{\text{LB}}) \quad (20)$$

Where $C$ is mean the child and $P$ mean parent For binary mutation, randomly change a particular gene in a chromosome, thus, 1 may be changed to a 0 or vice versa.

II.VI Elitism

In the process of the crossover and mutation- taking place, there is high chance that the optimum solution could be lost. There is no guarantee that these operators will preserve the fittest string. To avoid this, the elitist models are often used. Elitism refers to the process of ensuring that the best chromosome (or few best chromosomes) of the current population survive to the next generation. The best individuals are copied to the new population without being mutated. Elitism can rapidly increase the performance of GA, because it prevents a loss of the best found solution M. Obitko1998

II.VII Termination

The GA may be terminated by using the convergence criterion in order to get an acceptable approximate solution, the terminate if there is no improvement over a number of consecutive generation, by monitoring the fitness of the best individual if there is no significant improvement over a time, GA is to stop. Or if the objective function value of the fittest individual is 0 or very small number, which means that the optimal solution has been found.

In the present work the chromosome has two variables, the damage location and the stiffness reduction. The objective function generates an output from the set of input variables of a chromosome. The goal is to modify the output in some desirable fashion by finding the appropriate values of input variables. Fig.2 shows the flowchart of the method of damage detection using genetic algorithms.

III. NUMERICAL SIMULATION

The processes of damage detection are demonstrated using (in and out-of-plane) simply supported curved beam. The dimensions and material properties for the simply supported in and out-of-plane curved beam are shown in Table 1 and Table 2 respectively.

In and out-of-plane simply supported curved beam is divided into 30 finite elements of equal length, where the value of first natural frequency is used for convergent test for checking the stability of the results as shown in the Fig. 3 and Fig. 4 for in and out-of-plane respectively.

Six damage scenarios are investigated and are summarized in Table 3. In the first four
cases for single damage, the scenarios were simulate by reducing the stiffness of an element near the beam’s end and near the beam’s mid-span. The remaining damage cases D5 and D6 in the same table correspond to a multiple damage scenario and were simulated by reducing the stiffness of assumed elements at two different locations. The following parameters of the GA have been used: size of the population is 40, probability of crossover $P_c$ is 0.9, probability of mutation $P_m$ is 0.05, number of elitism is 2 and number of bit is 20.

IV. RESULTS AND DISSCUSSION

The frequency predictions from the FEM model of undamaged beam are validated by comparing with other researches as shown in Tables 4 and Table 5 for in and out-of-plane curved beam respectively.

IV.I Objective Function Based on Change in Natural Frequency.

The input first five natural frequencies of damage scenarios are shown in Table 6 and Table 7 for out-of-plane and in-plane curved beam respectively. A population of individuals is generated randomly then the natural frequencies and objective function are calculated for each individual. The GAs theory is used to find the optimal location and stiffness reduction by minimizing the eq. (16). For each scenario the algorithm is run from five different initial random population and the identified values for damage scenarios by using CGA and BGA are shown in Table 8 for out-of-plane and Table 9 for in-plane curved beam. In all scenarios there are no error recorded in prediction of damage element and the errors for the CGA are less than corresponding values for the BGA.

In all scenarios there are no error recorded in prediction of damage element and the errors for the CGA are less than corresponding values for the BGA, because in the CGA deals with real values without using any encoding method.

Fig. 5 show the typical objective function curve for out-of-plane at D4 by using CGA, it is see that the objective function value tends to zero with the increasing number of generations and reach zero at around 21 generations. The

Fig. 6 shows the objective function curves at same damage scenario but using BGA, the convergence occurs at 28 generation.

IV.II Objective Function Based on Modal Assurance Criterion (MAC)

The mode shapes are calculated numerically using finite element model for the damaged scenarios, these used as test inputs for the GA operator. A population of individuals is generated randomly then the objective function is calculated for each individual and the GAs theory is applied. For each scenario the algorithm is run in five different initial randomly generated populations and the average results obtained by CGA and BGA listed in Table 10 for out-of-plane and Table 11 for in-plane curved beam. The errors for CGA are less than corresponding values for the BGA.

For out-of-plane curved beam the objective function with multi damage for D5 using CGA is shown in Fig. 7 it can seen that convergence occurs at 15 generations.

V. CONCLUSIONS

The main conclusions from the present work may be stated as follows:

- The study shows that the genetic algorithm is effective in identifying positions and extents in single and multi damage.
- The results obtained from continuous genetic algorithms are more accurate then those obtained from binary genetic algorithms in damage assessment.
- The length of the run (in terms of generation number) and results depends on the initial randomly generated population and GA parameters and the test point.
- The objective function based on change in natural frequency is the best objective function, because the stiffness reduction has a relatively large
effect on the natural frequencies, as compared with mode shapes, it is insensitive of the modes to the damage.

REFERENCES


Notation

A  Sectional area (m²)

B  Bimoment (N .m)

E  Young modulus (N/m²)

G  Shear modulus (N/m²)

GA  Genetic Algorithm.

I_y  Area moment of inertia about y-axis (m⁴)

I_x  Area moment of inertia about x-axis (m⁴)

I_w  Warping moment of inertia (m⁶)

J  Area polar moment of inertia (m⁴)

K_T  St Venant constant of a straight member(m⁴)

l  Length of the finite element (m, cm)

M_{x,y}  Moment about x- and y-axis (N.m)

MAC  Modal Assurance Criterion

m_{x,y,z}  Uniform distributed moments about x-, y-, and z-axis

m_0  Uniform distributed bimoment

N_{POP}  Number of population

N_{var}  Number of variable

P_{siz}  Population size

q_{x,y,z}  Uniform distributed forces about x-, y-, and z-directions

R  Radius of initial curvature (m)

T  Kinetic energy (N.m)

U  Strain energy (N.m)

u_{x,y}  Displacement components of the shear center in x- and y-directions, respectively

V  Volume of body (m³)

\nu  Transverse shear forces (N)

\nu_{ave}  Average longitudinal displacement of cross-section

X_{UB}  Maximum value of variable

X_{LB}  Minimum value of variable

Greek letters

\rho  Mass density (Kg/m³)

\beta  Rotation of the cross-section about z-axis

\theta  Subtended angle (degree)

z_{i,j}  Components of strain tensor

\delta  Variation

\gamma_{i}  Nodal displacements

\epsilon_{i,j}  Components of stress tensor
### Table 1 Material properties of the in-plane curved beam

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of cross section (A)</td>
<td>$4 \times 10^{-3} \text{m}^2$</td>
</tr>
<tr>
<td>Radius of the arch (R)</td>
<td>2.438 m</td>
</tr>
<tr>
<td>Mass density ($\rho$)</td>
<td>7850 kg/m$^3$</td>
</tr>
<tr>
<td>Subtended angle ($\theta$)</td>
<td>97$^\circ$</td>
</tr>
<tr>
<td>Modulus of Elasticity (E)</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Modulus of Rigidity (G)</td>
<td>77 GPa</td>
</tr>
<tr>
<td>Moment of inertia (I)</td>
<td>$6.45 \times 10^{-6} \text{m}^4$</td>
</tr>
</tbody>
</table>

### Table 2 Material properties of the out-of-plane curved beam

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of cross section (A)</td>
<td>$9.3 \times 10^{-3} \text{m}^2$</td>
</tr>
<tr>
<td>Length (L)</td>
<td>10 m</td>
</tr>
<tr>
<td>Mass density ($\rho$)</td>
<td>7850 kg/m$^3$</td>
</tr>
<tr>
<td>Subtended angle ($\theta$)</td>
<td>89$^\circ$</td>
</tr>
<tr>
<td>Modulus of Elasticity (E)</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Modulus of Rigidity (G)</td>
<td>77 GPa</td>
</tr>
<tr>
<td>Moment of inertia (Ix)</td>
<td>$1.13 \times 10^{-6} \text{m}^4$</td>
</tr>
<tr>
<td>Moment of inertia (Iy)</td>
<td>$3.88 \times 10^{-6} \text{m}^4$</td>
</tr>
<tr>
<td>Warping moment of inertia (I$\omega$)</td>
<td>$5.56 \times 10^{-7} \text{m}^6$</td>
</tr>
<tr>
<td>Venant constant ($K_T$)</td>
<td>$5.38 \times 10^{-7} \text{m}^4$</td>
</tr>
</tbody>
</table>
Table 3 Damage scenario for in and out-of-Plane curved beam

<table>
<thead>
<tr>
<th>Damage Scenario</th>
<th>Damage Element</th>
<th>Stiffness reduction %</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>8</td>
<td>25</td>
</tr>
<tr>
<td>D2</td>
<td>11</td>
<td>50</td>
</tr>
<tr>
<td>D3</td>
<td>16</td>
<td>25</td>
</tr>
<tr>
<td>D4</td>
<td>25</td>
<td>50</td>
</tr>
<tr>
<td>D5</td>
<td>6,15</td>
<td>25</td>
</tr>
<tr>
<td>D6</td>
<td>8,25</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 4 Comparisons of modal frequencies for in-plane curved beam

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural Frequency(rad/sec)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[Ki. Young et al] Results</td>
<td>Present Numerical Results</td>
</tr>
<tr>
<td>1</td>
<td>396.98</td>
<td>396.936</td>
</tr>
<tr>
<td>2</td>
<td>931.22</td>
<td>930.94</td>
</tr>
<tr>
<td>3</td>
<td>1797.31</td>
<td>1796.67</td>
</tr>
</tbody>
</table>

Table 5 First natural frequencies for the simply supported out-of-plane curved beam

<table>
<thead>
<tr>
<th>Subtended Angle (degree)</th>
<th>Natural Frequency (rad/sec)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical Results[Ki-Young et al]</td>
<td>Numerical Results[Ki-Young et al]</td>
</tr>
<tr>
<td>0</td>
<td>53.3000</td>
<td>53.3000</td>
</tr>
<tr>
<td>10</td>
<td>31.8648</td>
<td>31.8699</td>
</tr>
<tr>
<td>20</td>
<td>19.9616</td>
<td>19.9614</td>
</tr>
<tr>
<td>30</td>
<td>13.9944</td>
<td>13.9931</td>
</tr>
<tr>
<td>40</td>
<td>10.5386</td>
<td>10.5372</td>
</tr>
<tr>
<td>50</td>
<td>8.2946</td>
<td>8.2888</td>
</tr>
<tr>
<td>60</td>
<td>6.7121</td>
<td>6.7012</td>
</tr>
<tr>
<td>70</td>
<td>5.5270</td>
<td>5.5090</td>
</tr>
<tr>
<td>80</td>
<td>4.5991</td>
<td>4.5707</td>
</tr>
<tr>
<td>90</td>
<td>3.8479</td>
<td>3.8048</td>
</tr>
</tbody>
</table>
Table 6 Natural frequencies for out-of-plane curved beam

<table>
<thead>
<tr>
<th>Damage Scenario</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
<th>$\omega_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>3.8583</td>
<td>45.456</td>
<td>168.575</td>
<td>381.32</td>
<td>661.097</td>
</tr>
<tr>
<td>D2</td>
<td>3.8452</td>
<td>45.113</td>
<td>168.525</td>
<td>373.46</td>
<td>647.75</td>
</tr>
<tr>
<td>D3</td>
<td>3.8664</td>
<td>45.532</td>
<td>168.139</td>
<td>381.2</td>
<td>658.47</td>
</tr>
<tr>
<td>D4</td>
<td>3.867</td>
<td>45.629</td>
<td>168.98</td>
<td>381.195</td>
<td>664.313</td>
</tr>
<tr>
<td>D5</td>
<td>3.8461</td>
<td>45.286</td>
<td>166.86</td>
<td>379.147</td>
<td>565.786</td>
</tr>
<tr>
<td>D6</td>
<td>3.8372</td>
<td>45.211</td>
<td>167.29</td>
<td>379.23</td>
<td>660.017</td>
</tr>
</tbody>
</table>

Table 7 Natural frequencies for in-plane curved beam

<table>
<thead>
<tr>
<th>Damage Scenario</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
<th>$\omega_3$</th>
<th>$\omega_4$</th>
<th>$\omega_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>392.685</td>
<td>926.68</td>
<td>1796.3</td>
<td>1980.6</td>
<td>2897.9</td>
</tr>
<tr>
<td>D2</td>
<td>389.175</td>
<td>929.03</td>
<td>1747.9</td>
<td>1963.6</td>
<td>2869.4</td>
</tr>
<tr>
<td>D3</td>
<td>396.876</td>
<td>921.37</td>
<td>1795.6</td>
<td>1980.4</td>
<td>2885.7</td>
</tr>
<tr>
<td>D4</td>
<td>386.261</td>
<td>905.81</td>
<td>1770.2</td>
<td>1960.9</td>
<td>2901.9</td>
</tr>
<tr>
<td>D5</td>
<td>386.182</td>
<td>878.912</td>
<td>1767.5</td>
<td>1923.3</td>
<td>2825.8</td>
</tr>
<tr>
<td>D6</td>
<td>374.657</td>
<td>893.524</td>
<td>1768.9</td>
<td>1926.9</td>
<td>2858.3</td>
</tr>
</tbody>
</table>

Table 8 Identified stiffness parameters for out-of-plane curved beam based on change in natural frequency

<table>
<thead>
<tr>
<th>Test Element No.</th>
<th>Actual</th>
<th>Identified by CGA</th>
<th>Error %</th>
<th>Identified by BGA</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.75</td>
<td>0.7491003</td>
<td>0.11</td>
<td>0.749992</td>
<td>0.001</td>
</tr>
<tr>
<td>11</td>
<td>0.5</td>
<td>0.5002577</td>
<td>0.051</td>
<td>0.500381</td>
<td>0.076</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>0.7503839</td>
<td>0.051</td>
<td>0.7500715</td>
<td>0.01</td>
</tr>
<tr>
<td>25</td>
<td>0.5</td>
<td>0.4999999</td>
<td>0.00001</td>
<td>0.50000152</td>
<td>0.003</td>
</tr>
<tr>
<td>6.15</td>
<td>0.75</td>
<td>0.7555629</td>
<td>0.48</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8.25</td>
<td>0.75</td>
<td>0.7334992</td>
<td>2.2</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 9: Identified stiffness parameters for in-plane curved beam based on change in natural frequency

<table>
<thead>
<tr>
<th>Test Element No.</th>
<th>Actual</th>
<th>Identified by CGA</th>
<th>Error %</th>
<th>Identified by BGA</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.75</td>
<td>0.74999999</td>
<td>0.00001</td>
<td>0.7500007</td>
<td>0.00009</td>
</tr>
<tr>
<td>11</td>
<td>0.5</td>
<td>0.5001478</td>
<td>0.03</td>
<td>0.4999847</td>
<td>0.003</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>0.74999999</td>
<td>0.00001</td>
<td>0.7499988</td>
<td>0.0001</td>
</tr>
<tr>
<td>25</td>
<td>0.5</td>
<td>0.5000117</td>
<td>0.002</td>
<td>0.5000152</td>
<td>0.003</td>
</tr>
<tr>
<td>6.15</td>
<td>0.75</td>
<td>0.7514097</td>
<td>0.18</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8.25</td>
<td>0.75</td>
<td>0.7481410</td>
<td>0.24</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 10: Identified stiffness parameters for out-of-plane curved beam based on MAC

<table>
<thead>
<tr>
<th>Test Element No.</th>
<th>Actual</th>
<th>Identified by CGA</th>
<th>Error %</th>
<th>Identified by BGA</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.75</td>
<td>0.7505467</td>
<td>0.07</td>
<td>0.753685</td>
<td>0.5</td>
</tr>
<tr>
<td>11</td>
<td>0.5</td>
<td>0.5023270</td>
<td>0.46</td>
<td>0.5076601</td>
<td>1.5</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>0.7517007</td>
<td>0.22</td>
<td>0.7468184</td>
<td>0.42</td>
</tr>
<tr>
<td>25</td>
<td>0.5</td>
<td>0.5046008</td>
<td>0.92</td>
<td>0.5088810</td>
<td>1.77</td>
</tr>
<tr>
<td>6.15</td>
<td>0.75</td>
<td>0.7340752</td>
<td>2.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8.25</td>
<td>0.75</td>
<td>0.7702559</td>
<td>2.7</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 11: Identified stiffness parameters for in-plane curved beam based on MAC

<table>
<thead>
<tr>
<th>Test Element No.</th>
<th>Actual</th>
<th>Identified by CGA</th>
<th>Error %</th>
<th>Identified by BGA</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.75</td>
<td>0.7430990</td>
<td>0.92</td>
<td>0.7757013</td>
<td>3.4</td>
</tr>
<tr>
<td>11</td>
<td>0.5</td>
<td>0.5044385</td>
<td>0.88</td>
<td>0.4923729</td>
<td>1.5</td>
</tr>
<tr>
<td>16</td>
<td>0.75</td>
<td>0.7547156</td>
<td>0.63</td>
<td>0.7459007</td>
<td>0.54</td>
</tr>
<tr>
<td>25</td>
<td>0.5</td>
<td>0.5123102</td>
<td>2.4</td>
<td>0.5155471</td>
<td>3.1</td>
</tr>
<tr>
<td>6.15</td>
<td>0.75</td>
<td>0.7256023</td>
<td>3.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8.25</td>
<td>0.75</td>
<td>0.7829436</td>
<td>4.3</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Fig. 1. Curved beam element
Fig. 2 Flowchart of suggested damage detection method using GAs
Fig. 3 Convergence test for in-plan curved beam
Fig. 4 Convergence test for out-of-plan curved beam
Damage Detection and Assessment of Stiffness and Mass Matrices in Curved Simply Supported Beam Using Genetic Algorithm

Asst. Prof. Dr. Nabil Hassan Hadi
Aveen A. Abdulkareem

![Graph 1](image1.png)

**Fig. 5** A Typical objective function curve of CGA for out-of-plane curved beam

![Graph 2](image2.png)

**Fig. 6** A Typical objective function curve of BGA for out-of-plane curved beam

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Fig. 7 A Typical objective function curve of CGA for out-of-plane curved beam