



MODELING OF INDUCTION HEATING PROCESS OF A CONDUCTING FERROMAGNETIC MATERIALS

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ABSTRACT

This research deals with a modeling of the heating process of an induction furnace with a conducting ferromagnetic core. This is done by performing a sequential coupling between two types of analysis; the harmonic electromagnetic analysis and the transient thermal analysis in order to furnish a complete numerical simulation of this device. The time elapsed to apply this procedure will be too long and unreasonable because of the consideration of the nonlinearity of the magnetic circuit. This problem is solved by the idea of the "Effective Relative Permeability", which is a constant relative permeability equivalent to the nonlinear one in calculating the eddy current losses.

The Finite Element Method (FEM) is considered in order to solve this problem using the "ANSYS" computer package. An efficient coupling program deals with heat treatment procedure for any axi-symmetrical non-linear work piece has been built. The obtained results show a good agreement with the published practical measurements. This algorithm is a computer aided design for a nonlinear induction furnace with rotational symmetry, hence it can be used to design such devices instead of the traditional trial and error, experience dependant, design methods which lead to a costly and time wasting design.

خلاصة

يعنى هذا البحث بنمذجة عملية التسخين في فرن حثي ذو قلب مكون من مادة فيرومغناطيسية. لقد تم ذلك بتنفيذ تعشيق متعاقب لنوعين من التحليلات هما: التحليل الكهرومغناطيسي التوافقي و التحليل الحراري العابر بغية الحصول على تمثيل رقمي متكامل لهذا الجهاز. أن الوقت المستغرق لتنفيذ هذه الخطوات سيكون طويل جدا وغير منطقي بسبب أخذ لاختية الدائرة المغناطيسية بنظر الاعتبار. عليه فقد تم حل هذه المعضلة بأعتماد فكرة "السماحية النسبية المكافئة"، والتي هي عبارة عن سماحية نسبية ثابتة القيمة ومكافئة لتلك للاختية عند احتساب خسائر التيارات الدوامة.

لقد أعتمدت طريقة العنصر المحدد (FEM) لغرض حل هذه المشكلة بأستخدام برامجيات الحاسبة (ANSYS)، حيث تم بناء برنامج كفاء للتعشيق يعنى بسياقات المعاملات الحرارية المختلفة للعينات للاختية المتناضرة محوريا. أن النتائج المستحصلة أظهرت أنسجما جيدا مع القياسات العملية المنشورة. لذا فأن هذا النظام يعتبر بمثابة تصميم بمساعدة الحاسبة للافران الحثية للاختية المتناضرة دورانيا، لذا يمكن استخدامه كبديل عن الطرق التقليدية التي تستخدم الخبرة و أعتداد أسلوب التجربة و أخطأ مما يؤدي الى تصاميم مكلفة وتستغرق وقتا طويلا.

INTRODUCTION

It is already known that the induced eddy currents inside the induction furnace core (the work piece or the specimen) is not uniform, and it depends on many factors like, the frequency of the excitation current, the shapes of the furnace and the specimen to be heated, and on the electrical conductivity and the permeability of the core material due to the equation [1]

$$\delta = \sqrt{2\rho_e / \mu f} \dots\dots\dots (1)$$

The mathematical simulation of a non-linear induction furnace is a complicated problem due to the following factors:

1. The induced eddy currents will heat the specimen gradually, and since the physical properties of the core material vary with temperature, then each part inside the core will have its different properties due to its temperature which will lead to change its behavior during the heating period, so the electromagnetic analysis should be followed by a transient thermal analysis to calculate the temperature inside the core for small period of time. This procedure abbreviated in the flow chart shown in Fig. 1
2. The non uniform distribution of the induced eddy currents inside the core leading to a non uniform heat distribution inside it, this fact leads to divide the core into small parts and evaluate the parameters of each part due to its temperature.
3. The permeability is a function of three variables, the flux density *B*, the magnetic intensity *H*, and the temperature *T* simultaneously in a non-linear way. This factor affects the analysis as follows:

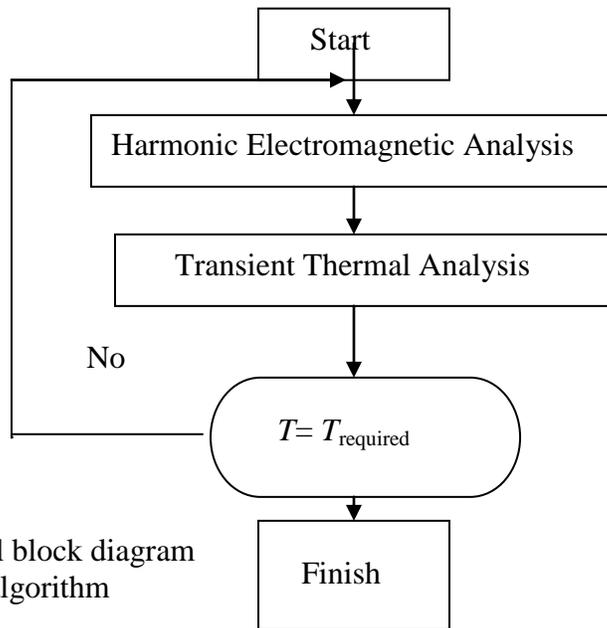


Fig. 1 General block diagram of the algorithm



- a. Considering the ($B-H$) nonlinearity as it is manifested in the relationship will make the electromagnetic solution very slow. Because it is necessary to march through time with a time step much smaller than the period of the driving magnetic field.
- b. The numerical analysis deals with sinusoidal functions only, but due to the nonlinear ($B-H$) relation of the ferromagnetic materials, if B is considered sinusoidal, then H will be non-sinusoidal and if H considered sinusoidal B will be not. So this conflict should be arranged for.
- c. The measurement of the ($B-H$) relation of the ferromagnetic material in high temperatures is a difficult task, so the researches often use an approximated formula to overcome this problem. Such calculations should be done separately in order to prepare the ($B-H$) relations of other temperatures
- d. To solve the problems in (a) and (b) above, the idea of the "Relative Effective Permeability" $\mu_{r,eff}$, which is a constant relative permeability effectively provides the same energy deposition that is produced when the true permeability is used. The quantity $\mu_{r,eff}$ will be a function of H and T simultaneously, as will be shown later on. So, this will lead to make the electromagnetic analysis itself an iterative process in order to update the value of the relative permeability with each temperature reached and for the right value of H consequently.

Then the real behavior of the core can be achieved during the heating period by dividing the core into a very small parts and a harmonic electromagnetic analysis is done taking into consideration the material parameters for each part alone to determine its induced eddy current losses, then these values considered as input data to a transient thermal analysis step to study the heat transfer for the core for a short period of time and determine the temperature distribution inside the core at the end of that period. Hence to calculate the temperature distribution for the next short period of time the above procedure must be repeated after updating the properties due to the new temperature of each part and so on until the heating period is finished by reaching the required temperature distribution inside the core.

This problem solved numerically by digital computer using the Finite Element Method (FEM). A macro program is written in (APDL) (ANSYS Programming Design Language) to satisfy all the non-linear electromagnetic and thermal analysis. This algorithm is done to model this process for any furnace with rotational symmetry. The algorithm can tell the following information about this process for a certain specimen in shape and type of material to be treated thermally in a certain way for certain purpose:

- (1) The excitation current required.
- (2) The frequency of that current.
- (3) The thermal distribution inside the specimen.
- (4) The thermal response of any point during the heating period.

Theoretical Background:

i. Electromagnetic Analysis

It is already known that the complex electromagnetic equations can be greatly simplified based on magnetic vector potential A . All quantities related to the induction Joule heat can be achieved, based on the ac current, I , input to the induction coil. A can be expressed with Biot-Savart's law [1]

$$A = \frac{\mu_o I}{4\pi} \int_s \frac{dl}{|r|} \dots\dots\dots (2)$$

The magnetic flux density B is defined as

$$B = \nabla \times A \dots\dots\dots (3)$$

From Faraday’s law, the electric field intensity E and magnetic field intensity H are related by the following equations:

$$\nabla \times E = -\frac{\partial B}{\partial t} \dots\dots\dots (4)$$

$$H = B / \mu \dots\dots\dots (5)$$

Based on Ampere’s circuital law, the current density, J , induced in work piece is

$$J = \nabla \times H - \frac{\partial(\epsilon \cdot E)}{\partial t} \dots\dots\dots (6)$$

After a mathematical manipulations of above formulas, the relationship between the magnetic vector potential, A , and the induced current density, J , becomes

$$J = \nabla \times \left(\frac{1}{\mu} \nabla \times A \right) + \sigma_e \frac{\partial A}{\partial t} \dots\dots\dots (7)$$

Finally, Joule heat generated from eddy currents is

$$Q_{induction} = \frac{J^2}{\sigma_e} \dots\dots\dots (8)$$

Where, μ, ϵ, σ , are the magnetic permeability, permittivity, and electrical conductivity of the medium respectively. These properties are all temperature dependent, and this classifies the electromagnetic analysis as a highly non-linear problem.

ii. Effective Relative Permeability

The calculation of $\mu_{r,eff}(H, T)$, must be started from the standard $B-H$ Curve at the ambient temperature. To do so, a function used by Vasiliev [2] for determination of the incremental relative permeability as a function of both T and H used as follows:

$$\mu_{ri}(H, T) = 1 + (\mu_{ri}(H)_{T=0} - 1) \cdot \left[1 - (T / T_{curie})^2 \right] \quad \text{when } T < 750^\circ C \dots\dots (9)$$

$$\mu_{ri}(H, T) = 1 \quad \text{when } T \geq 750^\circ C$$

Where: $\mu_{ri}(H, T)$: Is the incremental relative permeability for each H as a function of temperature T and field intensity H .

$\mu_{ri}(H)_{T=0}$: Is the incremental relative permeability for each H as a function of H when $T = 0^\circ C$.

Razzaq [3], provides the $B-H$ curve at room temperature for Steel C45 (SAE 1045) as a curve for B (in Tesla) Versus H (in Oersted). A transformation in units of H is done to be in MKS system (Where 1 Oe= 79.577 A/m) then the magnetization curve drawn to be as shown in Fig. (2). Curie temperature for steel C45 is 750°C.

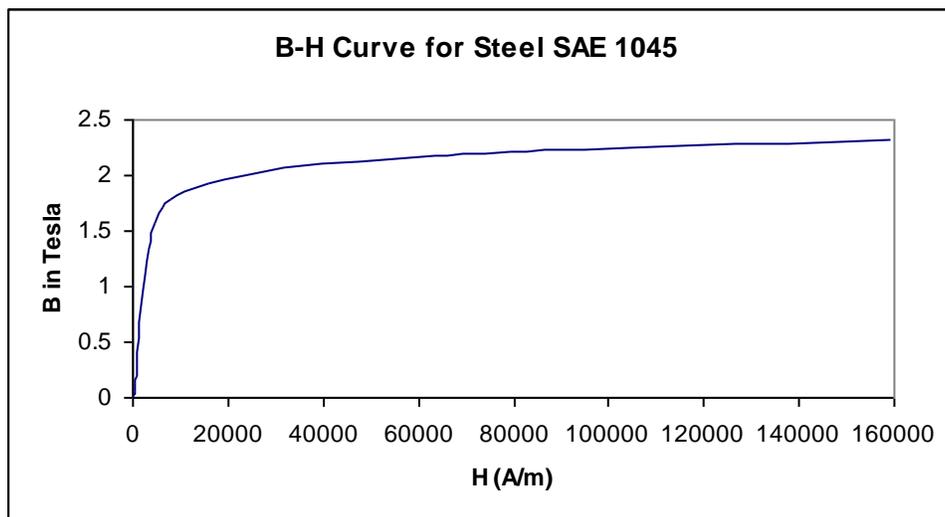


Fig. (2) The magnetization Characteristics of steel C45 at room temperature

The incremental relative permeability μ_{ri} calculated from the given $B-H$ data at $T=25^\circ C$ for certain value of H as $\mu_{ri}(H)_{T=0} = [(\Delta B / \Delta H) / \mu_o]$ and substituted in (9) for the required value of T the result will be $\mu_{ri}(T, H)$ for that value of H for the required temperature. Then ΔB for the required value of T can be determined since, $\Delta B_T = \mu_o * \mu_{ri}(H, T) * \Delta H$. Then, by dividing the main $B-H$ curve to many steps and the incremental relative permeability calculated for each step as shown in Fig. (3), then a new $B-H$ curve for any value of T can be achieved sequentially. Fig. (4) shows group of curves for different temperatures for the carbon steel C45.

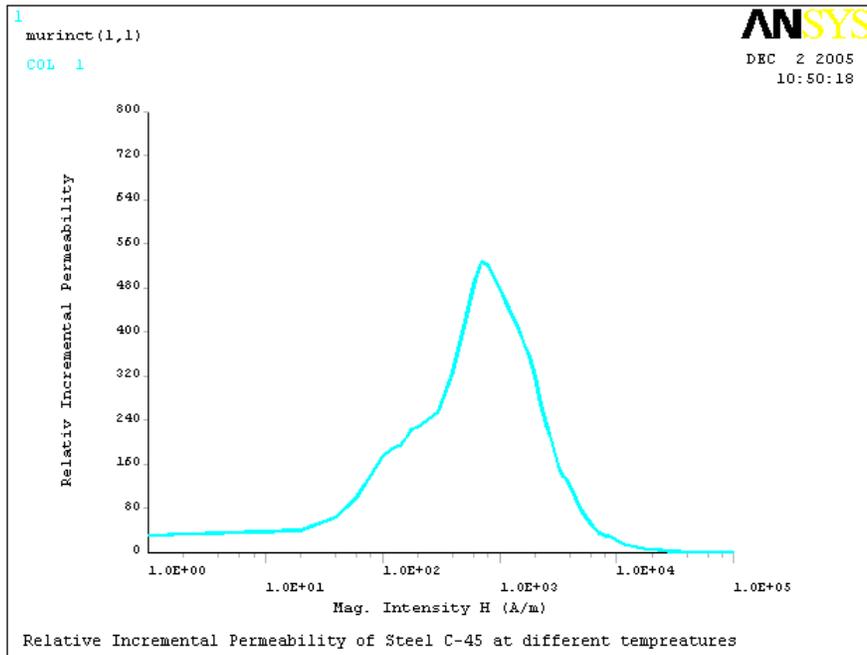


Fig. (3). Incremental relative permeability for steel C45

Fadhil [4] presents the idea of average permeability to achieve the effective relative permeability because neither B nor H is sinusoidal. Hence, for the case of considering B is sinusoidal only, while H will have a distorted shape due to B - H non linearity the average permeability will be

$$\mu_{ef1} = \frac{1}{T_c} \int_0^T \frac{B_{max} \sin \omega t}{H_B(t)} dt \dots\dots\dots (10)$$

Where $H_B(t)$: represents the distorted time variation of the field intensity.

T_c : represents the time period of one cycle.

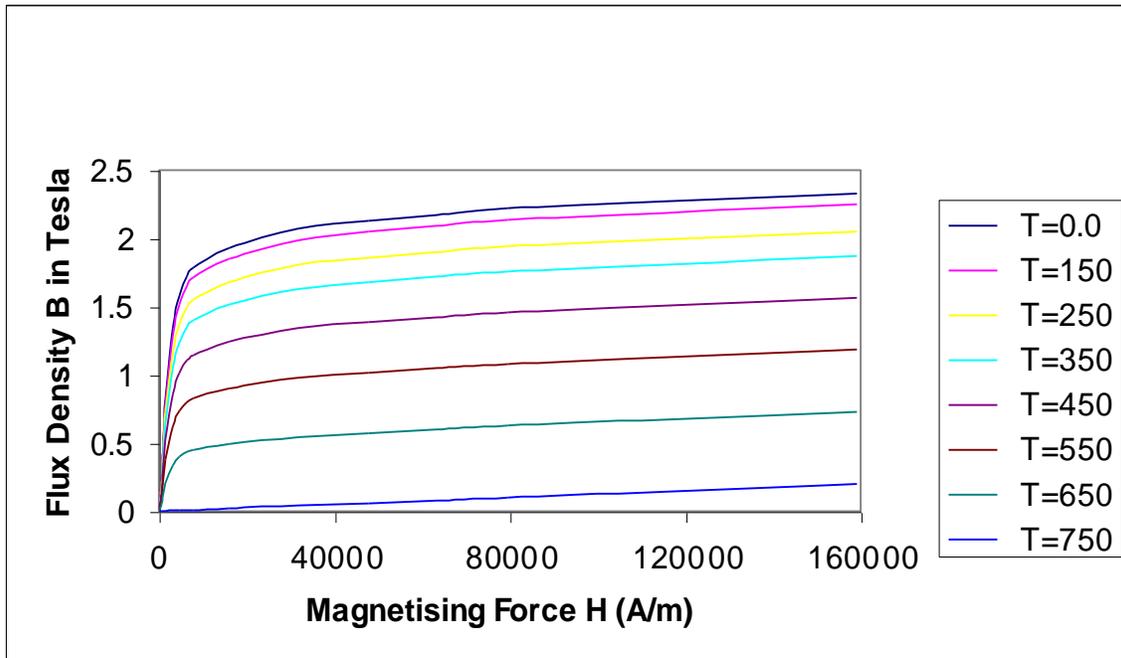


Fig. (4) The Magnetic Characteristics (*B-H* curves) for Steel C45 at different temperatures

For the case of considering *H* is sinusoidal only, while *B* will have a distorted shape due to *B-H* non linearity the average permeability will be

$$\mu_{eff2} = \frac{1}{T_c} \int_0^T \frac{B_H(t)}{H_{max} \sin \omega t} dt \dots\dots\dots (11)$$

Where $B_H(t)$: Is the distorted time variation of the flux density.

Since, in the real case *B* and *H* are non sinusoidal, therefore averaging both permeability would be a reasonable approximation to the effective permeability, then

$$\mu_{eff} = (\mu_{eff1} + \mu_{eff2}) / 2 \dots\dots\dots (12)$$

Using equation (12) the effective relative permeability as a function of *H* and *T* can be determined as shown in Fig. (5). It is clear that Fadhil’s method deals carefully with low intensity region.

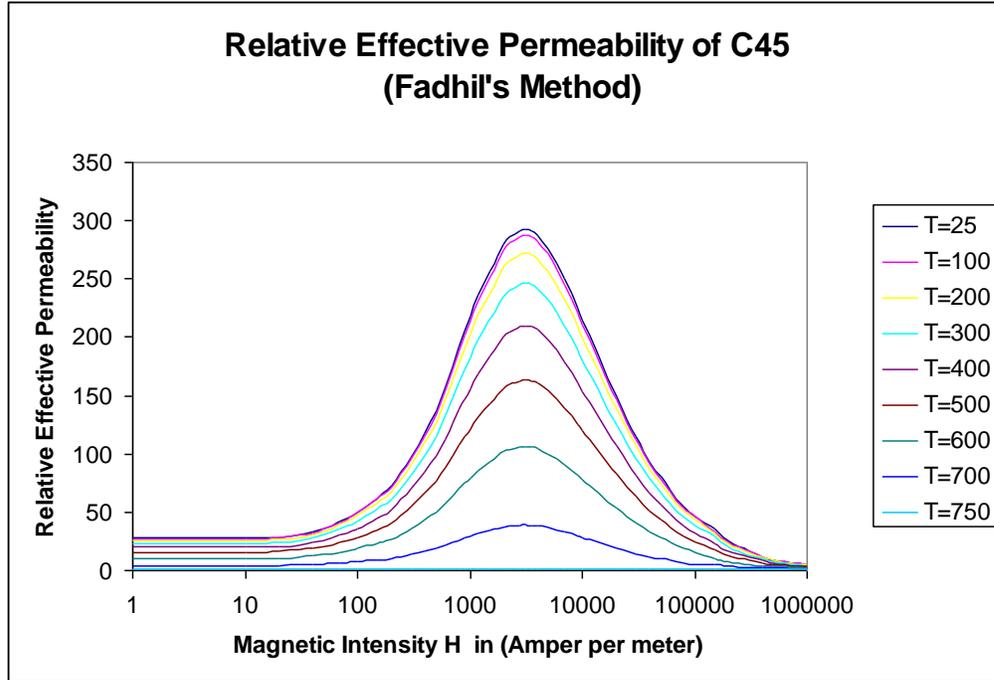


Fig. 5 The relative effective permeability of steel C45

iii. Thermal Analysis

Thermal analysis includes conduction, convection and radiation. The heating process is coupled with the electromagnetic conversion process. In the heating process, the heat conduction effect is the main issue to be considered because the heating cycle is generally very short as several seconds. The conduction equation is [2],

$$\rho_m \cdot C_p \cdot \frac{\partial T}{\partial t} = k \cdot \nabla^2 T + Q_{induction} \dots \dots \dots (13)$$

On surfaces of work piece, the heat conduction, radiation and free convection effects are all needed. The governing equation is,

$$\rho_m \cdot C_p \cdot \frac{\partial T}{\partial t} = k \cdot \nabla^2 T + Q_{induction} - \xi \cdot \sigma_s \cdot (T^4 - T_{air}^4) - \alpha \cdot (T - T_{air}) \dots \dots (14)$$

Where: T_{air} Is the ambient temperature

The Model:

Jorg Cstroeski [5], published a practical measurements on a ferromagnetic material. These measurements considered as a verification of this modeling. Hence the exact furnace used by Jorg is studied in this analysis.

The furnace under study in this case is no more than a "cylindrical shape core surrounded by an inductor with the shape of torus" [5], as shown in Fig. (6). "The cylinder was made of steel C45, it had a radius of 2cm, and it was 10cm long. The torus

was made of copper, it had an interior radius of 2.5cm, an exterior radius of 3.5cm, and its height was 1cm. Three currents were used for the excitation: 10500A, 7875A, and 5250A. The frequency was 10 kHz. In the first 0.2 seconds of the heating phase, only 50% of the exciting current is used, due to the heating regulation of the heating machine. The maximum surface temperature of the cylinder during the heating phase was measured with the aid of a thermo graphical camera. Of course, the maximum surface temperature is located at the center of the cylinder, close to the inductor." [5]

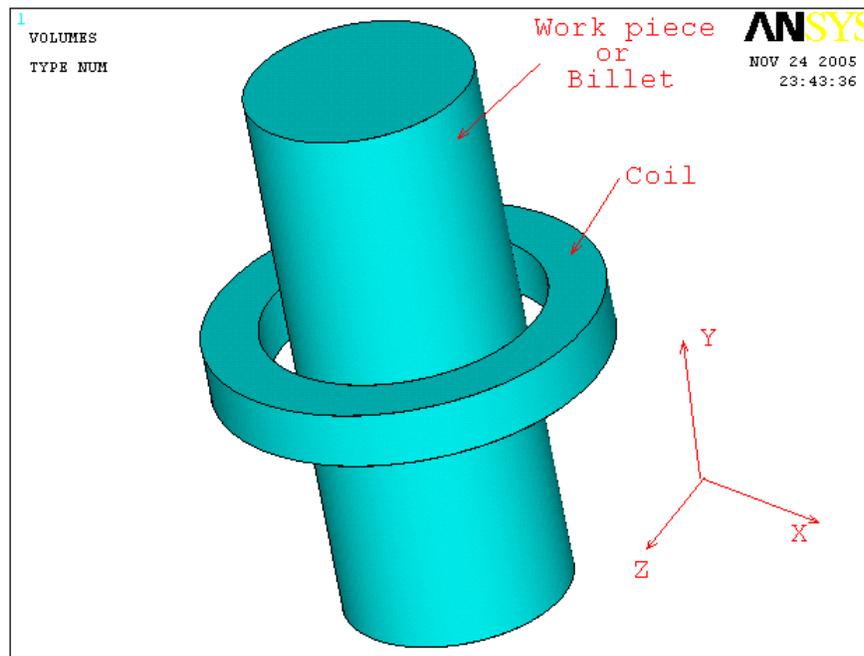


Fig. (6) Setting experiment

The electrical resistivity $\rho(T)$ [5] of steel C45 is shown in Fig. 7, The thermal properties, like the thermal conductivity and the enthalpy [5] are shown in Fig.(8) and Fig.(9).

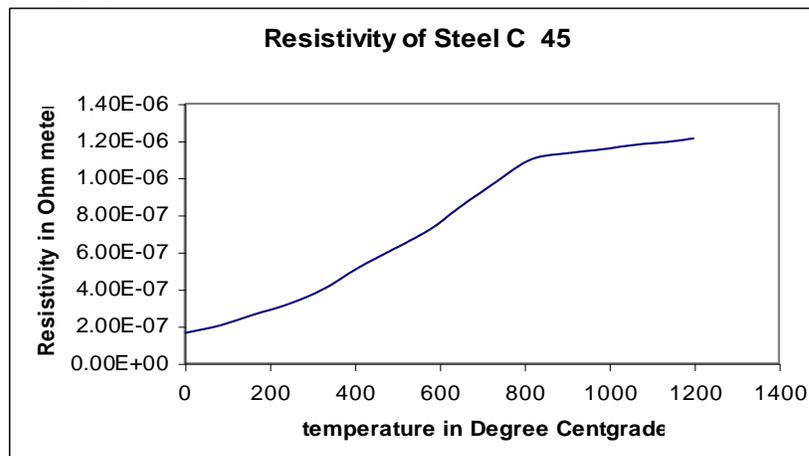


Fig. (7) The electrical resistivity of steel C45 as a function of temperature

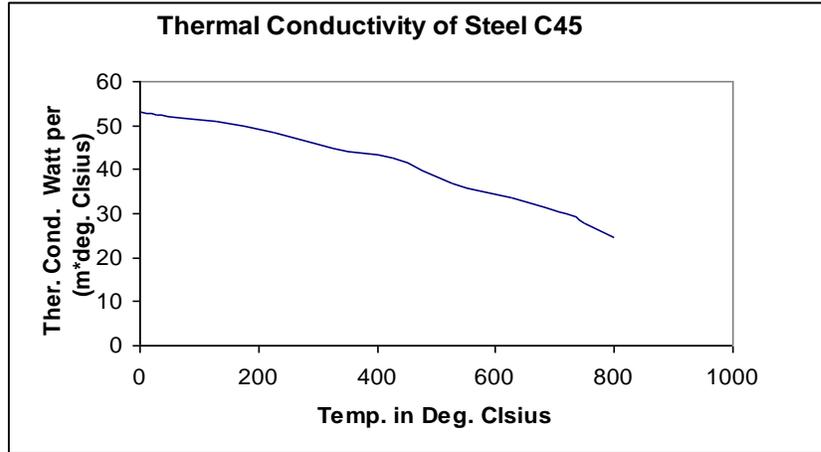


Fig. (8) Thermal Conductivity of Steel C45

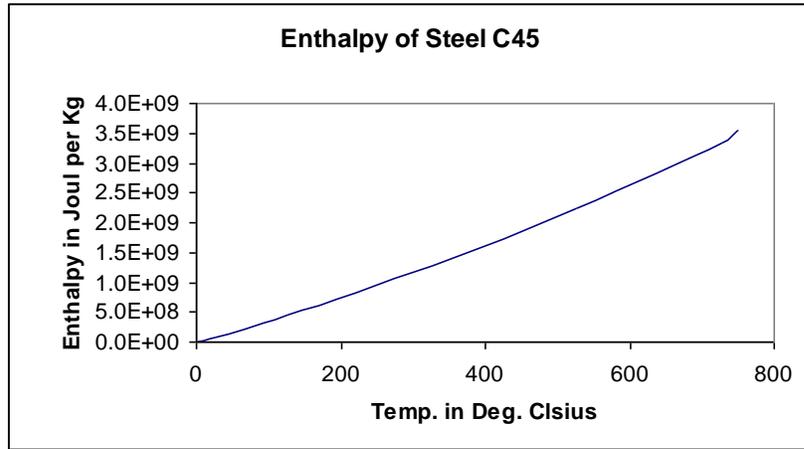


Fig. (9) Enthalpy of Steel C45 as a function of temperature

The Furnace Simulation using FEM

To simulate this furnace, A procedure of many steps should be followed. The first step is the determination of the system geometry and the boundary region, then meshing the different areas of the model and external surfaces of the billet in order to consider the dissipation of heat due to radiation. The model of furnace is an axi-symmetric one, so a quarter of its section is considered as shown in Fig.(10), which represent the areas of the Finite Element Model. The mesh used for this model is shown in Fig.(11), in which the surface element mesh used in the thermal analysis is cleared also. This part of the procedure is represented as step 1 in Fig. 12 which shows the detailed flow chart of the algorithm

In step 2 the program scans the main four tables of data $\rho_e(T)$, $\mu_{r,eff}(H,T)$, $\lambda(T)$, $Enth(T)$ for the core material. The initial temperature for the whole elements in the core is the ambient temperature. Step 3 is very important in using the $\mu_{r,eff}(H,T)$ three dimensional table of data in that the value of H is zero at any point at time zero before

applying the current, hence the determination of the $\mu_{r,eff}$ in this step will give a wrong results of H distribution

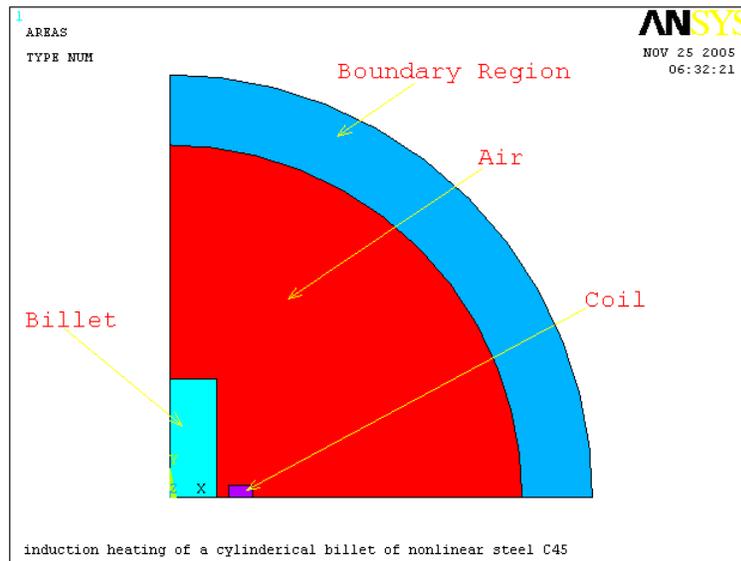


Fig. (10) The areas forming a quarter of the FE Model of the furnace

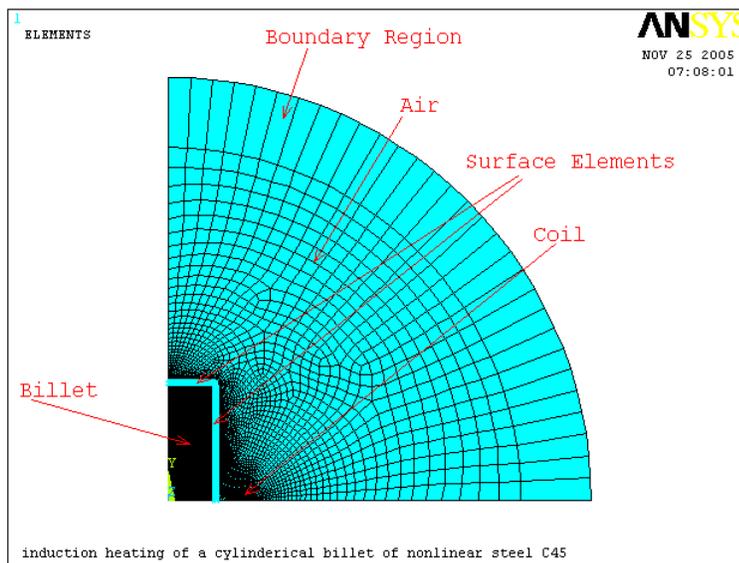


Fig. (11) The mesh of the whole system.

. Then this step is repeated assuming H for each element to be that determined in the previous step, then the resulted values of H is compared with that of the previous step and so on. If the difference is negligible then the updating step is finished. Step 4 is the determination of the values of heat generation at each element as results of applying the harmonic analysis, and feed them as input to the transient thermal analysis step. Steps 5&6 are the thermal steps in which the temperature of each node and element are determined after a specified period of time.

If the required temperature for the specified point is not reached, the steps (from 3 up to 6) are repeated for the new H and T of each element. This iteration continuous until the required temperature achieved,

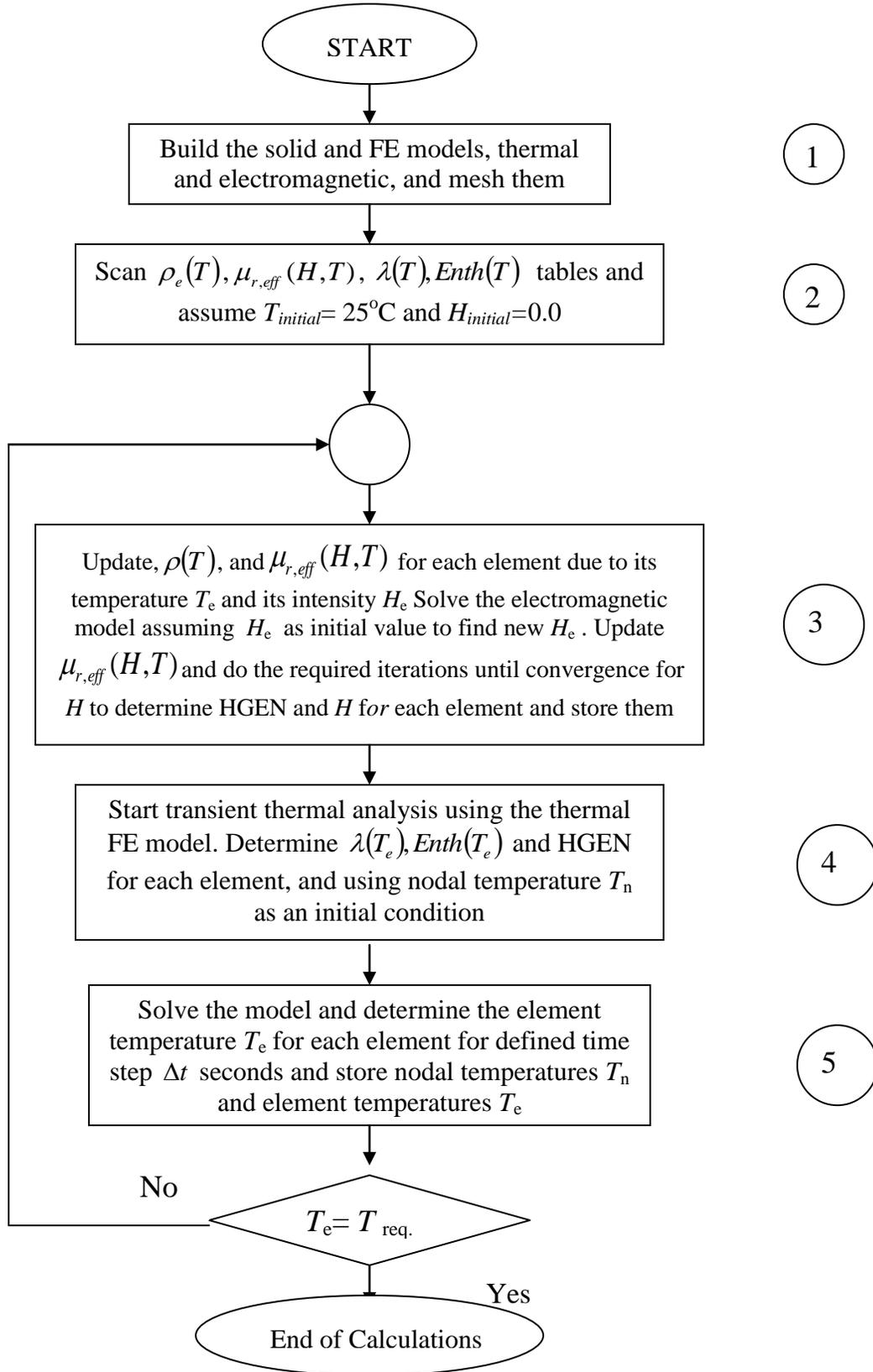


Fig. (12) Flow chart of the heating process

The obtained results

Results in Fig. (13) show the temperature response versus time for the central node at the surface close to the inductor for the three different excitation currents (10500A, 7875A, and 5250A). The curves for each case will be drawn separately.

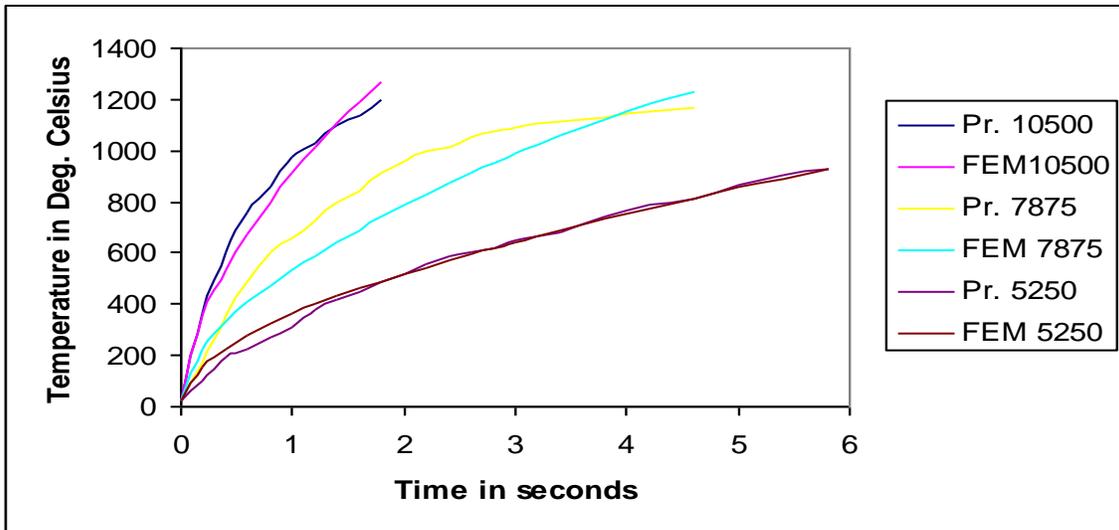


Fig. (13) The temperature response of the central node for three values of excitation current (10500A, 7875A, 5250A) as compared with practical measurements.

For 10500A case Fig.(14) shows the comparison between the practical measurements and the numerical calculation results.

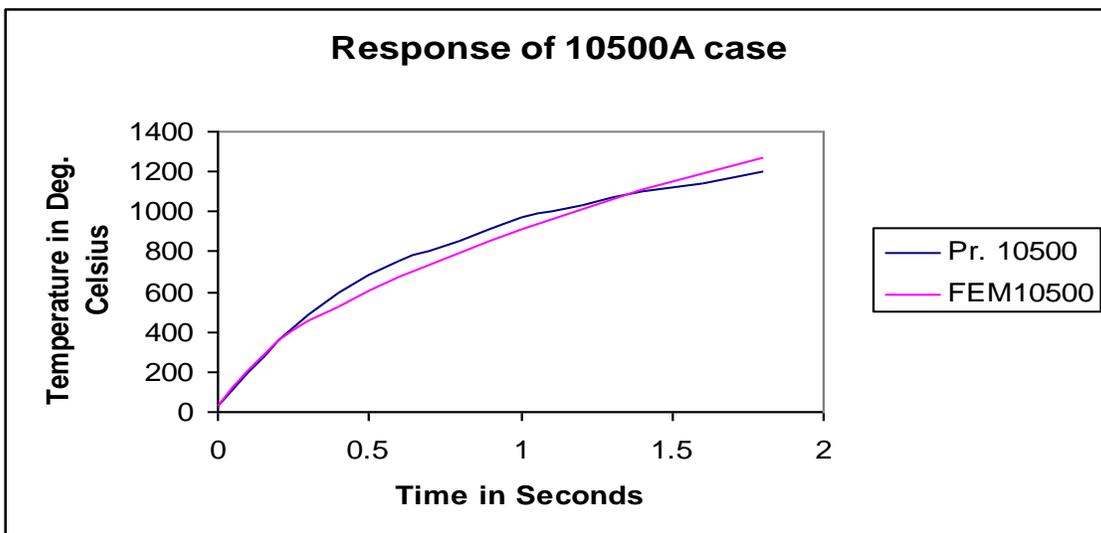


Fig. (14) The 10500A case results

The heat distribution inside the billet is shown in Fig. (15), the heat is concentrated in the central region near the inductor while that at the billet center is approximately not affected because the heating time is very short. Fig. (16) represents the case of 7875A.

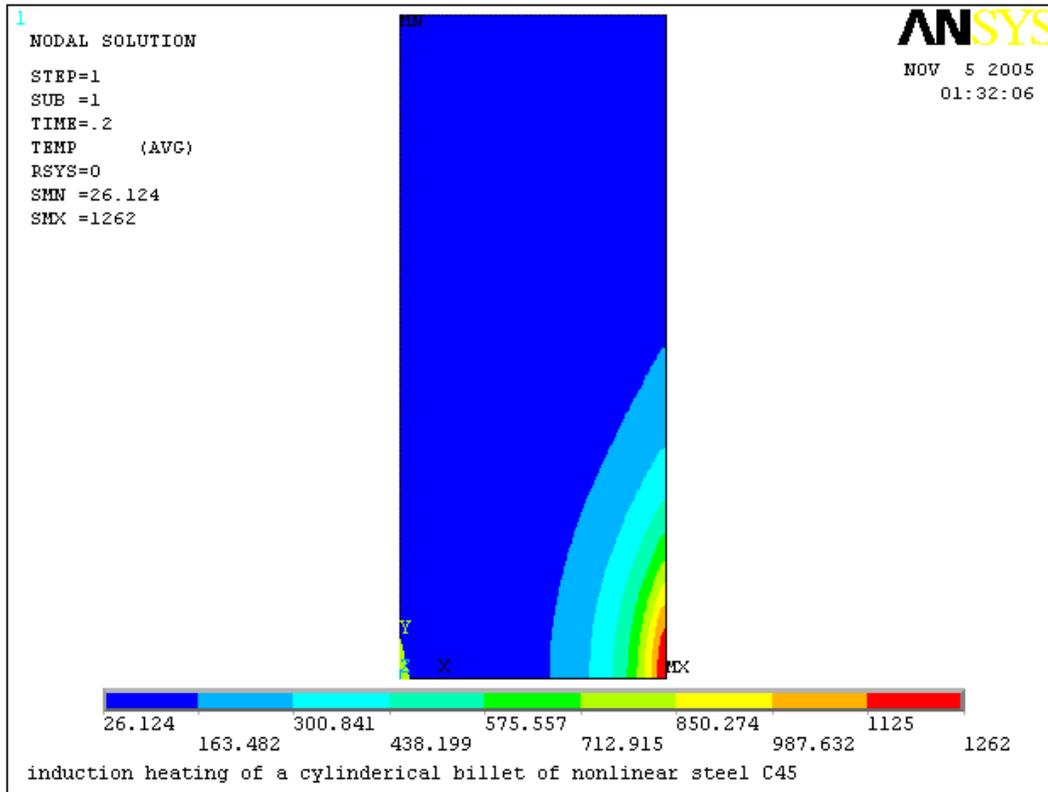


Fig. (15) A contour plot of the nodal temperature for 10500A case

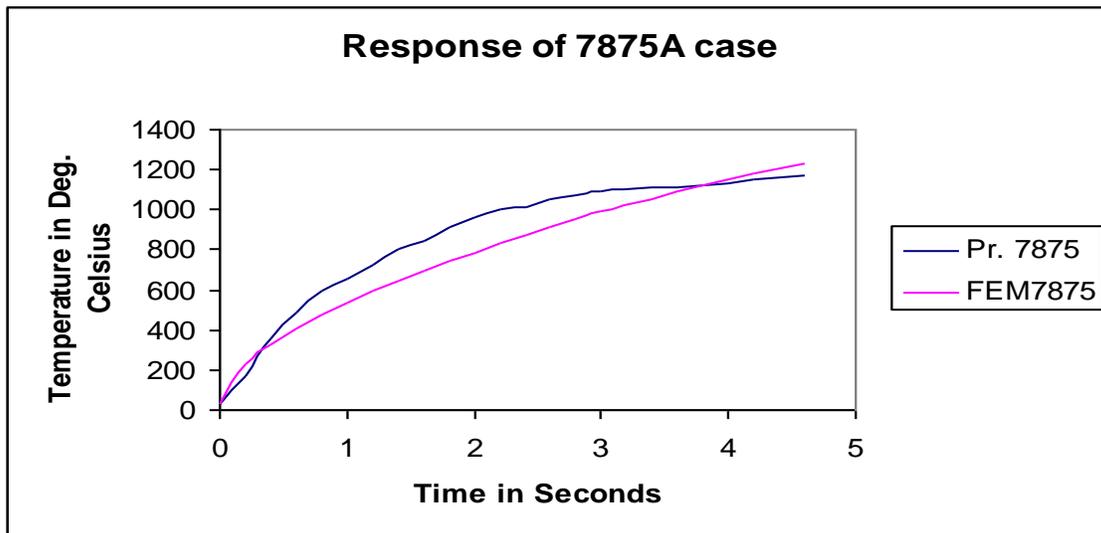


Fig (16) The 7875A case results



The heat distribution inside the billet in this case is shown in Fig. (17). The heat distribution shows more heat can reach the central region compared with the case of 10500A case, that is because the heating time of this case is 4.8 seconds, leads the heat to reach these regions by conduction.

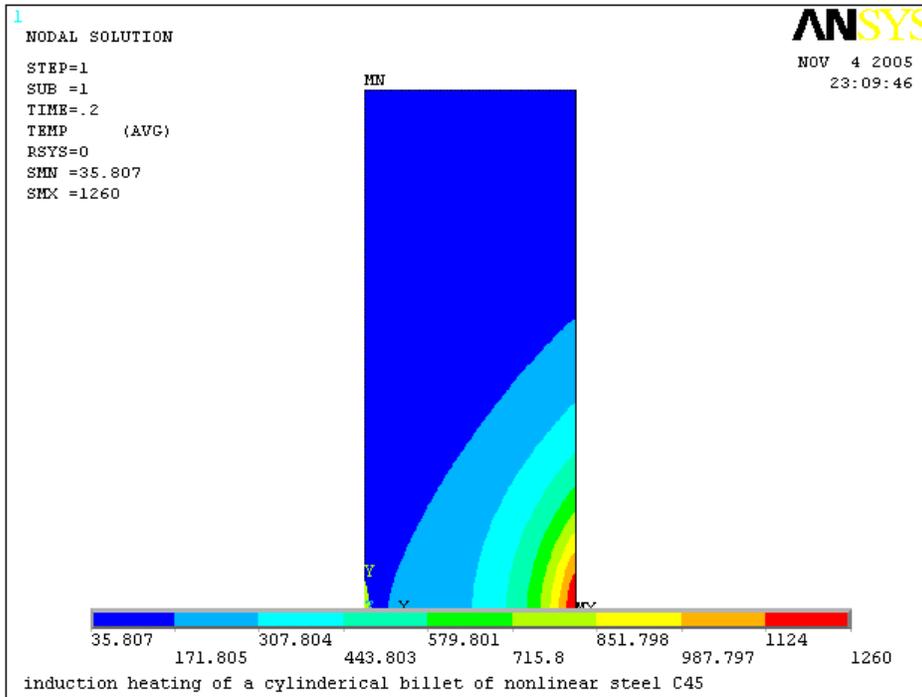


Fig. (17) A contour plot for the nodal temperature for the case of 7875A.

The third case is that of 5250A. It is the longest process among the three cases. The results in this case show a good agreement with the practical one. Fig.(18) shows the temperature-time response of the highest temperature node as compared with the practical measurements.

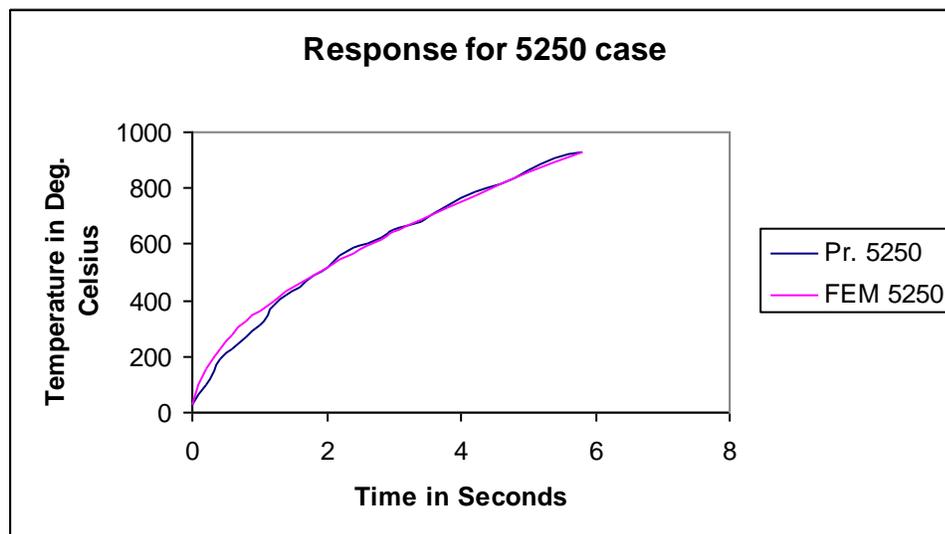


Fig. (18) The 5250A case results

The contour plot represents the heat distribution of this case. In spite of using half the current used in the first case, the heat reaches largest area inside the billet section, which happens due to the conduction inside the material along the heating time that elapses about six seconds. Fig. (19) shows the heat distribution inside the billet section for this case of exciting current.

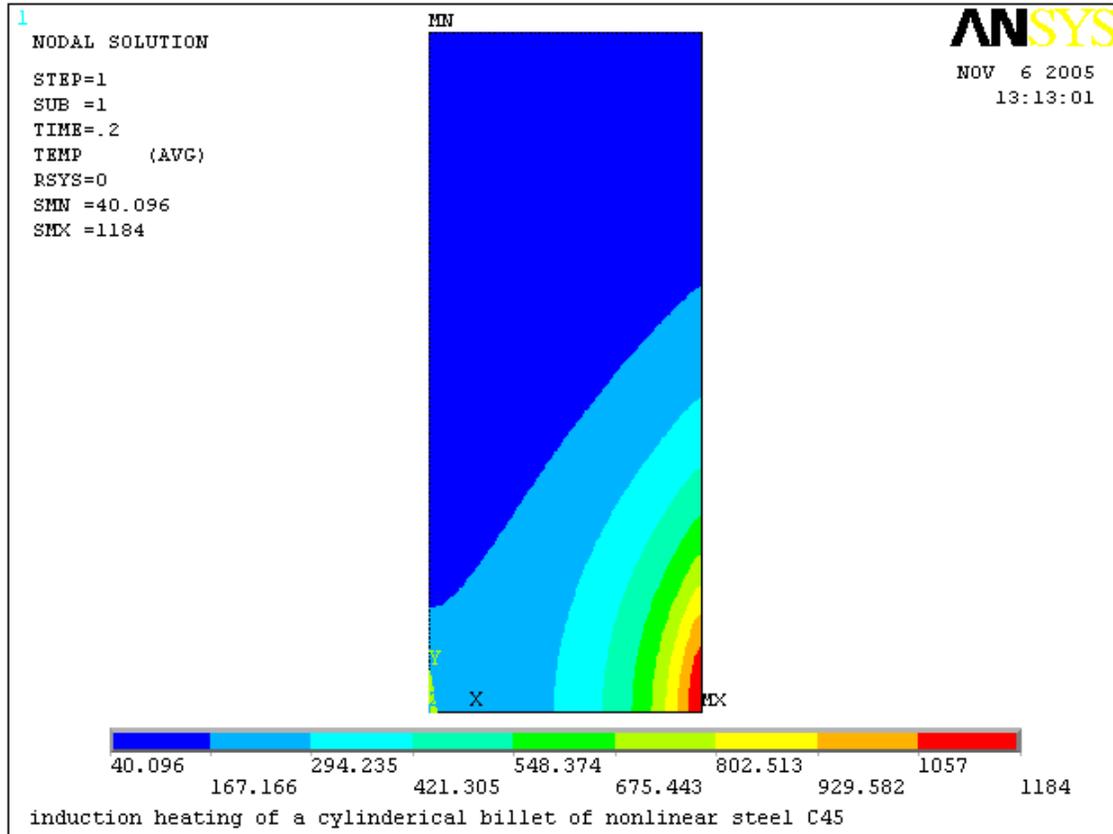


Fig. (19) A contour plot of the nodal temperature for 5250A case

Discussion and future work

The results obtained in this part of the research show many facts as follows:

- a. The penetration of power inside the billet is not a function of material properties and the supply frequency and current only, but it is a function of the heating period also.
- b. The algorithm used in this analysis gives an acceptable results for the three cases, which affirm the reliability of this type of numerical analysis, and assist the confidence to use it in future in industrial applications.
- c. The prediction of induction furnace performance is not an easy problem especially in the case when its core (the work piece) is an alloy of a ferromagnetic material because the following information are needed :
 - A precise description of the furnace geometry in details.
 - The behavior of the physical properties (of the different materials used in furnace itself and for that of its work piece) with temperature, which is a



difficult task especially when the alloys are used. These properties are precisely (Electrical Conductivity, The Effective Relative Permeability, The Thermal Conductivity, and The Enthalpy of the core material).

- The constants considered in the heat transfer analysis like the Stefan-Boltzman constant, Form Factor, and the Emissivity Coefficient, for the purpose of determination of the energy losses from the external surfaces of the billet by Radiation, and the Film Coefficient used in determining the effect of energy losses due to Convection. The determination of these coefficients is very difficult task because they depend on the surface softness and the geometry of the work piece and the furnace, and they are function of temperature also, and cannot be determined accurately and they are often determined empirically. So they are sources of error in this type of analysis.
- d. This algorithm proposed to be as a computer aided design method for the axi-symmetrical induction furnace. It can determine the required furnace geometrical shape and the excitation current and the frequency required to be supplied to its coil in order to do certain heat treatment for a work piece of a certain shape and manufactured from a certain material in the required period of time.

This study can be extended in future to determine the equivalent values of the inductance, resistance, and capacitance of the furnace and their variation with temperature during operation. If this analysis coupled to the circuit of the power supply will aid to study the whole system.

List of Symbols:

A	Magnetic vector potential	Wb.m^{-1}
B	Magnetic flux density	Tesla
C_p	Specific heat	$\text{kJ/kg.}^\circ\text{C}$
E	Induced electromotive force	Volt
$Enth$	Enthalpy	J/kg
H	Magnetic field intensity	Amp.m^{-1}
HGEN	Heat generation in each element	J.m^{-1}
I	Current	Amp
J	Current density	Amp.m^{-2}
k	Thermal conductivity	$\text{W/m.}^\circ\text{C}$
$Q_{induction}$	The Joule heat from the electromagnetic process	J.m^{-3}
T	Temperature	$^\circ\text{C}$
t	Time	Seconds
μ	Permeability	Henry.m^{-1}
α	Film coefficient	$\text{W.}^\circ\text{C}^{-1}$
ε	Permittivity	F.m^{-1}
ξ	Emissivity of the surface	
ρ_e	Electrical Resistivity	$\Omega.\text{m}$
ρ_m	Density	kg.m^{-3}
σ_e	Electrical conductivity	$(\Omega.\text{m})^{-1}$

F.A. Abood A. F. Batti A.H. Ahmad E. M. Abdul-Baki	Modeling of Induction Heating Process of A Conducting Ferromagnetic Materials
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σ_s	Stefan-Boltzmann constant	$W.m^{-2}.K^{-4}$
δ	The Skin depth	m
ω	The angular Frequency	Rad. Sec ⁻¹

REFERENCES

- J. Davies, and P.Simpson, "Induction Heating Handbook", *McGraw-Hill Book Company (UK)*, 1979.
- A. Vasiliev, I. Pozniak, V. Greshnov, "Modeling and Investigation Hardening Process", *International Scientific Colloquium*"Modeling for Electromagnetic Processing", Hannover, March (24- 26), 2003.
- Razzaq A. Marsuq, "Analysis and Study of Design Criteria of Induction Furnaces", *M.Sc. Thesis, University of Baghdad, Iraq.*(1994).
- Fadhil A. Abood, "Direct Circuit Coupled Based Finite Element Analysis of Three Phase Induction Motor", *Ph.D. Thesis, University of Technology. Baghdad, Iraq.* 2002.
- Jorg Cstroeski, "Boundary Element Methods for Inductive Hardening", *Ph.D. Thesis, University of Topengen, Germany*, 2003.