COMBINED ADJUSTED STEP SIZE LMS ALGORITHM AND ACTIVE TAP DETECTION TECHNIQUE FOR ADAPTIVE NOISE CANCELLATION

Ass.Prof.Dr Thamer M. Jamel
University of Technology, Department of Electrical Engineering, Baghdad –Iraq

ABSTRACT
In this paper, a new idea to combine Adjusted Step Size Least Mean Square (ASSLMS) algorithm with standard LMS active tap detection technique is presented. Then the combined method is used for estimating unknown time-invariant (stationary) and time varying (non-stationary) Finite Impulse Response (FIR) channel for adaptive noise cancellation (ANC) application. The focus of this paper is to improve the convergence rate and low level of the error in the steady state for the popular LMS adaptive filter. The simulation results have shown improvement on the convergence rate using the combined technique over both the ASSLMS algorithm and standard LMS active tap detection technique if they are used alone.

KEYWORDS: Adaptive Noise Cancellation, Active Tap Detecting Technique, Adjusted step size LMS algorithm.

INTRODUCTION
Adaptive Noise Cancellation (ANC) system is regarded as one of the method used for signal (speech) enhancement and attempts to reduce the additive noise which may arise from different sources. Noise canceling is an adaptive system that makes use of an auxiliary or reference input as shown in Fig.1 [Bernard Widrow, Samuel D. Strearns].
In this Fig., ANC has two inputs called primary and reference inputs. The reference input (n1) is filtered using adaptive FIR filter and subtracted from a primary input which is containing both desired signal and the noise (s+n0). As a result the primary noise is attenuated or eliminated by cancellation and the output signal is called error signal (e). The signal (n0) represents the noise that is filtered through the noise channel path a(n) which it is considered as FIR filter in this paper. This noise path channel has impulse response which may be stationary or non-stationary time varying. Filtering and subtraction of the ANC system must be controlled by an appropriate adaptive process that uses the error signal in order to obtain noise reduction with little risk of distorting the signal or increasing the noise level [Bernard Widrow, Samuel D. Strearns]. Many algorithms are used by ANC to adjust their impulse response. The LMS algorithm is regarded as a special case of the gradient search algorithm which was developed by Widrow and Hoff in 1959 [Bernard Widrow, Samuel D. Strearns]. This algorithm is often used for the adaptation of the ANC system because it is easy to implement and requires small number of calculations. But this algorithm suffers from slow convergence since the convergence time of LMS algorithm is inversely proportional to the step size [Bernard Widrow, Samuel D. Strearns]. However if large step size is selected then fast convergence will be obtained but this selection results in deterioration of the steady state performance (i.e. increased the misadjustment (error level)). Also if the channel to be estimated is time varying the LMS algorithm fails to track this type of channel. The LMS has also been cited and worked upon by many researchers and several modifications have been applied to it in order to optimize its performance for particular applications. Numerous modifications of the LMS algorithm have been reported [S.K., G. Zeng, J.J. Chen, R.R. Priemer, Bozo K., Zdravko U., and Ljubisa S., and R.H. Kang, E.W. Johnstone]. In these works, the optimization issue concerning the step size is discussed, and several methods of varying the step size to improve performance of the LMS algorithm especially in time varying environments are proposed. One of these modifications is to use adjusted step size LMS algorithm as proposed by [M.J. Al-Kindi, A.K. Al-Samarrie, and Th.M. Al-Anbakee]. Our proposed algorithm is called ASSLMS algorithm which is used for ANC application. Another modification of LMS algorithm is proposed by Dr. J. Homer [J. Homer]. He proposed active tap detection LMS technique for white inputs and invariant time channel. Then several improvements of this technique (i.e. active tap detection technique) are reported in order to improve the performance of this technique especially in non-stationary environments [Charles Q. Hoang, Boon L. Chock, Vanessa Edward, and Long Le, Ozgu Ozun, and Philipp Steurer]. This paper is organized as follows: first the concept of active tap detection LMS algorithm will be present, then ASSLMS algorithm will be present. After that, simulation results with different conditions will be present. Then this paper will enclose by the conclusions.
FUNDAMENTAL OF THE LMS ALGORITHM

Basically, the idea for the adaptive transversal filter (or FIR) is to model the noise path channel and attempt to match it. The tap delay line structure for the unknown modeled channel, which gives the desired response, is shown below in Fig.2.

The weights or the impulse response of the noise path is given as:

\[ W_k = [w_0 \ w_1 \ w_2 \ w_3 \ldots w_{n-1}] \]  

where \( n \) is the tap length, \( k \) is the time index. There is also the estimator (or adaptive filter) that has a similar structure:

\[ W_k = [w_0 \ w_1^* \ w_2^* \ w_3^* \ldots w_{m-1}^*] \]  

where \( m \) is the tap length. The adaptive filter has zero initial conditions as is done in practice. In general, the length of the filter \( m \) may be different from the length \( n \) of the channel. For simplicity in this paper, only the case of \( n=m \) was considered. Another important aspect is that the weights or coefficients of the FIR estimator will be adjusted using the LMS algorithm to reduce mean squared error (MSE) which is a measure of the accuracy of the estimated channel. However, since the MSE requires a large amount of memory, the instantaneous squared error is used, which gives an estimate of the gradient of the MSE surface [Bernard Widrow, Samuel D. Strearns]. The estimation error can be found as:

\[ e_k = (d_k + n_k) - y_k \]  

Where noise or additive disturbance \( n_k \) is added to the desired signal \( d_k \) (not shown in the Fig. (2)).

From equation 3, the adaptive filter output is

\[ y_k = W_k^*N_k \]  

where \( n_k \) is the input noise signal and * denotes convolution operation.

The LMS algorithm adapts the adaptive filter’s vector \( W_k^* \) according to [Bernard Widrow, Samuel D. Strearns],

\[ W_{k+1} = W_k + \mu N_k e_k \]  

Where \( N_k = [n_k \ n_{k-1} \ n_{k-2} \ldots \ n_{k-m+1}] \), which represents the input noise signal to the taps of the adaptive FIR filter, and \( \mu \) is fixed step size \( 0<\mu<1 \).
CONCEPT OF LMS ACTIVE TAP DETECTION ALGORITHM

In adaptive estimation applications, the channel is characterized by a time domain impulse response. Using the LMS algorithm alone, extended regions of negligible response or “inactivity” may be included in the calculation of this response [Charles Q.Hoang]. The idea proposed by Dr. John Homer is to apply a consistent LMS algorithm that performs estimation of nonzero or “active” taps only. This will improve the channel estimation performance (accuracy) and convergence rate. In order to detect a tap, a formula known as the least squares (LS) activity measure is used [J. Homer]:

$$X_i = \frac{\sum_{k=i+1}^{D}[v_k n_{k-1}]}{\sum_{k=i+1}^{D}n_{k-1}^2}$$  \hspace{1cm} (6)

Where $v_k= d_k$, $i =$ tap index, $D$ is the number of samples.

Now in order for the tap to be determined as active (as opposed to inactive), the value of $X_i$ must be above a certain minimum value called the active tap threshold condition ($T_d$). This is shown in the following threshold formula:

$$T_d = X_i (D) \sigma_v (D) \log (D)$$  \hspace{1cm} (7)

Where $\sigma^2_v$ is the variance of $d_k$ or $v_k$.

If the value is found to be below this criterion, then it can be discarded or multiplied by a forgetting factor (i.e. fraction) just in case it is an actual spike in the next iteration. So in summary, the LS activity criterion (6), and (7) works out or detects the position of the active taps in the unknown channel or noise path and the LMS algorithm works out the strength of the active taps [J. Homer].

STANDARD LMS ACTIVE TAP DETECTION WITH SLIDING WINDOWS

By theory for a stationary channel, the length of the window which tracks the channel is the length of the number of samples. However, channels that have a time-varying nature require a window which must be adjusted to the recent channel. There are several fundamental considerations that must be understood in the implementation of the standard LMS active tap detection algorithm for time-varying channels. First of all, the LMS algorithm uses the most recent channel estimation error in (5). This, in turn, means it is not severely affected by time-varying channels. Secondly, the LS activity measure equation uses all past signal ($d(k)$, $n(k)$) samples. This leads to the LS activity measure algorithm being affected by time-varying channels. Also, since the more distant past samples are no longer relevant to the system, these particular samples can bias the value obtained for the activity measure. And so, the basic idea of the system design is to include only the more recent samples in the LS activity measure equation. This is equated to applying a sliding window to the signals [Charles Q.Hoang].

There are several types of windows that can be used to track the non-stationary taps. These can be rectangular, triangular, exponential, and such. In this paper, an exponential sliding window is considered to detect and estimate the non-stationary channels due to its superiority over the other types of windows for this type of the channel [Charles Q.Hoang]. This exponential sliding window is shown in Fig.3.
The equivalent equations to (6), and (7) for the exponential sliding window approaches are [Charles Q. Hoang]:

\[ X_i = \frac{\sum_{k=i}^{D} \beta^{D-k} v_k n_{k-1}}{\sum_{k=i}^{L} \beta^{D-k} n_{k-1}^2} \]  \hspace{1cm} (8)

\[ \sigma_v^2 = \frac{\sum_{k=i}^{D} \beta^{D-k} v_k^2}{\sum_{k=i}^{L} \beta^{D-k}} \]  \hspace{1cm} (9)

And

\[ X_i > 2\sigma_v \log\left(1 - \frac{D-k}{L}\right) \]  \hspace{1cm} (10)

Where \( \beta \) is the exponential factor and obeys the limit of \( 0 < \beta < 1 \) and \( L \) is the length of the window. The main advantage of the exponential approach is more importance is placed on the most recent samples. The smaller the size of \( \beta \) is equivalent to a smaller effective window length (L). This means that more importance to the more recent samples will be achieved. In contrast, if the exponential factor \( \beta \) is large, the effective window length will be larger and importance to recent samples will be less [Charles Q. Hoang].

**PROPOSED ASSLMS ALGORITHM**

The proposed algorithm which is developed by us [M.J. Al-Kindi, A.K. Al-Samarrie, and Th.M. Al-Anbakee] used recursively adjusted adaptation step size based on the performance surface gradient square. This algorithm will be referred to as Adjusted Step Size LMS algorithm (ASSLMS) all over in this paper. However this algorithm is the modified version of Variable Step Size LMS algorithm that is developed in [R.H. Kang, E.W. Johnstone]. The step size of the proposed algorithm is varied according to the following equation [M.J. Al-Kindi, A.K. Al-Samarrie, and Th.M. Al-Anbakee]--:

\[ \mu_{k+1} = \alpha \mu_k + \sigma(e_k N_k)^2 \]  \hspace{1cm} (11)

Where \( 0 < \alpha < 1 \) and \( \sigma > 0 \), then

\[ \mu_{k+1} = \mu_{\text{max}} \quad \text{if} \quad \mu_{k+1} > \mu_{\text{max}} \]

And

\[ \mu_{k+1} = \mu_{\text{min}} \quad \text{if} \quad \mu_{k+1} < \mu_{\text{min}} \]  \hspace{1cm} (12)

Otherwise

\[ \mu_{k+1} = \mu_{k+1} \]
Then eq. (5) can be rewritten as follows:

\[ W_{k+1}^\wedge = W_k^\wedge + \mu_k N_k \ e_k \]  \hspace{1cm} (13)

From equations (11) and (13), \( \mu_k \) is time varying step size, which is adjusted according to the gradient square of the performance surface \( (e_k N_k)^2 \). The motivation of this algorithm is that a large gradient square will cause the step size to increase to provide faster tracking while small gradient square will decrease the step size to yield smaller error level. The value of \( \mu_{\text{min}} \) is chosen to provide minimum level of error level at steady state, and \( \mu_{\text{max}} \) ensures the stability of this algorithm.

**THE MODELING OF A NON-STATIONARY CHANNEL**

There are various ways in which the non-stationary or time-varying channel can be modeled. The channel can vary with respect to either the strength of each active tap, the position of each active tap, or both. Also, the rate at which the channel changes with time is another parameter [Charles Q.Hoang]. The variation of the strength of the known channel taps is one approach for modeling a non-stationary condition. There are two standard models which incorporate these ideas:

- The random walk process.
- 1st autoregressive process.

However in this paper only the first one is used. The random walk theory is defined by the following [Charles Q.Hoang]:

\[ W_{k+1} = \sum W_k \cdot \text{random\_number} \cdot \sigma_\delta \]  \hspace{1cm} (14)

Where \( \cdot \) denotes multiplication, \( W_k = [w_{0,k}, w_{1,k}, \ldots, w_{n-1,k}] \) is the response of the channel at time index \( k \).

\[ W_{k+1} - W_k = \delta_k = [\delta_{0,k}, \delta_{1,k}, \ldots, \delta_{n-1,k}] = W(k) \cdot \text{random\_number\_2} \cdot \sigma_\delta \]

\( \delta_{i,k} \) is zero mean with variance \( \sigma_\delta^2 \) (measure the speed of parameters change) and independence of \( \delta_{j,k} \ (i \neq j) \); \( \sigma_\delta \) is sometimes known as a Gaussian White Noise [Charles Q.Hoang]:

\[ \sigma_\delta = \frac{\text{random\_number} \cdot \sigma_\delta - 1}{\text{random\_number\_2}} \]

\[ E[\delta_{i,k}] = 0; \quad E[\delta_{i,k}^2] = \sigma_\delta^2 \quad \forall i; \quad E[\delta_{i,k} \delta_{j,k}] = 0 \]

**SIMULATION RESULTS**

Fig.1 is simulated under the following conditions:

- The desired input signal (S) is sine wave.
- The reference input (n1) is White Gaussian Noise with variance of 1.0 and zero mean.
- The signal to noise ratio is equal to approximately to (-15 db) for all simulation tests.
- The tap vectors (W) of the noise path a(n) is stationary and is shown in Table (1) and Fig.4. There are three active taps in the system: at the 4th tap, the strength is 1; at the 15th tap, the strength is -2; and at the 26th tap, the strength is 3. The channel also had a total tap length of 36 (i.e. \( m = 36 \) taps and then \( n = 36 \) as discussed previously).
Table (1) The tap vectors (W) of the noise path a(n) to be estimated.

<table>
<thead>
<tr>
<th>Tap Number</th>
<th>Type</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th</td>
<td>Active</td>
<td>1</td>
</tr>
<tr>
<td>15th</td>
<td>Active</td>
<td>-2</td>
</tr>
<tr>
<td>26th</td>
<td>Active</td>
<td>3</td>
</tr>
</tbody>
</table>

- Standard LMS alone with step size equal to (0.005) is applied to the ANC. Then the estimated channel taps are estimated very well as shown in Table (2) and Fig.5, but this algorithm provides noisy estimates of the 7, 19, 22, 24, and 35 inactive taps.
Table (2) The estimated tap vectors (W) of the noise path a(n) using LMS algorithm only

<table>
<thead>
<tr>
<th>Tap Number</th>
<th>Type</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>4th</td>
<td>Active</td>
<td>1</td>
</tr>
<tr>
<td>7th</td>
<td>inactive</td>
<td>0.3</td>
</tr>
<tr>
<td>15th</td>
<td>Active</td>
<td>-2</td>
</tr>
<tr>
<td>19th</td>
<td>inactive</td>
<td>0.5</td>
</tr>
<tr>
<td>22th</td>
<td>inactive</td>
<td>0.7</td>
</tr>
<tr>
<td>24th</td>
<td>inactive</td>
<td>0.2</td>
</tr>
<tr>
<td>26th</td>
<td>Active</td>
<td>3</td>
</tr>
<tr>
<td>35th</td>
<td>inactive</td>
<td>0.5</td>
</tr>
</tbody>
</table>

- Combined ASSLMS algorithm and LMS with active taps detector is applied then the estimated channel response is obtained in Fig. (6-a) which is similar to desired response of the stationary channel noise path and no estimation error occurred in this case.
- Also Fig. (6-b) shows that Active taps detector technique success to detect the number of active taps which it is equal to (3) after 100 iterations.
- The values of the parameters used for ASSLMS are as follows: $\mu_{\text{max}} = 0.005$, $\mu_{\text{min}} = 2 \times 10^{-6}$, $\alpha = 0.0001$, and $\sigma = 0.1$. 
Fig. (6-a) Estimated channel using ASSLMS algorithm and LMS with Active taps detected

Fig. (6-b) Number of Active taps detected for stationary channel

- Fig. (7) shows the channel estimation error \((W - W^*)^2\) for all algorithms that are used in this paper with the stationary channel. As shown in this Fig. ASSLMS algorithm has fast convergence time (600 iterations) than standard LMS with active taps detector (700 iterations) but it suffers from high error in the steady state compared to the standard LMS with active taps detector. This is because optimum parameters must be chosen carefully. However, if combined ASSLMS and standard LMS with active taps detector is used then both fast convergence (400 iterations) and low level of error is achieved and the performance of the system is improved compared with that if only standard LMS with active taps detector is used lonely.

- Fig. (8) shows the same result as in Fig. (7) when non-stationary channels that is modeled using random walk process. As shown in this Fig. when combined ASSLMS and standard LMS with active taps detector with an exponential sliding window is used then fast convergence time (1100 iterations) and low level of the error at the steady state is achieved, while ASSLMS has convergence time equal to (1300 iterations) and standard LMS with active taps detector has convergence time equal to (1500 iterations).

- The slight “bumps” in the beginning area of the curves between (200 and 800 iterations) in the Fig. (8) are due to high noise effect of the non-stationary channel.
In this simulation the rate change (speed) of the non-stationary channel model parameters is equal to 50 iterations. And since the total number of the iterations is 2000, then the total number of the change is 40 times. Also the exponential factor $\beta$ of the exponential sliding window is equal to 0.98.

Fig. (9-a) shows the last 300 samples (i.e. from 1700 to 2000 samples) of the input sinewave that is applied to the ANC for the non-stationary channel. Fig. (9-b) shows the corresponding output error signal of the ANC for the last 300 samples using the combined ASSLMS and LMS active taps detector with an exponential sliding window. As shown in this Fig. the output error signal has good estimate of the desired input signal with very low level of the noise at this steady state region.
CONCLUSIONS
This paper presents new idea which has combined ASSLMS algorithm that is developed by us in 1997 and standard LMS algorithm active taps detector technique with an exponential sliding window that developed by dr. J. Homer. Then this combined method is applied to the ANC system to measure its performance for both stationary and non-stationary environments. Although dr. J. Homer suggests his method for only white input noise in adaptive echo canceller, but here in this paper ANC sinewave input signal is applied instead of white noise. The simulation results shows that the combined method gives faster convergence time than the method that suggested by dr. homer, which it is important factors in the adaptive systems. Also the combined method provides lower level of the error in the steady state. However, ASSLMS algorithm alone also has faster convergence time than the method of dr. J.Homer but it is suffers from high level of error at the steady state.

REFERENCES


