DYNAMIC BEHAVIOR OF NON - RETURN VALVES OPERATING AT SMALL OPENING

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ABSTRACT

In the present work a general dynamic behavior of non return valves subjected to jet flow is presented. The differential equations of valve motion and discharge were developed in a non-dimensional from, in terms of suitable dimensionless variables and parameters of the valve system. The derived equations are coupled nonlinear differential equations. Thus, a computer program was developed using a package called (MatLab) to solve these equations. The study shows that there are three types of the valve responses depending on the overall hydrostatic pressure difference and it is found that the valve vibrating at a constant limit cycle, which is leading to the failure of the system. It is also shown that the limit cycle frequency decreases with increasing the stiffness parameter and inertia factor. Finally the study shows that the losses factor has negligible effect on valve vibration and discharge.

KEY WORDS: - Dynamic, Return valves, Nonlinear equations
NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Units</th>
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<tbody>
<tr>
<td>a</td>
<td>Acceleration</td>
<td>m/s²</td>
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<tr>
<td>A</td>
<td>Area of the pipe</td>
<td>m²</td>
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<tr>
<td>A2</td>
<td>Area of the pipe upstream of valves</td>
<td>m²</td>
</tr>
<tr>
<td>A3</td>
<td>Area of vena contract in the valve gap</td>
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<td>A4</td>
<td>Area of the pipe downstream of valves</td>
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<td>Ao</td>
<td>Area of the orifice</td>
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<tr>
<td>C</td>
<td>Damping</td>
<td>N.s/m</td>
</tr>
<tr>
<td>Cc</td>
<td>Valve contraction coefficient</td>
<td>-</td>
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<tr>
<td>Cd</td>
<td>Valve velocity coefficient</td>
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<td>Cv</td>
<td>Valve discharge coefficient</td>
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<td>g</td>
<td>Gravity acceleration</td>
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<tr>
<td>hL</td>
<td>Head losses over the length (L)</td>
<td>m</td>
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<tr>
<td>Lij</td>
<td>Inertia of the fluid between any section i-j</td>
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<tr>
<td>k</td>
<td>Stiffness</td>
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<td>Leq</td>
<td>Equivalent pipe length</td>
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<tr>
<td>Lij</td>
<td>Pipe length over section i-j</td>
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<td>L0</td>
<td>Length of jet through valve</td>
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<tr>
<td>P</td>
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<tr>
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<td>v</td>
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<td>X</td>
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Greek Symbols

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<td>Α</td>
<td>Dimensionless inertia factor</td>
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<td>α0</td>
<td>Dimensionless inertia factor of the jet</td>
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</tr>
<tr>
<td>μ</td>
<td>Dimensionless mass ratio</td>
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<td>τ</td>
<td>Dimensionless time</td>
<td>-</td>
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<tr>
<td>τo</td>
<td>Shear on the pipe wall</td>
<td>N/m</td>
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<tr>
<td>ζ</td>
<td>Damping factor</td>
<td>-</td>
</tr>
<tr>
<td>θ</td>
<td>Dimensionless downstream pipe area</td>
<td>-</td>
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<tr>
<td>η</td>
<td>Dimensionless down steam pipe</td>
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<td>γ</td>
<td>Specific weight</td>
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<tr>
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<td>Reference frequency</td>
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<td>Λ</td>
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INTRODUCTION

Check or non return valves are widely used in many power or process plants. They are unique because in their main mode they operate without receiving any outside help usually do not give indication of their setting. Furthermore, they must be reliable and must be able to operate for an extended period, thus they must be carefully designed. Various types of these valves and their application have been described by Siikonen 1983, in the technical research center of Finland, suggested a computational method for the valve dynamics. The method consists of a hydraulic part and a five equations model describing valve dynamics. These equations are coupled ordinary differential equations. Some important parameters for the boundary condition were determined experimentally, such as losses, discharge coefficient etc. The hydraulic equations were solved using the method of characteristics. The method gives good indication of behavior of non – return valves under unsteady flow condition.

Renold and Soung 1976 examined the hydraulic performance characteristics of large diameter tilting desk check valves. In particular, design parameters of pressure drop during normal operation, maximum permissible flow during valve closure, and maximum hammer surge are considered. Equations and coefficients were provided, for evaluation of pressure losses, as a function of desk angle and line Reynolds number. A method of calculating fluid torques on a moving disk. In general, all the design techniques developed are general and can be applied to the most check valves design.

Kubie 1982 studied the performance and design of plug type check valves. Full nonlinear equations of system, which are consisting of pipeline, pumps and check valve, were developed Using Newton second law, effect of different parameters such as discharge coefficient and inertia was developed. In particular, the work demonstrated that the check valves could not be properly designed without having enough information about the system in which they are to operate where the valves vary sensibly to the system components specially inertia effect.

Weaver and Dubi 1978 studied experimentally the flow-induced vibration of a check valve with a spring damper to prevent slamming. Both prototype and two-dimensional experiments were conducted to develop an understanding of the mechanism of self-excitation as well as the phenomenon was studied is considered to be the same as that causing vibration in numerous other flow control devices.

More research is being done and attempts are being made to develop an understanding of mechanisms of excitation. However, in many cases it is still necessary to use cut and try methods.

MATHEMATICAL MODELING

The basic system considered in the analysis is consisting of check valve, constant presser reservoir and connecting pipeline system. The excitation mechanism nature can be understood through the physical modeling and flow visualization. Through the physical model and flow visualization studies, an understanding of the excitation mechanism was developed. Furthermore, it was established that the effect of the unsteady separated flow around the valve is not an important part of the mechanism. Thus it was felt that the behavior of the valve might to be modeled satisfactory using simple one dimensional fluid mechanism Weaver and Ziada 1980. Therefore, the valve can be represented as an orifice with time varying area, in general pipeline system as show in Fig. (2). Water hammer occurred on each closure but was found to die out before valve recluse and, hence, it is not important phenomenon of the valve.
Weaver et al. 1975. Vortex shedding occurred in the wake of valve clapper during each cycle but this also played on role in the excitation mechanism Weaver et al. 1975. The pressure difference between up stream and down stream orifice is the same pressure acting on the up and down valve. Dynamic behavior in general pipeline system is influenced by the characteristics of rest of the system, especially the fluid inertia effects.

Referring to Fig. (1), which shows the forces acting on an element in the pipeline system, using Newton's second law and remembering that the flow is incompressible fluid flow, gives .

\[
F_i = P_i A \\
F_j = (P_j + dp) A
\]

Fig.(1). Forces acting on an element in the pipeline system.

\[
\sum F = m.a \\
P_i A - (p_j + dp) A - \tau_o (2\pi r) ds = \rho A ds \left( \frac{dv}{ds} + \frac{dv}{dt} \right)
\]  
\[
(1)
\]

Re – arranging eq. (1), and dividing by \( \gamma = \rho g \), \( A = \pi r^2 \), then

\[
- \frac{dp}{\gamma} - \frac{2\tau_o ds}{\gamma r} = v \frac{dv}{g} + \frac{ds}{g} \frac{dv}{dt}
\]

Substituting for \( v dv = l/2 dv^2 \), then eq. (2) becomes

\[
- \frac{dp}{\gamma} - \frac{dv^2}{2g} = \frac{2\tau_o ds}{\gamma r} + \frac{ds}{g} \frac{dv}{dt}
\]

Eq. (3) applies to unsteady flow of both compressible and incompressible real fluid.

For incompressible flow that are considering here \( \gamma \) is constant, so it can be integrate directly between point i and j and substituting for the distance between them \( L \), then eq.(3.3) becomes:

\[
\frac{p_i - p_j}{\gamma} + \frac{v_i^2 - v_j^2}{2g} = \frac{2\tau_o L_{ij}}{\gamma R} + \frac{L_{ij}}{g} \frac{dv_j}{dt}
\]

\[
(4)
\]
Where term represents the head losses ($h_L$) over the length ($L$), thus the Eq. (4) becomes:

$$\frac{p_i}{\gamma} + \frac{v_i^2}{2g} = \frac{p_j}{\gamma} + \frac{v_j^2}{2g} + h_L + \frac{L_{ij}}{g} \frac{dv_j}{dt}$$

Eq. (5) is the same as the steady flow equation, with addition of the last term, \[\left[\frac{L_{ij}}{g} \frac{dv_j}{dt}\right]\] which is called the accelerative head.

The head losses term can be expressed in terms of velocity, $v$, which is defined as:

$$h_L = \phi_j \frac{v_j^2}{2g}$$

Where:

$\phi_j = \text{loss factor}$

Substituting for eq.(6) in eq.(5), then

$$\frac{p_i}{\gamma} + \frac{v_i^2}{2g} = \frac{p_j}{\gamma} + (1 + \phi_j) \frac{v_j^2}{2g} + \frac{L_{ij}}{gA_j} \frac{dq}{dt}$$

where in general $q = vA$

Equation above is the unsteady Bernoulli equation, where $P$, $\gamma$, $v$, $\phi$ , and $q$ are the pressure, specific weight, mean velocity, turbulent losses factor and discharge respectively.

Applying eq.(7) between sections 1 & 2 as shown in Fig.(2) to get the equation of motion of the water column in the upper stream pipeline system, thus:

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + (1 + \phi_{1-2}) \frac{v_2^2}{2g} + \frac{L_{1-2}}{g} \frac{dv_2}{dt}$$
Applying Eq. (7) between section 2 & 3 to get equation of motion through the orifice, then

\[
\frac{p_2}{\gamma} + \frac{v_2^2}{2g} = \frac{p_3}{\gamma} + (1 + \phi_{2-3}) \frac{v_3^2}{2g} + \frac{L_{2-3}}{gA_3} \frac{dq}{dt}
\]

(9)

Where:

\text{L}_{2-3} \text{ is the jet length.}

Referring to Fig. (2), which shows the orifice in the pipeline system, it can be noted that the streamlines continue to converge for short distance downstream of the plane of the orifice. Hence the minimum-flow area is actually smaller than the area of the orifice. To relate area of minimum flow, which is often, called the contracted area of the jet, or vena contract, to the area of the orifice \(A_o\), using the contraction coefficient, which is define as:

\[
A_3 = C_c A_o
\]

(10)

Where:

\text{Cc: contraction coefficient.}

The velocity beyond the orifice section,\(v_3\), can be eliminated by means of the continuity equation. Solving eq. (9) for \(v_3\), then

\[
V_3^2 = \frac{2g}{1 - \left( \frac{A_3}{A_2} \right)^2} \left[ \frac{p_2}{\gamma} - \frac{p_3}{\gamma} - \frac{L_{2-3}}{gA_3} \frac{dq}{dt} \right]
\]

(11)

The discharge, is given by \(V_3A_3\), or, in terms of eq.(10) & eq.(11) is given by:
\[ q = C_d A_o \sqrt{\frac{2g}{1 - \left(\frac{C_d A_o}{A_2}\right)^2}} \left( \frac{P_2 - P_3}{\gamma} - \frac{L_{2-3}}{gA_3} \frac{dq}{dt} \right) \]  \hspace{1cm} (12)

Equation (12) describes the discharge for the flow of an incompressible fluid through an orifice; however, it is valid for relatively high Reynolds number John et.al. 1990. For low and moderate values of Reynolds numbers, viscous effects are significant and, an additional coefficient of viscosity must be applied to the discharge equation to relate the ideal flow to the actual flow. Thus for viscous fluid flowing through an orifice, the following discharge equation can be established:

\[ q = C_d A_o \sqrt{\frac{2g}{1 - \left(\frac{C_d A_o}{A_2}\right)^2}} \left( \frac{P_2 - P_3}{\gamma} - \frac{L_{2-3}}{gA_3} \frac{dq}{dt} \right) \]  \hspace{1cm} (13)

Where \( C_d \) is the discharge coefficient and it is given by \( C_d = C_v C_c \)

It is should be noted that the contraction and discharge coefficient in eq. (13) will generally depend on valve’s geometry, position and velocity in a way that cannot be predicated theoretically Daily and McCloy.

Referring to Fig.(2) and applying eq.(8) between sections 3 and 4 to get the equation of motion for the expanding jet, then:

\[ \frac{p_3}{\gamma} + \frac{v_3^2}{2g} = \frac{p_4}{\gamma} + (1 + \phi_{3-4}) \frac{v_4^2}{2g} + \frac{L_{3-4}}{gA_4} \frac{dq}{dt} \]  \hspace{1cm} (14)

Applying eq.(8) between sections 3 & 4, thus:

\[ \frac{p_4}{\gamma} + \frac{v_4^2}{2g} = \frac{p_5}{\gamma} + (1 + \phi_{4-5}) \frac{v_5^2}{2g} + \frac{L_{4-5}}{gA_5} \frac{dq}{dt} \]  \hspace{1cm} (15)

Equations (14) & (15) represent the equations of motion of water column in the down stream pipeline system.

As the valves dynamic behavior is strongly influence by characteristics of the rest of the system, especially the fluid inertia effects \( L_{ij} \), it is useful to include theses in the expression for the discharge through the valve. This may be done by substituting for \( (P_2-P_3)/\gamma \) in eq.(13) after assuming that the velocity \( V_1 = V_5 = 0 \), thus:
\( q = C_d A_o \sqrt{\frac{2g}{1 - \left( \frac{C_f A_o}{A_2} \right)^2}} \left[ \frac{P_1 - P_3}{\gamma} - \sum h_L - \sum \frac{L_{ij}}{gA_{ij}} \frac{dq}{dt} - \frac{V^2}{2g} \right] + \sum \frac{V^2}{2g} \)  

(16)

In this equation and those, which follow, the summation signs for the values of losses and interfaces exclude those values of the valve, section 2-3 and this attributed to the fact that those coefficients at the valve section are depending on the valve displacement while all others are constants.

Eq. (6) may be substituting for the turbulent losses in terms of losses coefficient, \( \phi_{ij} \), and the velocity head in the pipe just downstream of the valve, \( (V_4^2/2g) \), thus:

\[
q = C_d A_o \sqrt{\frac{2g}{1 - \left( \frac{C_f A_o}{A_2} \right)^2}} \left[ \frac{P_1 - P_3}{\gamma} - \frac{A_4^2}{A_j^2} \sum \phi_{ij} \frac{V_4^2}{2g} - \sum \frac{L_{ij}}{gA_{ij}} \frac{dq}{dt} \right] - \frac{L_{2-3}}{gA_3} \frac{dq}{dt} - \frac{A_4^2 V_4^2}{A_3^2 2g} + \frac{A_4^2 V_3^2}{A_3^2 2g} \]

(17)

The eq.(17) can be simplified by putting losses coefficients , \( \phi_{ij} \), in terms of overall losses coefficient, \( \psi \), which is define as:

\[
\psi = A_j^2 \sum \frac{\phi_{ij}}{A_j^2} \]

(18)

Substituting for eq. (18) in eq.(17), then

\[
q = C_d A_o \sqrt{\frac{2g}{1 - \left( \frac{C_f A_o}{A_2} \right)^2}} \left[ \frac{P_1 - P_3}{\gamma} - \psi \frac{V_4^2}{2g} - \sum \frac{L_{ij}}{gA_{ij}} \frac{dq}{dt} \right] - \frac{L_{2-3}}{gA_3} \frac{dq}{dt} - \frac{A_4^2 V_4^2}{A_3^2 2g} + \frac{A_4^2 V_3^2}{A_3^2 2g} \]

(19)

Assume \( (L_{eq}) \) is the length of pipe, of constant cross section-area \( A_4 \) that have the same inertia effect as the actual pipe. If the pipe consists of section of different cross-sectional area, then the equivalent length \( (L_{eq}) \) may be defined as:

\[
L_{eq} = A_4 \sum \frac{L_{ij}}{A_{ij}} \]

(20)
Therefore it can be simplified the inertia term in Eq.(19) by substituting eq.(20) and substituting for \( q = A_4 v_4 \), thus: to get:

\[
q = C_d A_o \left[ 1 - \left( \frac{C_c A_o}{A_2} \right)^2 \right] \left[ 2g\Delta H - \frac{1}{A_4^2} \left( \psi + \frac{A_4^2}{A_2^2} - \frac{A_4^2}{C_c A_o} \right) q^2 \right]
\]

(21)

Where:

\[
\Delta H = (P_1 - P_3)/\gamma \]

the total pressure drop cross the system and \( L_0 \) is the length of the jet \( (L_{2,3}) \) of area \( C_c A_o \) through the valve orifice. However, it is difficult to estimate the length of this jet and in many applications, the inertia of the jet may be neglected in comparison with rest of the system.

Assuming that the valve can be represented by single degree of freedom system [13] and it is equation of motion is given by:

\[
m \frac{d^2 x}{dt^2} + C \frac{dx}{dt} + k(x - x_o) + F = 0
\]

(22)

Where:

\( m \) is total effective mass, and \( C \) is the system damping including that due to fluid, \( k \) is the elastic restoring force of the valve, \( x_o \) is the zero load opening displacement of the valve, and \( F \) is the dynamic fluid loading on the valve, and it can be determined by integrating the dynamic pressure, \( \Delta p \), over the surface of the valve:

\[
F = \int \Delta p \, ds
\]

(23)

where

\( s \) : surface area of the valve.

\( \Delta p \) : dynamic pressure.

If the pumping action of the valve is neglected and the dynamic pressure is assumed to act uniformly over the upstream and downstream faces of the valve, so that the dynamic load is given by:

\[
F = \lambda s (p_2 - p_3)
\]

(24)

Where \( \lambda \) is an integration factor depends on the valve geometry and arrangement.

Substituting eqs. (10, 14, and 15) for \((p_2-P_3)\) in eq.(13) and simplifying the resulting equation in a manner similar to that followed for simplifying eq.(21), and assuming that \( V_1 = V_5 = 0 \), thus

\[
F = \lambda s \left[ (p_1 - p_3) - \frac{\rho}{2A_4^2} \left( \psi + \frac{A_4^2}{A_2^2} - \frac{A_4^2}{C_c A_o} \right) q^2 - \frac{\rho}{A_4} L_{eq} \frac{dq}{dt} \right]
\]

(25)

Where \( \rho \) is the fluid density
For the purpose of analysis it is convenient to represent eq.(21) and eq.(22) in terms of dimensionless form. In addition for most flow control devices operating at small opening, the flow area is linearly related to the valve displacement, thus the dynamic discharge equation and the valve displacement (for more details see Appendix A) are given by:

\[
Q = \frac{C_d X}{\left[\eta^2 - \left(\frac{C_r X}{\theta}\right)^2\right]^{1/2}} \left[\Delta P - \left(\psi + \frac{1}{\theta^2} - \frac{\eta^2}{C_r^2 X^2}\right) Q^2 \right]^{1/2}
\]

\[
d^2 X \over d\tau^2 + 2\zeta \frac{dX}{d\tau} + K(X - X_o) + \frac{1}{2} \mu \left[\Delta P - \left(\psi + \frac{1}{\theta^2} - \frac{\eta^2}{C_r^2 X^2}\right) Q^2 \right] - \left(\alpha + \frac{\eta \alpha_o}{C_r X}\right) \frac{dQ}{d\tau} = 0
\]

(26)

(27)

It should be noted that the discharge (Q) and the inertia factor (\(\alpha_o\)) represent the dynamic effects and approach zero faster than the displacement X and are equal to zero when the valve is closed.

- RESULTS & DISCUSSION

The governing differential equations obtained in the present study are coupled nonlinear differential equations in terms of valve discharge and displacement and until this time there is no way to be solved analytically. Thus, a numerical solution was adopted to obtain the solution for displacement and discharge. For this purpose computer simulation programs are developed using a package which is called MAT LAB. The flow chart of the program is shown in appendix (B). The general results are presented below for the most instructive and interesting cases. Also, for the purpose of comparison, the present results are compared quantitatively with the experimental data for specific case of a swing check valve.

In parametric study, the valve and discharge contraction coefficients are assumed constant, because there is no way to predict their variation with accelerating and decelerating flows or as a function of the valve motion.

The computations were executed show that there are three types of response depending on the overall hydrostatic pressure difference across the valve:

1. The valve undergo small damped oscillations and come to rest at an opening for which the static pressure drop across the valve is balanced by the valve elastic restraint. This occurs for \(\Delta P \leq 2.6\).
2. The valve closes for several times and come to rest in a closed position. This occurs for \(\Delta P \geq 40\).
3. The valve oscillates at a limit cycle of constant amplitude. This occurs for \(2.6 \leq \Delta P \leq 40\).

The latter case will be carefully discussed because it represents greatest insight into the system's behavior. Also, such results offer the possibility of comparison with experimental observations of vibrating structures.

The numerical values of the system parameters adopted in the present study are of typical values and are shown in table (1).
Table (4.1). Dimensionless parameters of the system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>Stiffness</td>
<td>K</td>
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<tr>
<td>Initial opening</td>
<td>X0</td>
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<tr>
<td>Pressure difference</td>
<td>ΔP</td>
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<td>Inertia factor</td>
<td>α</td>
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<tr>
<td>Losses factor</td>
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<tr>
<td>Contraction coefficient</td>
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**- EFFECT OF STIFFNESS PARAMETER (K)**

Figs (3), (4), and (5) show the variation of the valve displacement against the dimensionless time for different values of stiffness parameter and hydrostatic pressure difference.

It can be see that for low hydrostatic pressure difference (ΔP=1.8) the increase in stiffness leads to increase in the maximum overshoot. In control system the over shoot must be not exceeding 20 percent for design specification Weaver and Dubi [5], this makes the stiffness more than 0.9 unacceptable for the valve application. For high hydrostatic pressure difference (Δp=42.6) the valve opens for a several time and closed and remains closed. In this case, the increase in stiffness only increases the amplitude. For intermediate hydrostatic pressure difference (Δp=26.7), the behavior is similar to that observed in both the prototype valve model experiments reported by Weaver and Dubi [5].

Further insight is obtained by examining the discharge variation as a function of the valve displacement as shown in Fig (6). The plot is similar to the experimental curve reported by [6] and show that the maximum discharge occurs after the valve has reached its maximum opening.

**- EFFECT OF INERTIA FACTOR (Α)**

Figs (7), (8) and (9) show the variation of the displacement against dimensionless time for different values of inertia factor and hydrostatic pressure difference. Whereas Fig. (7) shows that the increase in inertia factor has no effect on the amplitude of oscillation for low hydrostatic pressure difference (Δp=1.8), the effect only appears at small inertia on the final rest of the valve. While, Figs. (8) and (9) show there are no effect of inertia factor on the amplitude at moderate and high hydrostatic pressure differences.

It also has been shown in Fig.(10) that for intermediate hydrostatic pressure difference the flow reaches its maximum discharge very shortly after the valve reaches its maximum displacement for small fluid inertia. Significantly, the discharge also reduces much more gradually during closure so that the rate of change of discharge and dynamic pressure is considerably reduced. This resulting in that the valve oscillates at a higher frequency (near its natural frequency) and the motion is much more nearly simple harmonic motion.
EXPERIMENTAL VERIFICATION OF MODEL

Figs (11-a) and (11-b) show comparison of the theoretical predictions with experimental results of limit cycle amplitude and frequency respectively. The spring stiffness and the frequency has been normalized with respect to the natural frequency of the valve in quiescent fluid in order to emphasize that these self-excited vibrations do not generally occur at the structural natural frequency.

While the results of the foregoing section shown that the displacement and discharge characteristics agree qualitatively with experimental observation, these curves demonstrate that the quantitative agreement is reasonable as well. Apparently, the theory underestimates the limit cycle amplitude by about 20 percent while overestimating the frequency by similar amount. It is thought that the difference is primarily due to the assumption of constant discharge coefficient.

- CONCLUSIONS

The main conclusions can be summarized as follows:

- A theoretical model has been derived for self-excited vibrations of valves subject to the jet flow mechanism of instability is sufficiently general that it is considered applicable to a large variety of flow control devices operating at small openings.

- When the valve is considered to be elastically restrained by support stiffness K at small initial opening (zero hydrodynamic load), its response to some disturbance can be classified into three categories:
  - It may undergo small damped oscillation and come to rest at an opening for which the static pressure drop across the valve is balanced by the valves elastic restraint. Such behavior is dynamically stable and is the desired response for flow control structures.
  - It may open, perhaps bounce several times, and come to rest in a closed position.
  - Valve may repeatedly open and close again and thus oscillate at some constant limit cycle amplitude. This represents a dynamically unstable system.

- Motion of the flow control device is far from simple harmonic and the discharge does not reach its maximum value until the valve has completely opened.

- Numerical results show that for systems in which the discharge variations are large and the fluid inertia is significant the system behaviors highly nonlinear.

- For the systems which have little fluid inertia the self-excited motion is more nearly simple harmonic and a much simpler analysis may be applicable.
Fig.(4) Effect of stiffness variation on valve displacement, (Δp=42.6).

Fig.(5) Effect of stiffness variation on valve displacement at (Δp=26.7).
Fig.(6) Effect of stiffness on valve dynamic discharge at (Δp=26.7).

Fig.(7) Effect of inertia factor variation on valve displacement at (Δp=1.8).
Fig. (8) Effect of inertia factor variation on valve displacement at ($\Delta p=42.6$).

Fig. (9) Effect of inertia factor variation on valve displacement at ($\Delta p=26.7$).
Fig.(10) Effect of inertia factor on valve dynamic discharge at ($\Delta p=26.7$).
Fig. (11). Comparison between theoretical and experimental results for a swing check valve.
REFERENCES

- J. Kubie “Performance and Design of Plug-Type Check Valves” Engineering Sciences Division 1982 Vo. 196 pp. 47-56.
APPENDIX (A)

In addition, for most flow control device operating at small opening, the flow area is linearly related to the valve displacement, thus:

\[ A_0 = Wx \]  

(A.1)

Where:

\( W \): area of the valve and \( x \) is valve displacement.

Consequently, the following dimensionless parameters are defined by:

Damping

\[ \zeta = \frac{c}{2m \omega} \]

Frequency

\[ \omega^2 = \frac{k_r}{m} \]

Displacement

\[ X = \frac{x}{d} \]

Zero load opening

\[ X_o = \frac{x_o}{d} \]

Stiffness

\[ K = \frac{k}{k_r} \]

Time

\[ \tau = \omega x \]

Down stream pipe area

\[ \eta = \frac{A_4}{Wd} \]  

(A.2)

Upstream pipe area

\[ \theta = \frac{A_2}{A_3} \]

Over all pressure difference

\[ \Delta P = \frac{2 \Delta p}{\rho (\omega \lambda d)^2} \]

Discharge

\[ Q = \frac{q}{A_4 (\omega \lambda d)} \]

Inertia factor of pipe

\[ \alpha = \frac{2L_{eq}}{\lambda d} \]

Inertia factor of the jet

\[ \alpha_v = \frac{2L_v}{\lambda d} \]

Mass ratio

\[ \mu = \frac{\lambda^3 s \rho d}{m} \]
The dynamic discharge equation (23) is given in terms of dimensionless parameters and displacement, by substituting Eqs. (A.1) and (A.2) thus:

\[ q = \frac{C_d W X d}{\sqrt{1 - \left(\frac{C_d W X d}{A_2}\right)^2}} \left( \frac{2(P_i - P_o) - 1}{A_i} \left( \psi + \frac{1}{\theta^2} - \frac{A_i^2}{(C_d W X d)^2} \right) \right) q^2 \]  
(A.3)

Multiplying and dividing Eq.(A.3) by \((\lambda \omega d)\), then:

\[ q = \frac{C_d W X d}{\sqrt{1 - \left(\frac{C_d W X d}{A_2}\right)^2}} \left( \frac{(\lambda \omega d)^2}{(\lambda \omega d)^2} \right) \left( \frac{2(P_i - P_o) - 1}{A_i} \left( \psi + \frac{1}{\theta^2} - \frac{A_i^2}{(C_d W X d)^2} \right) \right) q^2 \]
\[ - \frac{2\omega (L_{eq} + \frac{A_i L_o}{(C_d W X d)})}{A_i (L_{eq} + \frac{A_i L_o}{(C_d W X d)})} \frac{dq}{d\tau} \]
(A.4)

Or

\[ q = \frac{C_d W X d}{\sqrt{1 - \left(\frac{C_d W X d}{A_2}\right)^2}} \left( \frac{(\lambda \omega d)^2}{(\lambda \omega d)^2} \right) \left( \frac{2(P_i - P_o) - 1}{A_i} \left( \psi + \frac{1}{\theta^2} - \frac{A_i^2}{(C_d W X d)^2} \right) \right) q^2 \]
\[ - \frac{2}{A_2 \lambda^2 (\omega d)^2} \left( L_{eq} + \frac{A_i L_o}{(C_d W X d)} \right) \frac{dq}{d\tau} \]
(A.5)

Substituting Eq.(A.2), in Eq.(A.5), and dividing by \((A4\omega \pi)\), then:

\[ Q = \frac{C_d X}{\left[ \eta^2 - \left( \frac{C_d X}{\theta} \right)^2 \right]^{\frac{1}{2}}} \left[ \Delta P - \left( \psi + \frac{1}{\theta^2} - \frac{\eta^2}{C_d X^2} \right) Q^2 \right]^{\frac{1}{2}} \]
\[ - \left( \alpha + \frac{\eta \alpha_o}{(C_d X)} \right) \frac{dQ}{d\tau} \]
(A.6)

In order to obtain the valve equation of motion (displacement) in terms of dimensionless parameter using Eqs. (A.2) and (A.6) then Eq. (22) becomes:

\[ \frac{d^2 X}{d\tau^2} + 2\zeta \frac{dX}{d\tau} + K(X - X_o) + \frac{1}{2} \mu \left[ \Delta P - \left( \psi + \frac{1}{\theta^2} - \frac{\eta^2}{C_d X^2} \right) Q^2 \right] \]
\[ - \left( \alpha + \frac{\eta \alpha_o}{(C_d X)} \right) \frac{dQ}{d\tau} \] = 0  
(A.7)

Or

\[ \frac{d^2 X}{d\tau^2} + 2\zeta \frac{dX}{d\tau} + K(X - X_o) + \frac{1}{2} \mu \Delta P^* = 0 \]  
(A.8)

Where \(\Delta P^*\) is given by

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\[
\Delta P' = \left[ \Delta P - \left( \psi + \frac{1}{\theta^2} - \frac{\eta^2}{C_c X^2} \right) Q^2 - \alpha \frac{dQ}{d\tau} \right]
\]  
(A.9)

APPENDIX (B)

Fig. (B.1). Flow chart of computer simulation program.