The Linear Vibrational Behavior of Thick Plates Including the Effects of Shear and Rotary Inertia

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Abstract

In this work, a suggested analytical solution for static and dynamic analysis of (fiber-reinforced) composite laminated thick plate is developed by using the single layer theory and first-order shear deformation theory (FSDT) theory. The dynamic analysis for equations of motion for those theories is presented and solved by using the modal analysis method of forced vibration. A computer program was built for this purpose for anti-symmetric cross-ply and angle-ply and simply supported thick laminated plate and the developed equations are solved by using (MATLAB V.7) program. The numerical solution by using finite-element technique is also adopted using (ANSYS V5.4) package, to compare the analytical results. Both above approaches use (FSDT) and include the effect of shear deformation and rotary inertia. The results are the deflection, stress in each layer and (through thickness) inter-laminar shear stress for thick laminated plates with different boundary conditions subjected to the static and dynamic loading conditions. The results presented show the effect of plate thickness-to-length ratio (h/a), aspect ratio (a/b), number of layers (N), the degree of orthotropy ratio (E_1/E_2), fiber orientation, boundary conditions, lamination scheme, and the effect of shear deformation and rotary inertia on the thick laminated plate.

Key Words:- Thick Plates, Composite Laminated Thick Plate, (FSDT) theory

Nomenclature

a, b	Length dimensions of rectangular plate in x and y direction respectively.	m
A	Total cross section area of lamina.	m^2
E_1, E_2, E_3	Young's modulus for lamina in 1, 2 and 3 directions respectively.	N/m^2
G_{12}, G_{13}, G_{23}	Shear modulus for lamina in 12, 13 and 23 plane respectively.	N/m^2
h	Total plate thickness.	m
k_4, k_5	Transverse shear correction factors.	Unit less
M_x, M_y, M_{xy}	Normal and twisting moments per unit length.	N.m /m
N	Number of laminate's layer.	Unit less
N_i	Number of terms in the expansion.	Unit less
N_x, N_y, N_{xy}	Normal resultants and shear forces per unit length.	N/m
q(x,y)	General loading traction pressure subjected on plates.	N/m^2
Q_x, Q_y	Shear forces in the normal faces to the x and y axis per unite length respectively.	N/m
t	Time	sec
u_1, u_2, u_3	Displacements in plates in x, y and z directions respectively	m
u, v, w	Displacements along the coordinate lines of a mid-plane on the xy-plane.	m
x, y, z	Rectangular coordinates.	x, y, z
Z_k, Z_{k-1}	Distance from plate middle surface to the lower and upper surface of k^{th} layers plates respectively.	m

$[A_{ij}]$, $[B_{ij}]$, $[D_{ij}]$	Extension, bending-extension (coupling) and bending stiffnesses elements respectively.
[f]	Load vector.
[k]	Laminated plate stiffness matrix.
[m]	Laminated plate mass matrix.
[P]	Modal matrix.
$\lfloor \tilde{P} \rfloor$	Weighted modal matrix.
$\vec{Q}_p$	Generalized forces vector.
$\lfloor \overline{Q}_{ij} \rfloor$	Transformed reduced stiffnesses.
$[\varepsilon]_{1,2,3}$	Strain vector in 1, 2 and 3 directions.
$[\varepsilon]_{x,y,z}$	Strain vector in x, y and z directions.
$[\sigma]_{1,2,3}$	Stress vector in 1, 2 and 3 directions.

Introduction

In several mechanical and civil engineering structures, such as automobiles, aircrafts, ships, fluid–storage tanks, bridges and building slabs fall into the category of plate–shell composed system. Composite materials are materials that are made from combined two or more material "a selected filler or reinforcing elements and compatible matrix binder" that have quite different properties, that when combined offer properties which are more desirable than the properties of the individual materials.

Thick composites plates are used also in a marine hull and minesweeper hull because thier low density, non-magnetic, good resistance to corrosion and marine fouling and good resistance to fatigue and stress corrosion cracking. So, studies involving the assessment of the transient analysis of thick laminated composite plates and the effect of transverse shear deformation and rotary inertia are receiving the attention of designers and researchers. **Whitney** and **Sun [1]** have developed a laminated plate theory which was applicable to fiber reinforced composite materials under impact loading. In addition to the usual bending and extensional motion, the theory also includes the first symmetric thickness shear and thickness stretch motions as well as the first anti-symmetric thickness shear mode.

Reddy [2] employed a shear flexible finite element to investigate the transient response of isotropic, orthotropic and layered anisotropic composite plates. Numerical convergence and stability of the element is established using Newmark's direct integration technique. Numerical results for deflections and stresses are presented for rectangular plates under various boundary conditions and loading. The parametric effects of the time step, finite element mesh, lamination scheme and orthotropy on the response are investigated.

Kant and **Mlikarjuna [3]** formulated a refined higher-order theory for free vibration analysis of unsymmetrical laminated multilayer plates. The theory accounts for parabolic of the transverse shear strain through the thickness of the plate and rotary inertia effects. A simple finite element formulation is presented and the nine-nodded Lagrangian element is chosen with seven degrees-of-freedom per node. The adopted theory predicts the frequencies more accurately when compared with classical plate theories.

Yin [4] presented a variational method involving Lekhnitskii's stress functions is used to determine the inter-laminar stresses in a multilayered strip of laminate subjected to arbitrary combinations of axial extension, bending, and twisting loads. The stress functions in each layer are approximated by polynomial functions of the thickness coordinate.

khras et al [5] developed a finite strip method for the vibration and stability analyses of thick anisotropic laminated composite plates according to the higher–order shear deformation theory. This theory accounts for the parabolic distribution of the transverse shear strains through the thickness of the plate and for zero transverse shear stresses on the plate surface. In comparison with the finite strip method based on the first–order shear deformation theory, the present method gives improved results for every thick plate while using approximately the same number of degrees of freedom.

The main objective of the current work is to determine the linear behavior of thick laminated composite plates including the effect of transverse shear deformations and rotary inertia, under general loading condition static and dynamic. To achieve the above objectives a suggested analytical solution is developed for linear dynamic analysis of thick laminated composite plates under transient loading by using the theories of laminated plates, solved analytically by designed computer program, built using (**MATLAB V.7**) program. In addition, a numerical solution for static and dynamic equations of thick laminated composite plates is achieved by using a finite element method software **(ANSYS V5.4)** program and comparison of the dynamic results is made between analytical and numerical solutions.

Theoretical Investigation

Classical Laminated Plate Theory (CLPT)

The two–dimensional theory of extensional and flexural motions of heterogeneous an isotropic plates is deduced from the dynamical equations of three–dimensional elasticity

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + Fx = \rho \frac{\partial^2 U_1}{\partial t^2} \quad (1a)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + Fy = \rho \frac{\partial^2 U_2}{\partial t^2} \quad (1b)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + Fz = \rho \frac{\partial^2 U_3}{\partial t^2} \quad (1c)$$

where: $\rho = \rho(x, y, z)$ is the material density.
Fi = is the body force in i-axis component.
These equations are converted to plate–stress equations by method of Yang, Norris and Stavsky theory; [6].

First–Order Shear Deformation Theory (FSDT)

The equations of motion for (FSDT) obtained in the same method are applied to obtain the general equations of motion for (CLPT); [6].

For (FSDT) the following equations of motion are:

$$\begin{aligned}
&\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \psi_x}{\partial t^2} \\
&\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_1 \frac{\partial^2 v}{\partial t^2} + I_2 \frac{\partial^2 \psi_y}{\partial t^2} \\
&\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = I_1 \frac{\partial^2 w}{\partial t^2} + q(x,y,t) \\
&\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2 \frac{\partial^2 u}{\partial t^2} + I_3 \frac{\partial^2 \psi_x}{\partial t^2} \\
&\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = I_2 \frac{\partial^2 v}{\partial t^2} + I_3 \frac{\partial^2 \psi_y}{\partial t^2}
\end{aligned} \quad 2$$

Second and Higher Order Shear Deformation (ESL) Laminated Plate Theories

The second and higher shear deformation theories use higher order polynomials (quadratic or cubic) in the expansion of the displacement components through the thickness of the laminate, i.e., (a parabolic variation of the transverse shear strains throughout the thickness). The more important theory that must be treated with it, is the general third order shear deformation laminated plate theory (GTOT).

General Third-Order Theory of Reddy (GTTR)

In this section, the displacement field for (GTTR) will be derived. This theory is a special case from the (GTOT), which was developed by Reddy, who made it imply the following conditions; [7 and 8].

$$\phi_x = -\frac{1}{2}\frac{\partial \phi_z}{\partial x} \quad , \quad \theta_x = -\frac{1}{3}\left[\frac{\partial \theta_z}{\partial x} + \frac{4}{h^2}\left(\psi_x + \frac{\partial w}{\partial x}\right)\right]$$

$$\phi_y = -\frac{1}{2}\frac{\partial \phi_z}{\partial y} \quad , \quad \theta_y = -\frac{1}{3}\left[\frac{\partial \theta_z}{\partial y} + \frac{4}{h^2}\left(\psi_y + \frac{\partial w}{\partial y}\right)\right] \quad (3)$$

Putting the above conditions in equation (3) leads to the following displacement field:

$$u_1 = u + Z\psi_x + Z^2\left(-\frac{1}{2}\frac{\partial\phi_z}{\partial x}\right) + Z^3\left[-\frac{1}{3}\left(\frac{\partial\theta_z}{\partial x} + \frac{4}{h^2}\left(\psi_x + \frac{\partial w}{\partial x}\right)\right)\right]$$
$$u_2 = v + Z\psi_y + Z^2\left(-\frac{1}{2}\frac{\partial\phi_z}{\partial y}\right) + Z^3\left[-\frac{1}{3}\left(\frac{\partial\theta_z}{\partial y} + \frac{4}{h^2}\left(\psi_y + \frac{\partial w}{\partial y}\right)\right)\right] \quad (4)$$
$$u_3 = w + Z\phi_z + Z^2\theta_z$$

Inter–Laminar (through–thickness) Stresses

In the laminated plates, no account is taken of inter–laminar stresses such as $\square_z$, $\square_{xz}$ and $\square_{yz}$. Inter–laminar stresses are one of the failure mechanisms uniquely characteristic of composite materials and a source of damage in stressed laminates.

The inter–laminar shear stresses are determined from:

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} = \rho\frac{\partial^2 u_1}{\partial t^2}$$
$$\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + + \frac{\partial\tau_{yz}}{\partial z} = \rho\frac{\partial^2 u_2}{\partial t^2} \quad (5)$$

Classical Laminated Plates Theory (CLPT)

In classical laminated plates theory only the stresses in the plane of laminate, $\square_x$, $\square_y$ and $\square_{xy}$ are considered, because no account is taken of inter laminar (through–thickness) stresses such as inter laminar shear stresses $\square_{xz}$ and $\square_{yz}$, that is, a plane stress state is assumed.

First–Order Shear Deformation Laminated Theory (FSDT)

In the first–order shear deformation laminated theory, the inter-laminar shear stresses τ_{xz} and τ_{yz} are calculated in the same way in the (CLPT).

The Dynamic Response

The equations of motion of a multi-degree of freedom system under external forces are given by:

$$[M][\ddot{\Delta}(t)] + [C][\dot{\Delta}(t)] + [K][\Delta(t)] = [F] \quad (6)$$

To solve equation (6) by modal analysis, it is necessary first to solve the eigen-value problem:

$$\omega^2[M].\vec{\delta} = [K].\vec{\delta} \quad (7)$$

And find the natural frequencies ω_1, ω_2… ω_n and the corresponding normal weighted modal .

Computer Program

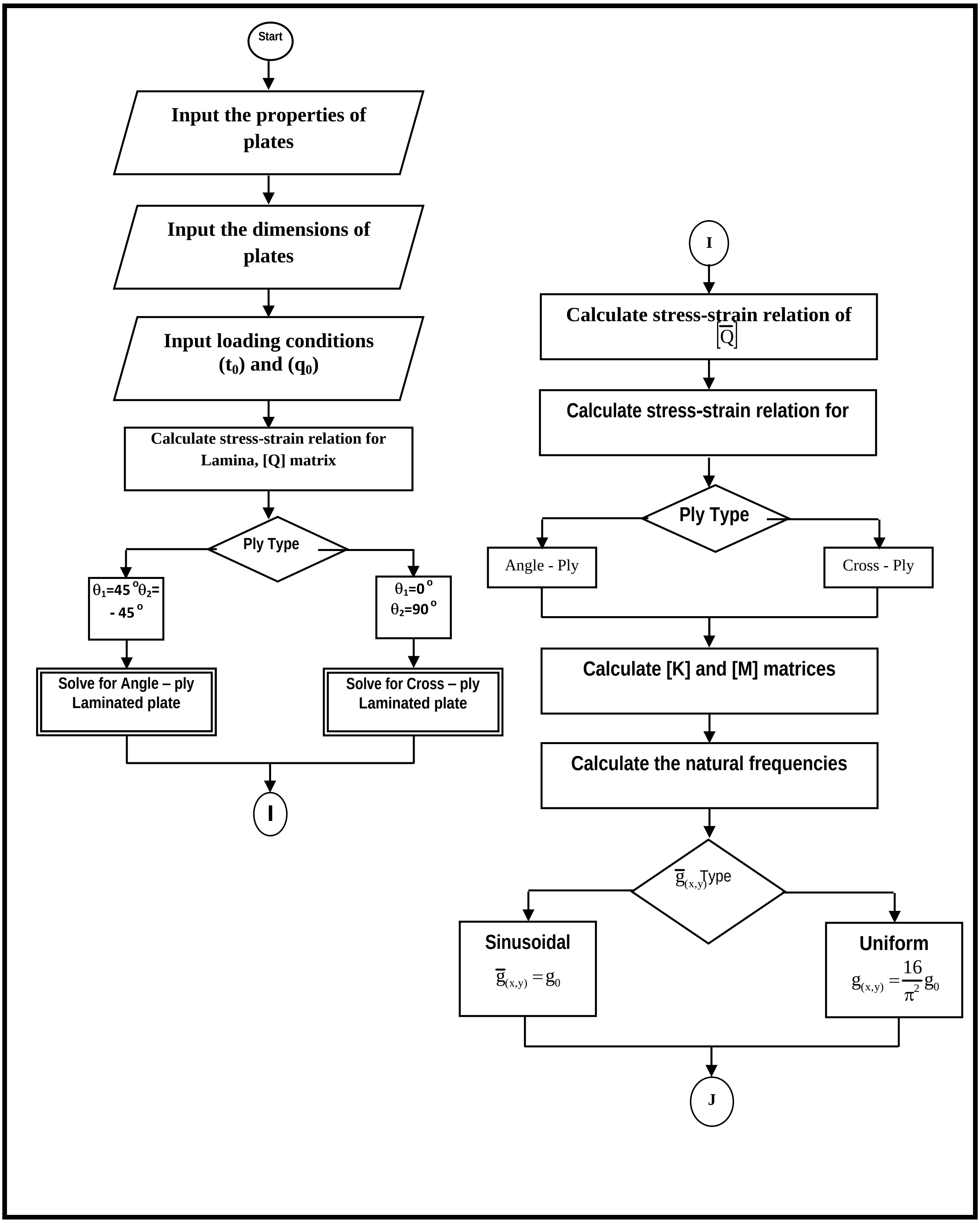
The sequence in Fig. (1) shows the flow chart for computer program for dynamic analysis of composite laminated plate. The program is built for solving the developed by using first order shear deformation theory (FSDT), for anti-symmetric (cross-play) and (angle-ply) simply supported laminated plates subjected to uniformly, sinusoidal distribution for pulse and ramp dynamic loading.

The Input of Program

1. The properties of composite laminated plate in (1,2,3) directions of lamina (E_1, E_2, E_3,G_{12},G_{23},G_{13},ν_{12}, ν_{23}, ν_{13}, and ρ) .
2. The geometry or (the dimensions) of thick composite laminated plate , length of plate (a), width of plate (b) and the thickness of plate (h).
3. The data of load (pressure) (q_0 and t_0).

The Output of Program are

1. The maximum deflection (Central deflection at x = a/2 and y = b/2) with time.
2. The stresses in each layer of laminate with time



Start
Input the properties of plates
Input the dimensions of plates
Input loading conditions (t_0) and (q_0)
Calculate stress-strain relation for Lamina, [Q] matrix
Ply Type
θ_1=45°θ_2= - 45°
θ_1=0° θ_2=90°
Solve for Angle – ply Laminated plate
Solve for Cross – ply Laminated plate
I
I
Calculate stress-strain relation of $[\overline{Q}]$
Calculate stress-strain relation for
Ply Type
Angle - Ply
Cross - Ply
Calculate [K] and [M] matrices
Calculate the natural frequencies
$\overline{g}_{(x,y)}$ Type
Sinusoidal
$\overline{g}_{(x,y)} = g_0$
Uniform
$g_{(x,y)} = \frac{16}{\pi^2} g_0$
J

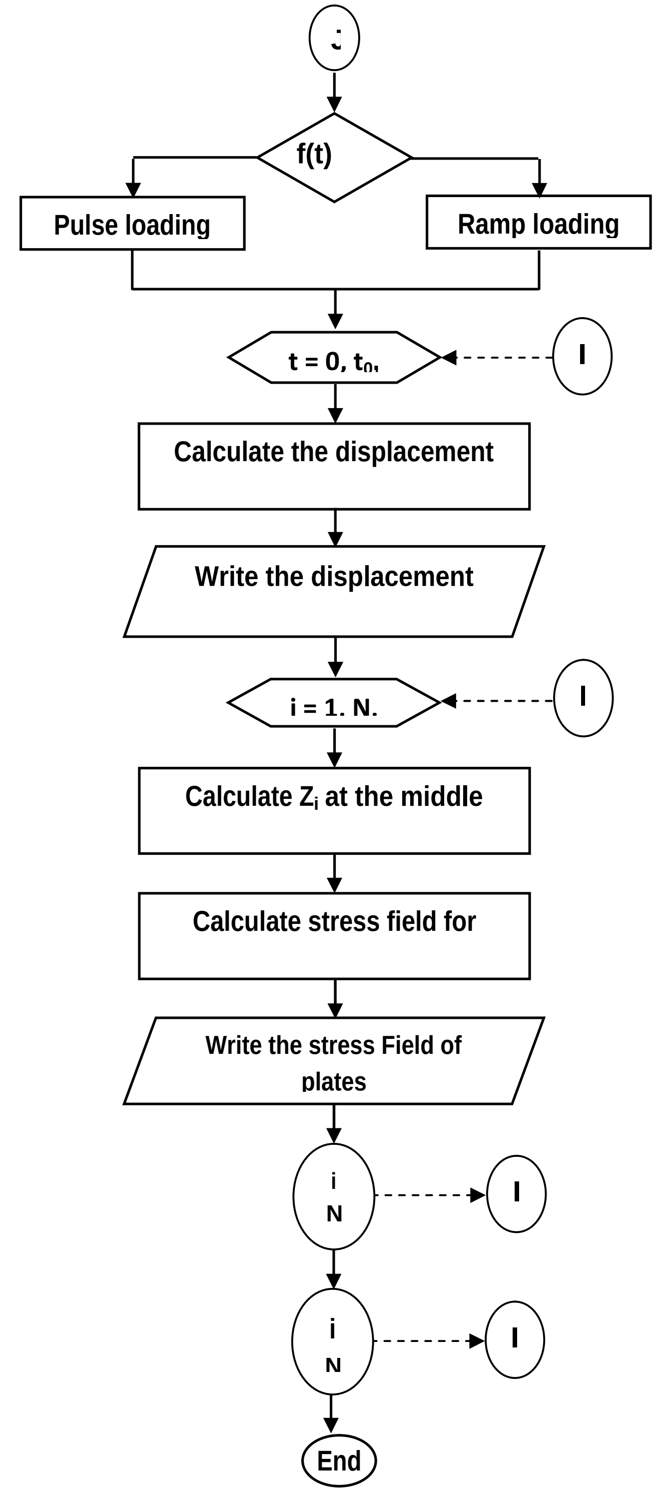


Fig. (1) Flow Chart of the Developed Computer Program

Application of ANSYS Program to Composite Materials

The ANSYS program allows to model composite materials by using specialized elements called layered elements. Once the model is built using these elements, any structural analysis can be done (including nonlinearities can be achieved such as large deflection and stress stiffening). (SHELL91 16-Layer structural shell) shown in Fig. (2) is used for application of laminated plates, SHELL91 which may be used for layered applications of a structural shell model or for modeling thick sandwich structures

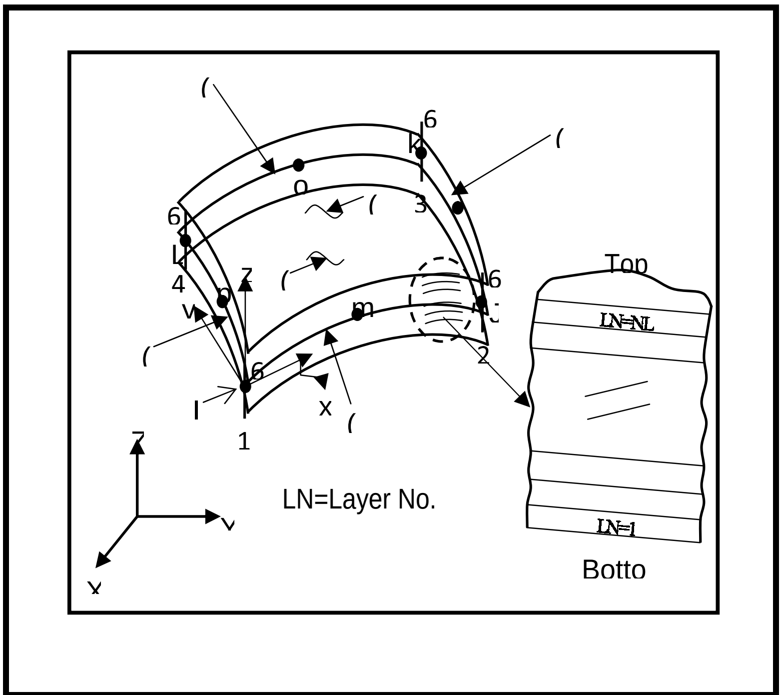


Fig. (2) SHELL91 16-Layer, Structural Shell.

Verification Case Study

The case study discussed here is comparison between the work of Reddy [2] of a numerical solution with the present analytical solution and numerical solution for a thick laminated plate . The case study discussed here is comparison between the work of Reddy [2] of a numerical solution with the present analytical solution and numerical solution for a thick laminated plate.

* Both solutions use two-layer (0/90) cross-ply square laminated plate of simply supported boundary conditions at edges as shown in Fig. (3) below.
* Employing the given geometry and material properties and adapted the first order shear deformation theory (FSDT) with mesh size (4x4) element

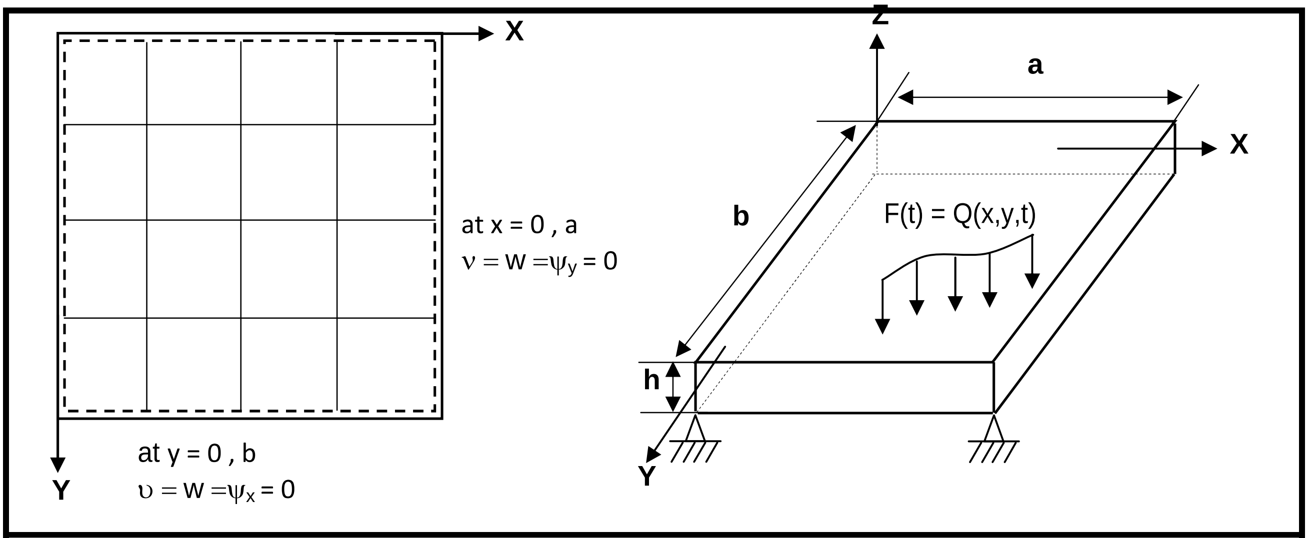


Fig. (3) Geometry and Boundary Conditions of Case Study

The two layer cross-ply laminated plate is subjected to sinusoidal pulse loading:

$$F(t) = Q(x, y, t) = \overline{q}(x, y).f(t)$$

where:

$$\overline{q}(x, y) = \frac{4}{ab}\int_0^b\int_0^a q(x, y).dx.dy$$

$$q(x, y) = q_0 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$$

$$q_0 = 100\ \frac{KN}{m^2}$$

I. **The Dimension of Plate**

a = b = 25 cm = 0.25 m

$a)$ $\quad h = 5cm = 0.05m \Rightarrow \frac{a}{h} = 5$

$b)$ $\quad h = 1cm = 0.01m \Rightarrow \frac{a}{h} = 25$

I. **The Properties of Plate**

$E_2 = E_3 = 21$ GPa

$$\frac{E_1}{E_2} = 25$$

$G_{12} = G_{13} = G_{23} = 0.5\ E_2$

$\rho = 800\ kg/m^3$

$\nu_{12} = \nu_{13} = \nu_{23} = 0.25$

Results and Discussions

Fig. (4) shows the effect of aspect ratio (a/b) on the central deflection of plate with different fiber orientations and boundary conditions. From the figure, the central deflection increases with increasing of the aspect ratio because increasing of the aspect ratio means increasing of the length of plate which leads to decrease the stiffness and increase the deflection.

Fig. (5) shows the effect of degree of orthotropy or modulus of elasticity ratio (E_1/E_2), of (E_2=8.96) Gpa, on central deflection of simply supported and clamped edges, anti-symmetric cross-ply and angle-ply thick laminated plates. From the figure, the central deflection of thick laminated plates decreases with increasing (E_1/E_2) ratio. This is because that with increasing of the (E_1/E_2) ratio means increasing of (E_1) with respect to (E_2), the stiffness of the plate or deflection resistance increases.

Fig. (6) shows the effect of fibers orientation angle (θ) on the central deflection for different number of layers (N) and boundary conditions for angle-ply thick laminated plates. The figure shows that, the deflection for simply supported and (N=2) layer plate increases with the increase in (θ) to (200), then decreases with increase in (θ) to (450), and symmetric about (450), because the effect of [Q] matrix calculation, and the central deflection of the plate will decrease with the increasing of the number of layer (N).

Fig. (7) shows the effect of aspect ratio at (a=1m) on the maximum stress (σ_x) for different fiber orientation and boundary conditions of thick laminate plates. The figure shows that the stress (σ_x) increases with increasing the aspect ratio and the stress (σ_x) for (same thick laminated plates) of clamped edges plates is more than that of simply supported plates for cross-ply lamination and the stress (σ_x) for (same thick laminated plates) of simply supported plates is more than that of clamped edges plates of cross-ply lamination because the fiber orientation and boundary conditions effects.

Fig. (8) shows the effect of (h/a) ratio at (a=1m), on the stress (σ_x) for cross-ply and angle-ply, with simply supported and clamped edges thick laminated plates. The figure shows that the stress (σ_x) decreases with increasing (h/a) ratio, that means it decreases with increase in thickness (h).

Figs. (9 and 10) show the inter-laminar shear stress (τ_{xz}) and (τ_{yz}) respectively, for four layers anti-symmetric angle-ply (45/-45/…) thick laminated plates subjected to uniformly ramp loading. The maximum shear stresses (τ_{xz}) and (τ_{yz}) occur at the middle plane (1.5 Mpa) and (0.82 Mpa) respectively, in addition the inter-laminar shear stresses (τ_{xz}) and (τ_{yz}) are symmetric about the middle plane for both cases of anti-symmetric cross-ply (0/90/0/…) and anti-symmetric angle-ply (45/-45/…) laminated plates, where the shear stress (τ_{xz}) and (τ_{yz}) between layer (1-2) is equal to the stress between layer (3-4) and less than the stress at the middle plane. The stresses (τ_{xz}) and (τ_{yz}) for anti-symmetric angle-ply (45/-45/…) are more than that of anti-symmetric cross-ply (0/90/0/…) (Figs. 11 and 12).

Fig. (13) shows the effect of number of layers on the inter-laminar shear stress (τ_{xz}) and (τ_{yz}) at the middle plane, i.e., between layers (2-3) for four layer anti-symmetric cross-ply (0/90/0/…) laminated plates subjected to uniform ramp loading. The figure shows that there is no effect to number of layers on the stress and the value of stresses (τ_{xz}) and (τ_{yz}). They remain constant when the number of layers increase because the distance (Z) of middle plane will remain constant and equal zero.

Fig. (14) represents the comparison of the inter-laminar shear stress (τ_{xz}) with (τ_{yz}) at middle plane for different aspect ratios (a/b), (a=1) for four layer anti-symmetric cross-ply thick laminated plates subjected to uniformly pulse loading. The shear (τ_{xz}) is greater than the (τ_{yz}) at (a/b=0.5) because the area under (τ_{xz}) is less than the area under (τ_{yz}), and (τ_{xz}) is less than (τ_{yz}) at (a/b=2) that because the area under (τ_{yz}) is less than the area under (τ_{xz}). The (τ_{yz}) increases with increasing of the aspect ratio.

Fig. (15) shows the effect of the degree of orthotropy ratio (E_1/E_2) of (E_2=10.6 Gpa) on the inter-laminar shear stress (τ_{xz}) and (τ_{yz}) at the middle plane, i.e., between layers (2-3) for four layer anti-symmetric cross-ply (0/90/0/…) thick laminated plates subjected to uniform ramp loading. The shear stresses (τ_{xz}) and (τ_{yz}) decrease with increasing of (E_1/E_2) ratio

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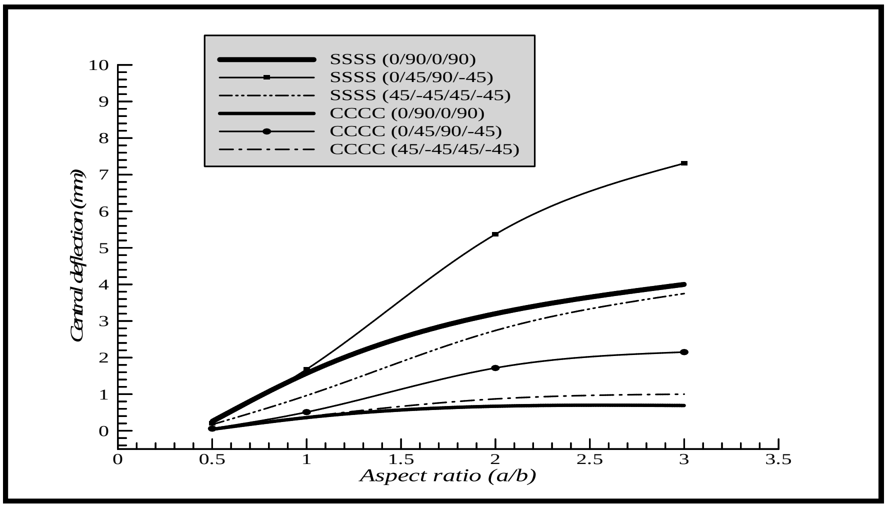

Fig. (4) Central Deflection for Different Boundary Conditions and Fiber Orientation of Thick Laminated Plate with Different Aspect Ratios (a/b) Under Sinusoidal Loading for (N=4).

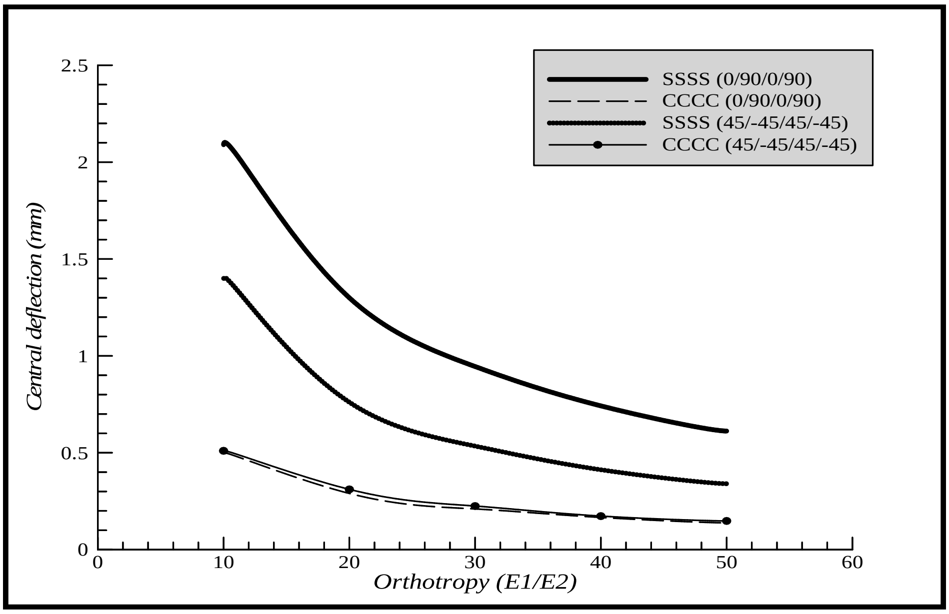

Fig. (5) Central Deflection for Different Fiber Orientation and Boundary Conditions of Thick Laminated Plate with Different Orthotropies (Modulus Ratio) (E_1/E_2) Under Sinusoidal Loading for (N=4).

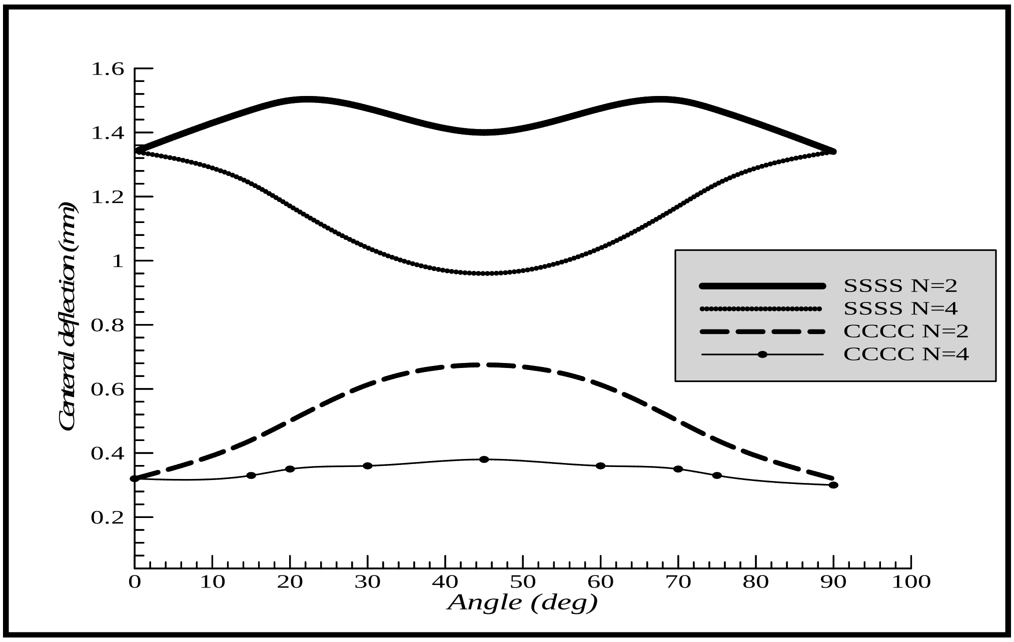

Fig. (6) Central Deflection for Different Number of Layer (N) and Boundary Conditions of Angle-Ply Laminated Plate with Different Angles (θ) Under Uniform Loading.

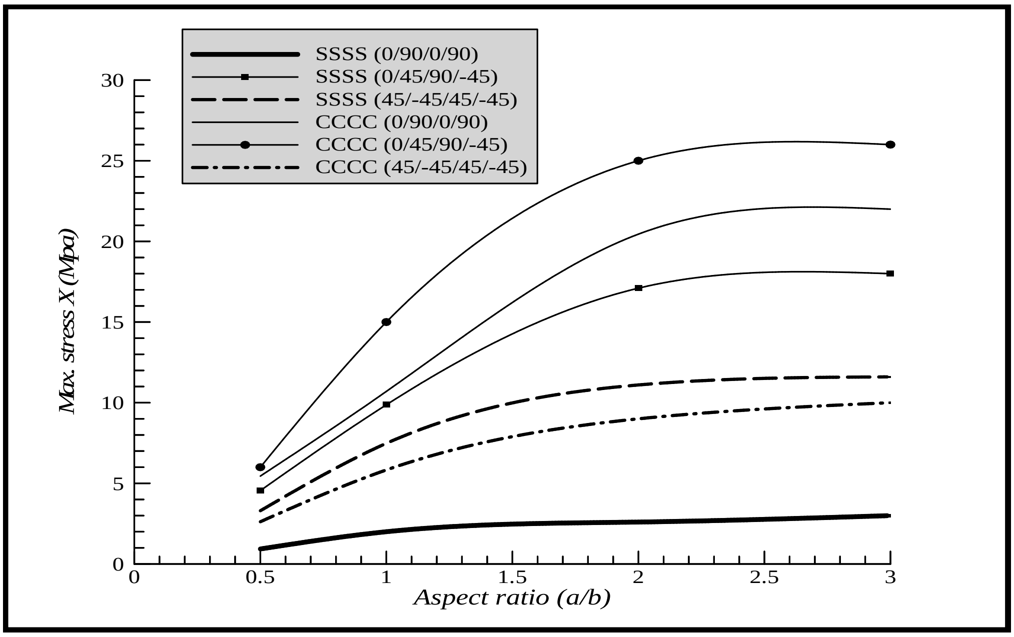

Fig. (7) Max. Stress (σ_x) for Different Fiber Orientation and Boundary Conditions of Thick Laminated Plate with Different Aspect Ratios (a/b) Under Sinusoidal Loading for (N=4).

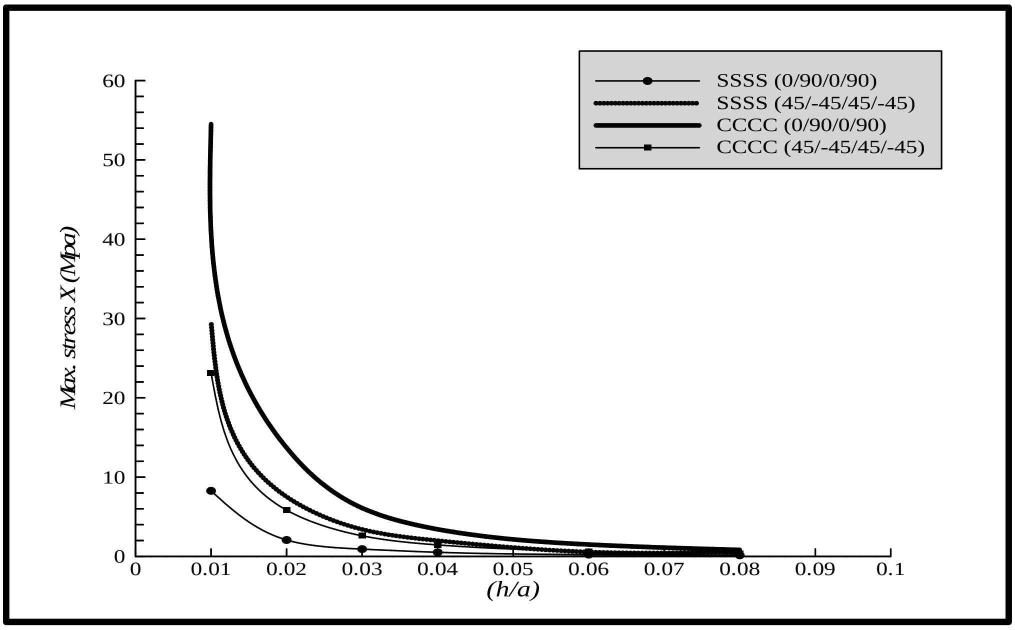

Fig. (8) Max. Stress (σ_x) for Different Fiber Orientation and Boundary Conditions of Thick Laminated Plate with Different Thickness Ratios (h/a) Under Uniform Loading for (N=4).

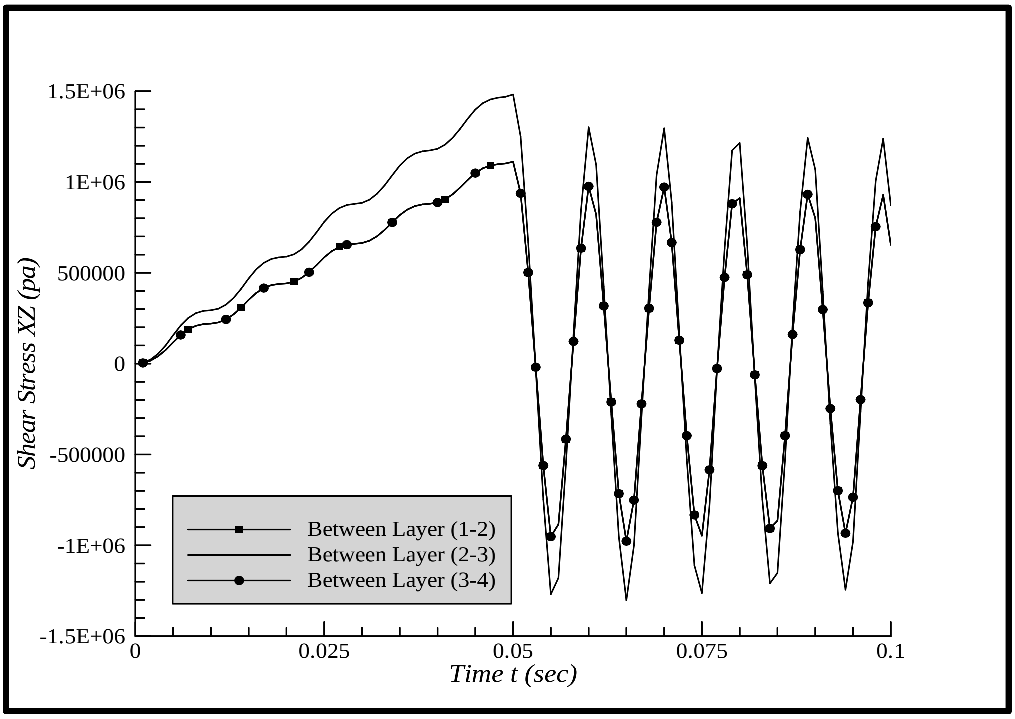

Fig. (9) Inter-Laminar Shear Stress (τ_{xz}) due to Uniform Ramp Loading For (4) Layers, Anti-Symmetric Angle–Ply (45^o/-45^o/…) Laminated Plates.

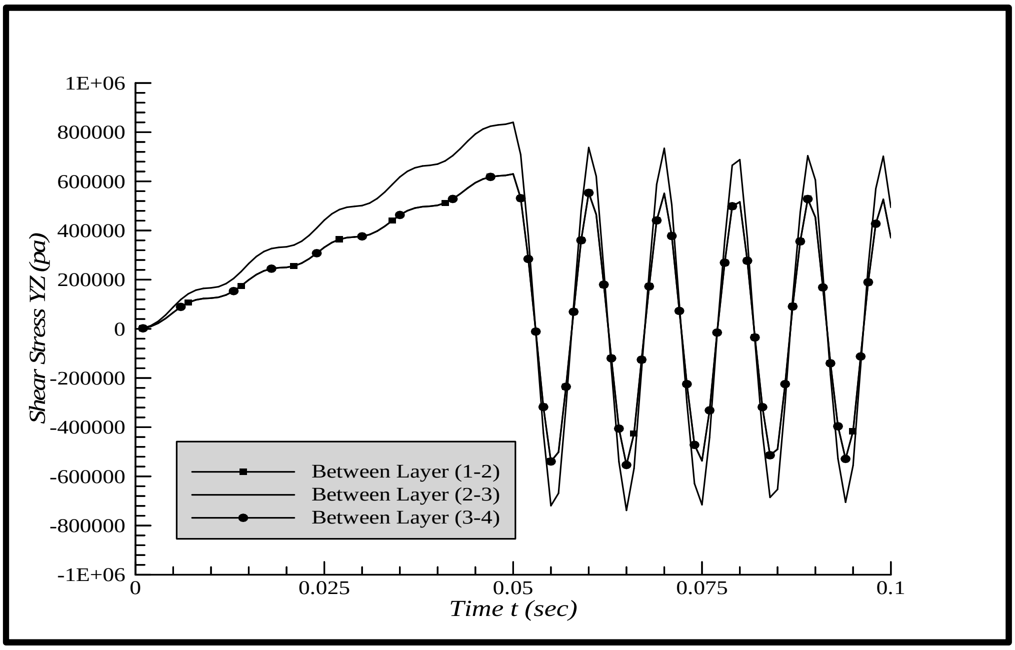

Fig. (10) Inter-Laminar Shear Stress (τ_{yz}) due to Uniform Ramp Loading For (4) Layers, Anti-Symmetric Angle–Ply (45^o/-45^o/…) Laminated Plates.

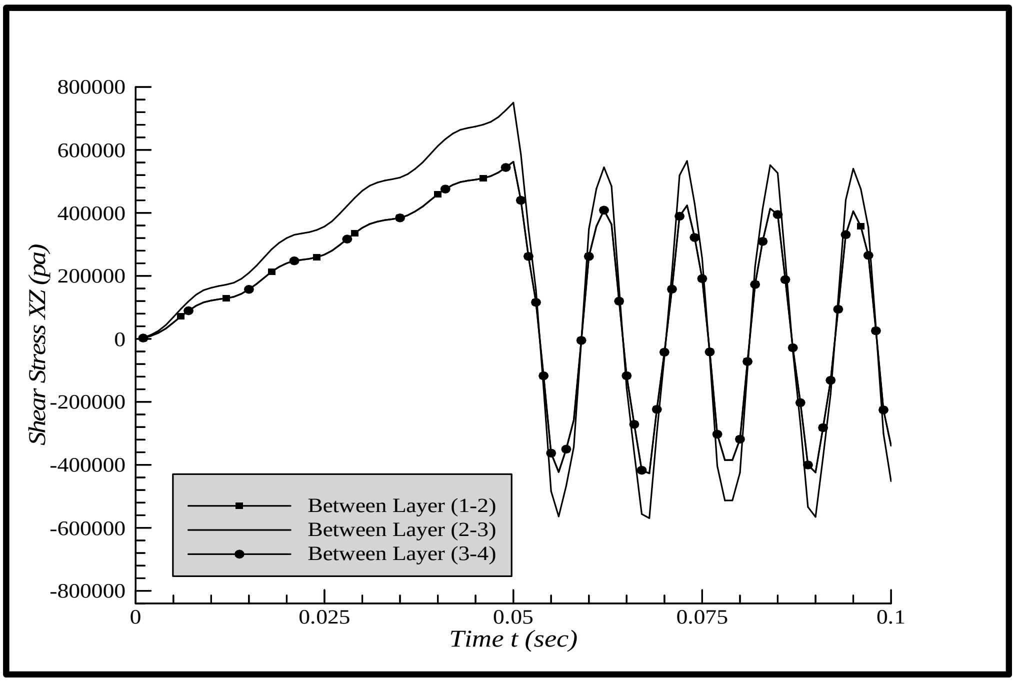

Fig. (11) Inter-Laminar Shear Stress (τ_{xz}) due to Uniform Ramp Loading For (4) Layers, Anti-Symmetric Cross–Ply ($0^o/90^o$/…..) Laminated Plates.

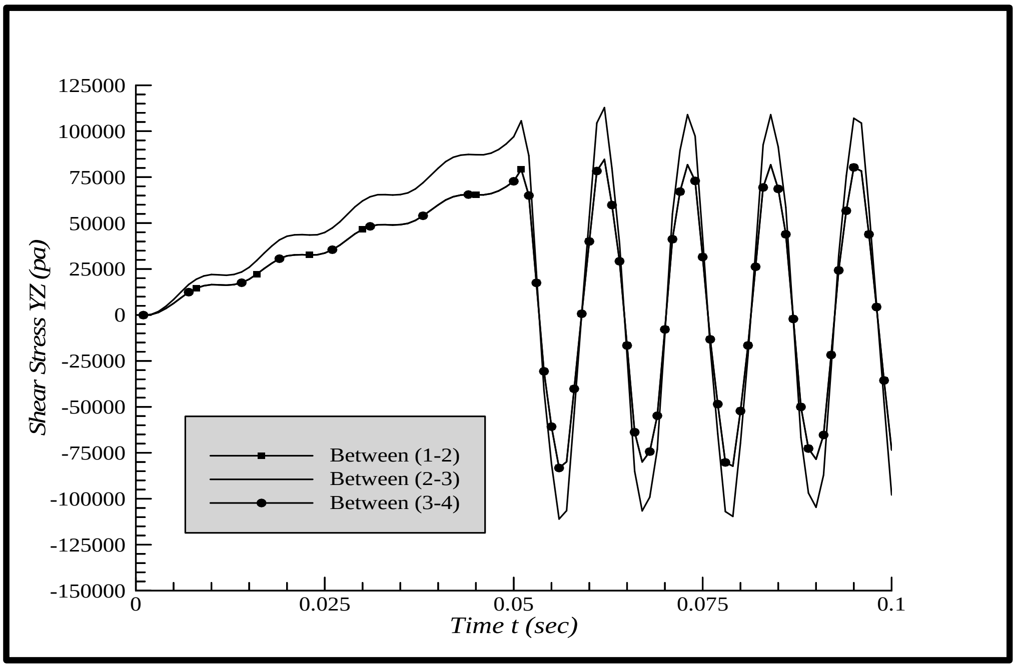

Fig. (12) Inter-Laminar Shear Stress (τ_{yz}) due to Uniform Ramp Loading For (4) Layers, Anti-Symmetric Cross–Ply ($0^o/90^o$/…) Laminated Plates.

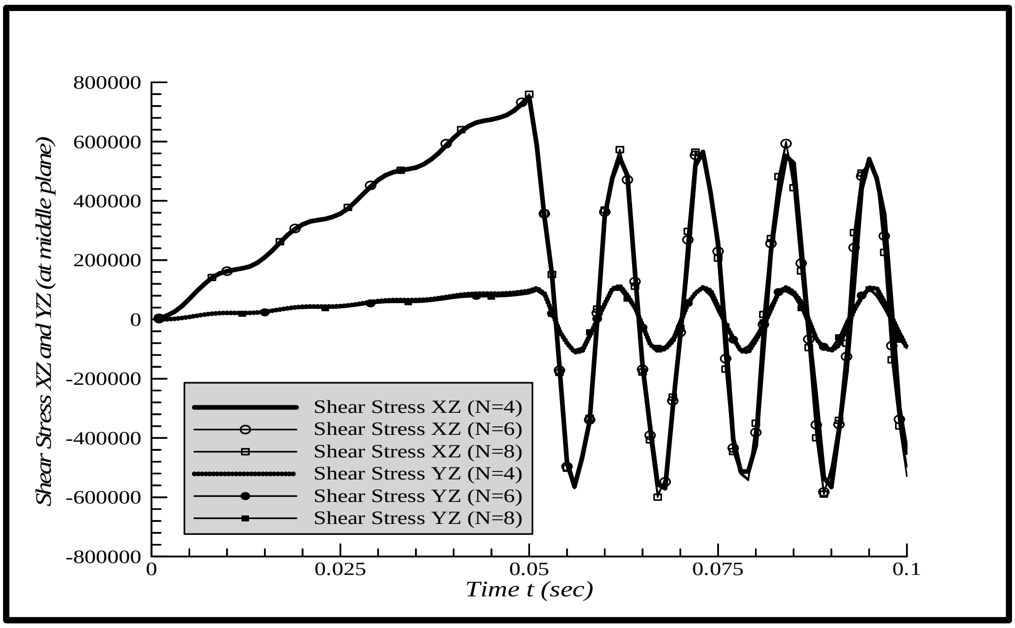

Fig. (13) Effect of Number of Layers (N) on Inter-Laminar Shear Stress (τ_{xz}) and (τ_{yz}) at Middle Plane of (4) Layers, Anti-Symmetric Cross–Ply Laminated Plates, Under Uniform Ramp Loading.

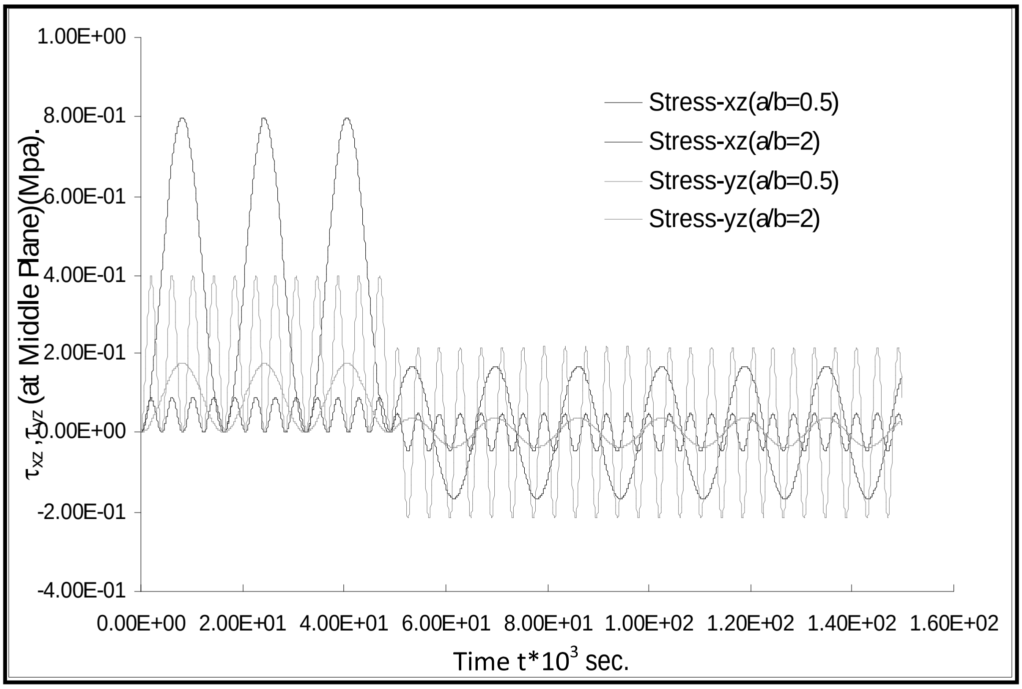

Fig. (14) Effect of Aspect Ratio (a/b) on Inter-Laminar Shear Stress (τ_{xz}) and (τ_{yz}) at Middle Plane of (4) Layers, Anti-Symmetric Cross–Ply Laminated Plates, Under Uniform Pulse Loading

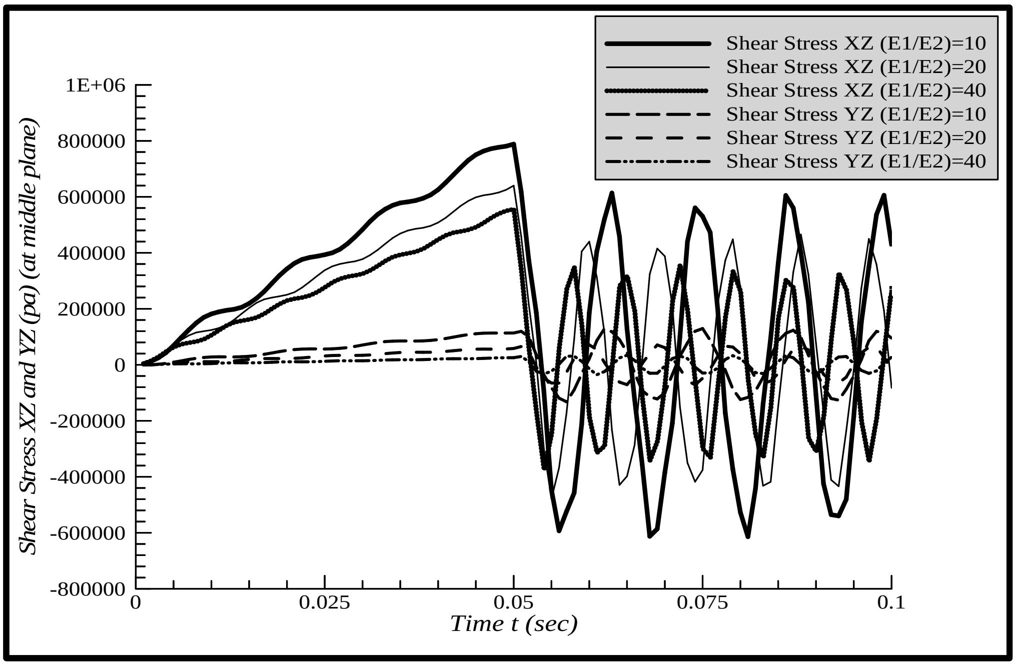

Fig. (15) Effect of Modules Ratio (E_1/E_2) on Inter-Laminar Shear Stress (τ_{xz}) and (τ_{yz}) At Middle Plane of (4) Layers, Anti-Symmetric Cross–Ply Laminated Plates, Under Uniform Ramp Loading.

Conclusions

1. The amplitude values of deflection and stress for angle-ply (45/-45/…) thick laminated plates are less than that for cross-ply (0/90/…) thick laminated plates in simply supported boundary condition and the situation is reversed in clamped boundary condition.
2. For the angle-ply thick laminated plates, it was found that, minimum deflection and stresses for simply supported laminated plates and maximum deflection for clamped laminated plates occurs at ($\theta=45^0$).
3. Increasing the number of layers (N), the degree of orthotropy ratio (E_1/E_2), and thickness-to-length (h/a) ratio of laminated plates decreases the deflection and stress in the plates. On the other hand they increase with the increase of the aspect ratio (a/b) of plates.
4. The inter-laminar shear stresses (τ_{xz}) and (τ_{yz}) in case of using anti-symmetric angle-ply (45/-45/-45…) lamination is more than using anti-symmetric cross-ply (0/90/0/…) lamination.
5. There is no effect of the number of layers on the inter-laminar shear stress (τ_{xz}) and (τ_{yz}) at the middle plane and the values of stress will remain constant in spite of the increase in the numbers of layers.
6. The inter-laminar shear stress (τ_{xz}) at middle plane decreases with increasing the aspect ratio (a/b) and the orthotropy ratio (E_1/E_2) ratio of plates.
7. The values of the deflection and stresses for simply supported thick laminated plates are increased if the effect of the shear deformation and rotary inertia is taken into account.

References

1. **M.Whitney** and **C.T.Sun**, "A high Order theory for extensional motion of laminated composites", J. Sound and Vibration, vol. 30, no. 1, (1973).
2. **J.N.Reddy**, "Dynamic (Transient) Analysis of layered Anisotropic composite–material plates", int. J. Numerical Methods in Engineering, vol. 19, (1983), P.P. (237–255).
3. **T.Kant** and **Mallikarjuna**, "A higher–Order theory for free vibration of unsymmetrically laminated composite and sandwich plates–finite element evaluations", J. Computers and Structures, vol. 32, no. 5, (1989).
4. **W.L.Yin**, "Free-Edge Effects in Anisotropic Laminates Under Extension, Bending and Twisting, Part–I: a Stress–Function–Based Variational Approach", J. Applied Mechanics, Vol. 61, No. 2, (June 1994).

5. **J.Petrolito** and **B.W.Golley**, "Vibration of Thick Plates Using Finite Strip–element" J. ANZIAM, vol. 42(E), (2000), P.P. C1137–C1155.
6. **P.C.Yang**, **C.H.Norris** and **Y.Stavsky**, "Elastic wave propagation In Heterogeneous plates", Int. J. Solids and Structures, vol. 2, (1966), P.P (655–684).
7. **P.Bose** and **J.N.Reddy** "Analysis of Composite Plates using Various Plate Theories Part I: Formulation and Analytical Solutions" J. Structural Engineering and Mechanics, Vol. 6, No. 6, (1998), P.P. (583-612).

P.Bose and **J.N.Reddy**, "Analysis of Composite Plates using Various Plate Theories. Part 2: Finite Element Model and Numerical Results" J. Structural Engineering and Mechanics vol. 6, No. 7, (1998), P.P (727-746).

السلوك الاهتزازي المرن للصفائح السميكة بوجود تأثيرات القص والقصور الدوار

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الخلاصة

في هذا االعمل تم اقتراح تحليل رياضي للتصرف الساكن "Static" و الحركي "Dynamic" للصفائح السميكه المركبه المقواة بالألياف و المصنوعه على شكل طبقات باستخدام نظريات الطبقه المفرده "single layer theories"، ونظرية تشوه القص من الرتبة الأولى. و من ثم، تم إيجاد التحليل الحركي لمعادلات الحركة عن طريق استخدام "Modal Analysis" للاهتزاز الناتج من تسليط قوى متغيرة مع الزمن. كما و تم بناء برنامج صمم لهذا الغرض، للصفائح المتناظره و الغير متناظرة و المسندة ببساطة (simply supported)، وتم حلها باستخدام برنامج (MATLAB V.7). و لدعم وأسناد النتائج التحليليه تم وضع الحل العددي باستخدام طريقة العناصر المحددة و باستخدام برنامج(ANSYS V5.4) . كلا الحلين تم بنائهما بأستخدام (FSDT) ويحتويان تأثير تشويه القص(shear deformation) وعزم القصورالدوراني (rotary inertia). النتائج المستنتجه هي التشوه، الإجهادات و الإجهادات ما بين الطبقات لمختلف انواع التثبيت والتحميل والتي منها تم دراسة تأثير نسبة الطول إلى السمك (h/a)، نسبة الباع (a/b)، نسبة درجة الارثوتروبية (E_1/E_2)، زاوية التركيب (Lamination Angle) و حالة التثبيت للصفيحة. من خلال النتائج التي تم التوصل إليها وجد بان التشويه، الإجهاد و الإجهاد بين الطبقات للصفائح يقل بزيادة عدد الطبقات، نسبة درجة الارثوتروبية و نسبة الطول إلى السمك للصفيحة. بالإضافة إلى إن الحل لها يزداد بزيادة نسبة الباع. بالإضافة إلى ذلك ان قيم التشويه ، الإجهاد و الإجهاد بين الطبقات للصفائح تكون أعلى أذا تم الأخذ بنظر الإعتبار تأثير تشويه القص وعزم القصور الدوراني.