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Reinforcement Design Algorithm For Concrete Shells

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AbstracT

The absence of universally accepted solutions in the structural concrete codes for the design of reinforcement in shells gives rise to the problem of calculating the required reinforcement in these structures. The constant development of the computer's performance and storage capacity combined with the powerful numerical methods reveal the need for a standard procedure to design shells subjected to membrane and flexural forces.

In this paper, the solution for the design of the required reinforcement in concrete shells is presented based on a complete iterative computational algorithm to design shell elements subjected to combined membrane forces and bending moments.

In the design equations, the reinforcement will contribute to tension and the concrete compression struts parallel to the crack direction will contribute to compression. The reinforcement is assumed to have two orthogonal layers placed in the top and bottom surfaces with appropriate covers. Each reinforcement layer has reinforcing bars placed orthogonally. For the concrete compression struts, the stress is assumed to be uniformly distributed in the depth of Whitney's stress block.

This design algorithm is achieved by developing a design code (**DRCSH)** based on a complete iterative computational algorithm. This program can be used as a stand-alone version, to determine the load carrying capacity of critical points in reinforced concrete panels, plates and shells; and to verify the design code on the element level, five experimental models are designed. The designed elements give calculated ultimate strengths from 7 to 18% higher than test results values, except one model, which confirms the adequacy of the design algorithm, and the developed design code.

Key words: Finite element, shells, concrete structure, stress analysis

1. Overview of Design Methods for Reinforced Concrete Shells:

At any point in the shell, as shown in Fig. (1), two different types of internal forces may occur simultaneously; those associated with membrane action (N_x , N_y and N_{xy}) and those associated with bending of the shell (M_x , M_y and M_{xy}).

Even though shells resist the applied forces primarily through in-plane membrane action, bending is still induced on the shell. Therefore, a more rational approach to the design process is to simultaneously include combined membrane forces and bending moments.

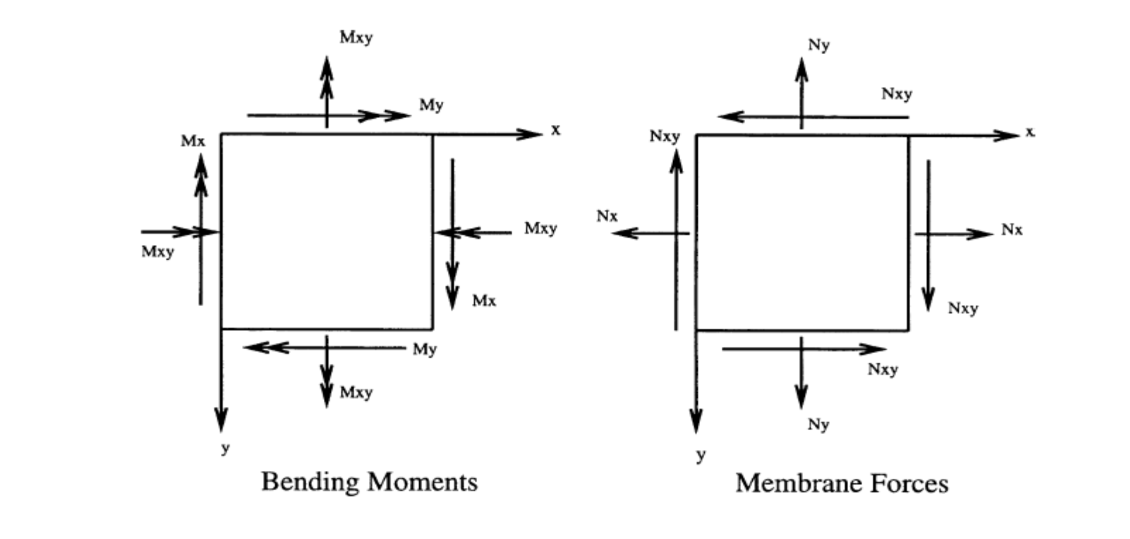

Figure (1) Applied forces and moments on an element

Currently, designers first perform the design with membrane forces only and later provide the reinforcement for bending in particular locations, such as near boundaries or near structural discontinuities. Design of reinforcement in shells for a combined membrane and bending state of stress is a complex problem, and till now, the complete solution for this problem has not been presented in the international codes and does not include any advance in this particular field.

The **ACI-Code** (ACI 318M-05) [1], contains a chapter on shells and folded plates without any clear design algorithm but only mentions that "any method of design which assures sufficient strength with equilibrium is considered applicable".

The **Model Code 90** ("CEB-FIP"1993) [2, 3], suggests the use of a three-layer model, "the plate may be modeled as comprising three layers. The outer layer provides resistance to the in-plane effects of both the bending and the in-plane loading, while the inner layer provides a shear transfer between the outer layers". The proposed model is only approximate as it does not model the different lever arms for concrete and steel forces. In addition, it does not give any procedure to design the element and it only states that an exact determination of the lever arm values for the internal forces "is complex and may require iteration since they depend on the levels of reinforcement and on the thickness of the concrete layers".

The **Euro code 2**("Design" 1991) [2,3], suggests a different method using the usual expressions for plates subjected to in-plane loading and slabs to bending and does not include any provisions for shells. These simplified expressions of general use are not safe and they are inconsistent, as shown by Gupta [4].

A general solution, however, has started to evolve in 1986 by Gupta [4]. Gupta developed an iterative trial-and-error design method using the principle of minimum resistance by dividing the shell into two imaginary concrete layers within each orthogonally placed reinforcing layer. He only considered the case in which reinforcement is needed in both outer layers; thus the method is inappropriate for any other case. With respect to the need of reinforcement four different cases must be analyzed and treated separately: reinforcement needed only in the bottom layer; reinforcement needed only in the top layer; and no reinforcement needed. Also, he showed a few sample design problems on the element level.

In 1993, Lourenco and Figueiras [2,3] presented an automated design of reinforced concrete plates and shells in accordance with the Model Code 90. The authors assumed initial lever arm $d = 0.8h$, referring that an iterative procedure might be adopted to calculate the lever arms, but no additional provisions is given. They implemented the design equations on a computer program, and performed several design examples, comparing the results with optimization module capable of minimizing the sum of the tensile forces and, hence, the required reinforcement. They found that the results changed and the reinforcement decreased by (3.5-6.0 %).

In 2004, Min [5] developed a complete iterative computational algorithm that accurately calculates the internal lever arms to design a plate or a shell element subjected to combined membrane forces and bending moments, in which the shell element is analyzed globally and not as two membrane outer layers in the three-layer model.

The algorithm is developed on the basis of Gupta's derivation (1986). Gupta obtained the design equations partly for the case of reinforcement required in the top and bottom layers, simultaneously. Three more cases are developed for the reinforcement required only in the bottom layer, for reinforcement required only in the top layer, and for no reinforcement required.

Min in his paper presented the complete design algorithm for the two cases: reinforcement required in both top and bottom layers, and reinforcement required in the bottom layer, the other two cases (reinforcement required in the top layer, and no reinforcements required) are derived and presented in the present work in a similar way on the basis of Gupta's and Min's derivation to reach the aim to provide a complete and clear design algorithm for reinforced concrete shells. Finally, the design code **(DRCSH)** is used to design several experimental examples, and to compare the present design algorithm with those for other design teams (i.e., Gupta, Lourenco and Figueiras and Min)

2 Formulation of Design Equations:

A typical shell element subjected simultaneously to membrane forces N_x, N_y, N_{xy} and bending and twisting moments M_x, M_y, M_{xy} per unit length, is shown in Fig. (1). In the ultimate state, the applied forces have to be in equilibrium with the tensile forces in the reinforcement, and the compressive forces in the concrete compression strut have to be parallel with the crack direction. In this limit state, concrete stress in compression is assumed to be distributed uniformly in the depth of Whitney's stress block. The tensile strength of concrete was ignored as in the current design philosophy (ACI 318M-05) [1].

Figure (2) shows a shell element with reinforcement represented as smeared layers. A rigid-plastic behavior is assumed for the reinforcement. It is assumed that the reinforcement consists of two orthogonal layers placed at the top and the bottom surfaces, with appropriate covers, and that each orthogonal reinforcement layer has reinforcing bars in the x-and y-directions, respectively. The capacity of these reinforcements can be designated as N^*_{xt} , N^*_{xb} , N^*_{yt} and N^*_{yb} where subscripts x and y designate the directions, and t and b stand for the top and bottom layers, respectively.

At the limit state, a vertical plane of crack, whose normal makes an angle θ_t and θ_b with the x-axis in the xy-plane, penetrates the top and bottom surfaces, Fig. (3). The concrete is in compression parallel to this crack; it is assumed that the depth of Whitney's stress block is a_t and a_b , respectively.

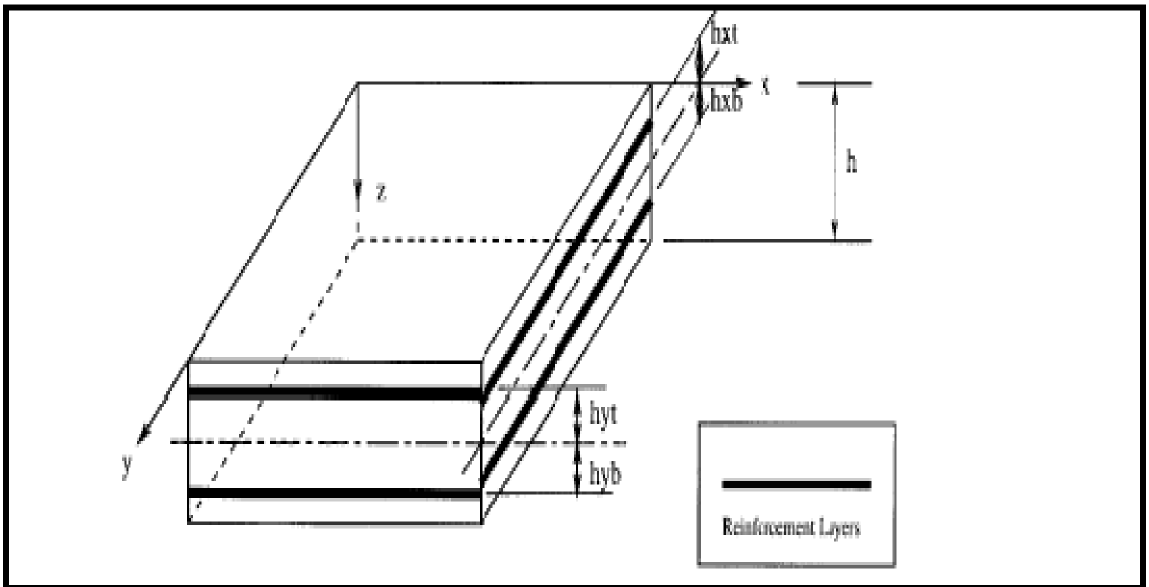

Figure (2) Four reinforcement layers (two for top and two for bottom) [5]

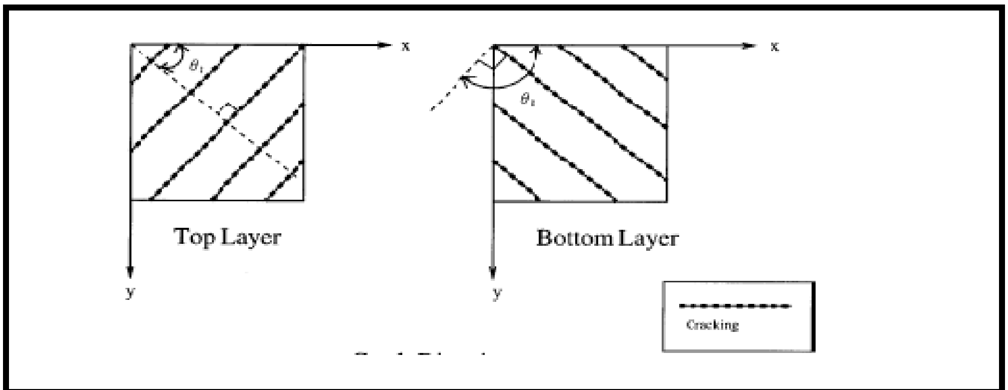

Figure (3) Crack directions [5]

2.1Reinforcement required in top and bottom layers:

The total forces and moments resisted by the reinforcement in the *x*- and *y*-directions are given by

$$N_x^* = N_{xt}^* + N_{xb}^* \qquad N_y^* = N_{yt}^* + N_{yb}^* \tag{1}$$

$$M_x^* = -N_{xt}^* h_{xt} + N_{xb}^* h_{xb} \qquad M_y^* = -N_{yt}^* h_{yt} + N_{yb}^* h_{yb} \tag{2}$$

The average compressive stress parallel to the crack direction in the concrete block is f^c the force and moment resultants of the top and bottom concrete blocks are

$$N_t^c = -a_t f_t^c \quad and \quad M_t^c = -h_t N_t^c \tag{3}$$

$$N_b^c = -a_b f_b^c \quad and \quad M_b^c = -h_b N_b^c \tag{4}$$

where;

h is the total thickness of the shell element

$$h_t = \frac{(h - a_t)}{2}$$

$$h_b = \frac{(h - a_b)}{2}$$

a_t and a_b are the depths of stress blocks

The resisting forces and moments given by Eqs. (1-4) should be in equilibrium with the applied forces and moments. Therefore, the equilibrium equations for a unit cracked element in the *x*- and *y*-directions are

$$N_x = N_x^* + N_t^c \sin^2\theta_t + N_b^c \sin^2\theta_b\ ,$$
$$N_y = N_y^* + N_t^c \cos^2\theta_t + N_b^c \cos^2\theta_b\ and$$
$$N_{xy} = -N_t^c \sin\theta_t \cos\theta_t - N_b^c \sin\theta_b \cos\theta_b \tag{5}$$

$$M_x = M_x^* + M_t^c \sin^2\theta_t + M_b^c \sin^2\theta_b$$
$$M_y = M_y^* + M_t^c \cos^2\theta_t + M_b^c \cos^2\theta_b\ , and$$
$$M_{xy} = -M_t^c \sin\theta_t \cos\theta_t - M_b^c \sin\theta_b \cos\theta_b \tag{6}$$

Therefore, the system of six equations, Eqs. (5) and (6), contain eight unknowns: four reinforcement capacities $N_{xt}^*, N_{xb}^*, N_{yt}^*$ and N_{yb}^*; crack directions θ_t and θ_b; the depths of compressive stress block a_t and a_b. Ideally, these quantities should be selected so that the total capacity of reinforcement is as minimum as possible. As discussed by Gupta [4] and Lourenco and Figueiras [2] the initial values of $\theta_t = \theta_b = \pm \pi/4$ give a satisfactory result with $a_t = a_b$ = 0.2*h*. These values are to be adjusted by an iterative procedure until the equilibrium conditions are satisfied.

From Eqs. (3) to (6), the top and bottom concrete block resultants can be written as

$$-N_t^c = \frac{2(h_b N_{xy} - M_{xy})}{h_c \sin 2\theta_t}\ , and$$
$$-N_b^c = \frac{2(h_b N_{xy} - M_{xy})}{h_c \sin 2\theta_b} \tag{7}$$

where;

$$h_c = h - \frac{(a_t + a_b)}{2}$$

The reinforcement capacities of top and bottom layers in the *x*- and *y*-orthogonal directions are given by Eqs. (1)-(7) as

$$N_{xt}^* = N_{xt} + N_{xyt} C_{xtt} \tan\theta_t + N_{xyb} C_{xtb} \tan\theta_b \tag{8}$$

$$N_{xb}^* = N_{xb} + N_{xyt} C_{xbt} \tan\theta_t + N_{xyb} C_{xbb} \tan\theta_b \tag{9}$$

$$N_{yt}^* = N_{yt} + N_{xyt} C_{ytt} \cot\theta_t + N_{xyb} C_{ytb} \cot\theta_b \tag{10}$$

$$N_{yb}^* = N_{yb} + N_{xyt} C_{ybt} \cot\theta_t + N_{xyb} C_{ybb} \cot\theta_b \tag{11}$$

in which

$$N_{xt} = \frac{h_{xb} N_x - M_x}{h_x}\ ,\quad N_{xb} = \frac{h_{xt} N_x + M_x}{h_x}$$
$$N_{yt} = \frac{h_{yb} N_y - M_y}{h_y}\ ,\quad N_{yb} = \frac{h_{yt} N_y + M_y}{h_y}$$
$$N_{xyt} = \frac{h_b N_{xy} - M_{xy}}{h_c}\quad and\quad N_{xyb} = \frac{h_t N_{xy} + M_{xy}}{h_c} \tag{12}$$

and

$$C_{xtt} = \frac{h_{xb} + h_t}{h_x}\ ,\quad C_{xtb} = \frac{h_{xb} - h_b}{h_x}$$
$$C_{xbt} = \frac{h_{xt} - h_t}{h_x}\ ,\quad C_{xbb} = \frac{h_{xt} + h_b}{h_x}$$
$$C_{ytt} = \frac{h_{yb} + h_t}{h_y}\ ,\quad C_{ytb} = \frac{h_{yb} - h_b}{h_y}$$
$$C_{ybt} = \frac{h_{yt} - h_t}{h_y}\quad and\quad C_{ybb} = \frac{h_{yt} + h_b}{h_y} \tag{13}$$

where,

$$h_x = h_{xt} + h_{xb}$$
$$h_y = h_{yt} + h_{yb}$$

The compressive forces in concrete can be obtained by Eqs. (.7) and (12), and are given by

$$-N_t^c = \frac{2N_{xyt}}{\sin 2\theta_t}\quad and\quad -N_b^c = \frac{2N_{xyb}}{\sin 2\theta_b} \tag{14}$$

When the values of θ_t or θ_b are very small, then the compressive forces in Eq. (14) will be very large and the iterative numerical method will become unstable.

Lourenco and Figueiras [2] used $10^\circ \le (\theta_t, \theta_b) \le 80^\circ$ criterion for the purpose of avoiding numerical instability. Min [5] found that all the elements converge within the maximum range of $-5^\circ \le (\theta_t, \theta_b) \le 5^\circ$. Therefore, he set $(\theta_t, \theta_b) = 0^\circ$ when $|(\theta_t, \theta_b)| \le 5^\circ$ to avoid numerical instability. In the cases of θ_t or θ_b are set to zero, then N_{yt}^* and N_{yb}^* are equal to zero, respectively, Eq. (14) can expressed, as

$$-N_t^c = \frac{M_y - h_{yb} - (h_b - h_{yb})N_b^c \cos^2\theta_b}{h_t + h_{yb}} \quad (15)$$

$$-N_b^c = \frac{M_y - h_{yt} - (h_t - h_{yt})N_t^c \cos^2\theta_t}{-h_b - h_{yt}} \quad (16)$$

When the calculated values of Eqs. (8) to (11) are negative, then no reinforcement is required in that direction. One can set the reinforcement capacity in that direction to zero and recalculate the values of θ_t or θ_b. Min in his study implemented the minimum reinforcement area ($A_{s,\min}$) of ACI 318M-05 [1] for limiting crack width and spacing under the service load condition. In each direction, the minimum capacity $N^*_{\min}$ can be obtained by $N^*_{\min} = A_{s,\min} f_y / 2$, where f_y is the yield stress of reinforcement.

Therefore, in Eq. (8), when $N^*_{xt} \le N_{\min}$, then set $N^*_{xt} = N^*_{\min}$ and calculate a new θ_t value as

$$\theta_t = \frac{\tan^{-1}(N^*_{\min} - N_{xt} - N_{xyb}C_{xtb}\tan\theta_b)}{N_{xyt}C_{xtt}} \quad (17)$$

Similarly, from Eqs. (9)-(11) if N^*_{xb}, N^*_{yt} and N^*_{yb} are smaller than $N^*_{\min}$, then N^*_{xb}, N^*_{yt} and $N^*_{yb} = N^*_{\min}$, respectively, and obtain θ_t or θ_b values accordingly.

2.2Reinforcement required only in top layer:

Reinforcement is required only for the top layers; thus, the total forces and moments resisted by the reinforcement in the *x*- and *y*-directions can be expressed as

$$\begin{aligned} N^*_x &= N^*_{xt} \\ N^*_y &= N^*_{yt} \\ M^*_x &= -N^*_{xt} h_{xt} \\ M^*_y &= -N^*_{yt} h_{yt} \end{aligned} \quad (18)$$

The force and moment resultants of the top concrete block are

$$N_t^c = -a_t f_t^c \quad and \quad M_t^c = -h_t N_t^c \quad (19)$$

where;

h is total thickness of the shell element

$$h_t = \frac{(h - a_t)}{2}$$

The bottom layer concrete forces are N^c_{xb} and N^c_{yb} in the *x*- and *y*-directions respectively, and the shear forces N^c_{xyb}. Then, the equilibrium equations become

$$\begin{aligned} N_x &= N^*_x + N^c_{xb} + N^c_t \sin^2\theta_t \ , \\ N_y &= N^*_y + N^c_{yb} + N^c_t \cos^2\theta_t \ and \\ N_{xy} &= -N^c_t \sin\theta_t \cos\theta_t - N^c_b \sin\theta_b \cos\theta_b \end{aligned} \quad (20)$$

$$\begin{aligned} M_x &= M^*_x + M^c_{xb} + M^c_t \sin^2\theta_t \ , \\ M_y &= M^*_y + M^c_{yb} + M^c_t \cos^2\theta_t \ and \\ M_{xy} &= M^c_{xyb} - M^c_t \sin\theta_t \cos\theta_t \end{aligned} \quad (21)$$

in which

$$\begin{aligned} M^c_{xb} &= h_b N^c_{xb} \ , \\ M^c_{yb} &= h_b N^c_{yb} \ and \\ M^c_{xyb} &= -h_t N^c_{xyb} \end{aligned} \quad (22)$$

Therefore, the system of six equations, Eqs. (20) and (21) contains eight unknowns: two reinforcement capacities N^*_{xt} and N^*_{yt} , crack direction θ_t , depths of compressive stress blocks a_t and a_b , and bottom layer concrete forces N^c_{xb}, N^c_{yb} and N^c_{xyb} .From Eqs. (19)–(21), the top concrete block resultant can be written as,

$$-N_t^c = \frac{2(h_b N_{xy} - M_{xy})}{h_c \sin 2\theta_t} \quad (23)$$

The principal force of the bottom layer and the depth of the compressive stress block can be expressed, respectively, as

$$N^c_{b1,b2} = \frac{N^c_{xb} + N^c_{yb}}{2} \pm \sqrt{\left(\frac{N^c_{xb} - N^c_{yb}}{2}\right)^2 + \left(N^c_{xyb}\right)^2} \qquad (24)$$

$$a_b = -\frac{N^c_{b2}}{f^c_b}$$

As before, the minimum reinforcement area ($A_{s,\min}$) of ACI 318M-05 [1] was implemented. Therefore, with the minimum reinforcement capacity $N^*_{\min}$ in the bottom layer, and using Eqs. (18)–(23), the tensile steel forces developed at the top layer can be given as,

$$N^*_{xt} = N_{xxt} + N_{xyt1}\tan\theta_t + D_{xt}N^*_{\min} \ \ and \qquad (25)$$

$$N^*_{yt} = N_{yyt} + N_{xyt2}\cot\theta_t + D_{yt}N^*_{\min}$$

in which

$$N_{xxt} = \frac{h_b N_x - M_x}{h_b + h_{xt}} \ , \quad N_{yyt} = \frac{h_b N_y - M_y}{h_b + h_{yt}} \qquad (26)$$

$$N_{xyt1} = \frac{h_b N_{xy} - M_{xy}}{h_b + h_{xt}} \ , \quad N_{xyt2} = \frac{h_b N_{xy} - M_{xy}}{h_b + h_{yt}}$$

$$D_{xt} = \frac{-h_b + h_{xb}}{h_b + h_{xt}} \quad and \ D_{yt} = \frac{-h_b + h_{yb}}{h_b + h_{yt}}$$

Also setting $\theta_t = 0^\circ$ when $|\theta_t| \le 5^\circ$ to avoid numerical instability, then $N^*_{yt} = 0$, and N^c_t is expressed as

$$-N^c_t = \frac{M_y - h_b N_y}{h_c} \qquad (27)$$

From Eq. (25), if N^*_{xt} is smaller than $N^*_{\min}$, then $N^*_{xt} = N^*_{\min}$, and θ_t is calculated as,

$$\theta_t = \tan^{-1}\left(\frac{\left[-N_{xxt} + (1 - D_{xt})N^*_{\min}\right]}{N_{xyt1}}\right) \qquad (28)$$

Similarly, when N^*_{yt} is smaller than $N^*_{\min}$, then $N^*_{yt} = N^*_{\min}$, and θ_t is calculated as,

$$\theta_t = \tan^{-1}\left(\frac{N_{xyt2}}{\left[-N_{yyt} + (1 - D_{yt})N^*_{\min}\right]}\right) \qquad (29)$$

The design equations for the reinforcement required only in the bottom and no reinforcement required are omitted here, and the complete design algorithms for the four cases are presented in details in Ref. [6]

3 Verification Examples

The design algorithms are implemented in design code **DRCSH** (standing for: **D**esign of **R**einforcement in **C**oncrete **SH**ell) developed in the present work, and to verify the design code on the element level several "design and experimental" examples are designed and compared with those of other design teams as shown in the next sections.

Design examples:

- ***Gupta's design problem*:**

Gupta [4] showed a design example problem in the case of reinforcement required in the top and bottom layers, simultaneously. The design variables of Gupta's example (which are the only input data required for **DRCSH**) are given in Table (1).

Table (1) Design variables of Gupta's example.
N_x = -350.16 kN/ m (-2000 lb/ in.); N_y = 297.636 kN/ m. (1700 lb/ in.); N_{xy} = 175.08 kN/ m. (1000 lb/ in.)
M_x = -60.048 kN.m/ m (-13500 lb-in/in.); M_y = 12.0096 kN.m/ m (2700 lb-in/in.); M_{xy} = 0.8896 kN.m/ m (200 lb-in/in.)
f^c = 6.895 MPa(1000psi) ; f_y = 41.37 MPa (60000 psi) ; h = 0.254m (10 in)
Initially assume $a_t = a_b$ = 0.2h = 0.0508m., $\theta_t = \theta_b = 45^\circ$ and $h_{xt} = h_{yt} = h_{xb} = h_{yb}$ = 0.1016m.

As shown in Table (2) the presented design code produce approximately the same total reinforcement capacity of Gupta's result for minimum reinforcement ratio ($N^*_{\min}$ =0.0), with very small minimum reinforcement ratio, the total reinforcement capacity is increased only by 2.419% with respect to Gupta's results.

Table (2) Comparison of the designs with and without setting minimum reinforcement ratios and the Gupta's result

	Gupta's Result	**Present Study**	
		$N^*_{\min}$ **=0.0**	$N^*_{\min}$ **=13.131 kN/m**
No. of iterations		6	6
a_t (m)	0.0254(1.0in)	0.021	0.0204
a_b (m)	0.0762(3.0in)	0.077	0.0792
θ_t (°)	45.00	45.00	45.00
θ_b (°)	45.00	78.45	78.76
N^*_{xt} (kN/m)	229.179(1309lb/ in)	229.447	233.049
N^*_{yt} (kN/m)	167.90(959lb/ in)	167.591	167.024
N^*_{xb} (kN/m)	0.0	0.0	13.131
N^*_{yb} (kN/m)	223.051(1274lb/ in)	222.492	221.959
Sum of Tensile Forces	620.13(3542lb/ in)	619.530	635.163

- ***Lourenco and Figueiras's design problem:***

Lourenco and Figueiras [2] showed two design examples, one for a case of the reinforcement required in top and bottom layers, simultaneously, the other one for a case of reinforcement required only in the top layer. Then, the resulting reinforcements in this work have been compared with those obtained by Lourenco and Figueiras [2] and with their optimization module [2] capable of minimizing the sum of tensile forces and, hence, the required reinforcement.

The design variables of the first design case (two tensile layers) are given in Table (3),

Table (3) Design variables of Lourenco and Figueiras's first design problem

N_x = -200 kN/ m.; N_y =300kN/ m.; N_{xy} =75 kN/ m.
M_x =-60 kN m/ m. ; M_y = 40 kN m/ m.; M_{xy} =-200 kN m/ m.
f^c =7.34 MPa ; f_y = 348 MPa ; h = 0.2 m
Initially assume a_t = a_b =0.2h = 0.04 m., $\theta_t = \theta_b$ = 45° and $h_{xt} = h_{yt} = h_{xb} = h_{yb}$ = 0.08m.

A comparison of the design for this case by Lourenco and Figueiras [2] and their optimization module with the present design code **DRCSH** is given in Table (4) and shows that the present design code provides tensile forces in the reinforcement less than those obtained by Lourenco and Figueiras, and very close to the reinforcement forces provided by the optimization module.

Table (4) Comparison with the designs results of Lourenco and Figueiras's first design problem

	Lourenco and Figueiras's		**Present study**
	Design results	**Optimization module**	
No. of iterations		-	7
a_t (m)	0.0495	-	0.490
a_b (m)	0.0816	-	0.075
θ_t (°)	45.00	45.6	45.00
θ_b (°)	-79.6	-78.9	-78.89
N^*_{xt} (kN/m)	526.8	509.0	505.573
N^*_{yt} (kN/m)	79.0	72.4	75.862
N^*_{xb} (kN/m)	34.7	0.0	0.0
N^*_{yb} (kN/m)	422.5	422.8	422.915

The design variables of the second design case(compression in top layer) are given in Table(5),

Table (5) Design variables of Lourenco and Figueiras's second design problem
N_x = -200 kN/ m.; N_y =300kN/ m.; N_{xy} =75 kN/ m.
M_x = 60 kN m/ m. ; M_y = 40 kN m/ m.; M_{xy} =-20 kN m/ m.
f^c =7.34 MPa ; f_y = 348 MPa ; h = 0.2 m
Initially assume $a_t = a_b$ =0.2h = 0.04 m., $\theta_t = \theta_b = 45^\circ$ and $h_{xt} = h_{yt} = h_{xb} = h_{yb} =$ 0.08m.

A comparison of the design for this case by Lourenco and Figueiras [2] and their optimization module with the present design code **DRCSH** is given in Table (6), which shows that the present design algorithm provides tensile forces in the reinforcement which are in agreement with those obtained by Lourenco and Figueiras and the optimization module

Table (6) Comparison of the designs results of Lourenco and Figueiras's second design problem

	Lourenco and Figueiras's		Present study
	Design results	Optimization module	
No. of iterations		-	5
a_t (m)	0.0474	-	0.048
a_b (m)	0.0236	-	0.015
θ_t (°)	-	-	71.11
θ_b (°)	-	-44.6	-45.0
N^*_{xt} (kN/m)	0.0	0.0	0.0
N^*_{yt} (kN/m)	0.0	0.0	0.0
N^*_{xb} (kN/m)	377.6	377.1	378.787
N^*_{yb} (kN/m)	493.7	494.2	494.206

Experimental examples:

Several experimental examples are designed and performed with nonlinear inelastic analysis [6] to show the adequacy of the design equations. If the calculated ultimate strength is larger than the ultimate strength obtained from the test, then the design method can be considered satisfactory, The experimental examples are: (1)Marti et al.'s [7] slab elements ML 7 and ML9 subjected to torsional moments, (2) Polak and Vecchio's [8] shell elements SM1, SM2 and SM3 models.

Table (7) shows a comparison of steel ratios between the original tested specimen, given by the Lourenco-Figueiras's design [3], given by Min's design [5] and those obtained by the present design code **DRCSH**. As shown in Table (7), the present design code provides reinforcement capacity that are almost identical to those obtained by Min [5].

Table (8) shows a comparison of ultimate strength obtained from the test and that calculated by the nonlinear inelastic analysis [6]. The designed elements give calculated ultimate strengths from 7 to 18% higher than test results values, except SM2 model because the nonlinear analysis failed to find a convergence due to a very large deformation of the mode and the behavior of it is somewhat similar to normal steel structure behavior, which confirms the adequacy of the design algorithm, and the developed design code.

Table (7) Comparison of steel ratios from original tested specimens, from design teams and from present design code (%)

Models	Design teams	Top layer (Steel ratio)		Bottom lay
		x-dir.	y-dir.	x-dir.
ML 7	Marti et al [7]	0.25	0.25	0.25
	Lourenco and Figueiras[3]	0.25	0.25	0.25
	Min [5]	0.26	0.26	0.26
	Present study	0.25	0.25	0.25
ML 9	Marti et al [7]	1.00	1.00	1.00
	Lourenco and Figueiras[3]	1.21	1.21	1.21
	Min [5]	0.96	0.96	0.96
	Present study	0.95	0.95	0.95
SM 1	Polak and Vecchio [8]	1.25	0.42	1.25
	Lourenco and Figueiras[3]	0.0	0.0	1.43
	Min [5]	0.01	0.01	1.59
	Present study	0.01	0.01	1.55
SM 2	Polak and Vecchio [8]	1.25	0.42	1.25
	Lourenco and Figueiras[3]	0.0	0.0	1.84
	Min [5]	0.01	0.01	1.86
	Present study	0.01	0.01	1.83
SM 3	Polak and Vecchio [8]	1.25	0.42	1.25
	Lourenco and Figueiras[3]	0.0	0.0	1.42
	Min [5]	0.01	0.01	1.55
	Present study	0.01	0.01	1.42

Table (8) Comparison of ultimate strength obtained from the test and calculated by nonlinear inelastic analysis			
Models	**Ultimate Moment Obtained from the test (kNm/m)**	**Calculated Ultimate Moment [6](kNm/m)**	**Ratio % [(2)*(1)]/100**
ML 7	42.5	45.208	107
ML 9	101.5	112.839	111
SM 1	477	652.86	118
SM 2	421	383.11	91
SM 3	488	546.56	112

4 Conclusions

In this paper, a complete design algorithm is achieved by developing a design code (**DRCSH)** based on a complete iterative computational algorithm. This program can be used as a stand-alone version, to determine the load carrying capacity of critical points in reinforced concrete panels, plates and shells; and to verify the design code on the element level. Five experimental models are designed. The designed elements give calculated ultimate strengths from 7 to 18% higher than those of test results values, except one model, which confirms the adequacy of the design algorithm, and the developed design code.

5 References

1) ACI Committee 318M, "Building code requirements for structural concrete", ACI standard 318M-05; and the commentary on building code requirements for reinforced concrete, American Concrete Institute, Detroit, 2005.
2) Lourenco, P.B. and Figueiras, J.A., "Automatic design of reinforcement in concrete plates and shells", Engineering Computation, Vol.10, 1993, pp. 519-541.
3) Lourenco, P.B. and Figueiras, J.A., "Solution for design of reinforcement concrete plates and shells", Journal of Structural Engineering, ASCE, Vol.121, No.5, May 1995, pp. 815-823.
4) Gupta, A.K., "Combined membrane and flexural reinforcement in plates and shells", Journal of Structural Engineering, ASCE, Vol.112, No.3, March 1986, pp. 550-557.
5) Min, C.S., "Design and ultimate behavior of RC plates and shells", Nuclear Engineering and Design, Vol.228, 2004, pp. 207-223.
6) Hussain, N.M.A., "Analysis and Optimum Reinforcement Design of Concrete Plates and Shells", PhD Thesis. Nahrain University, Iraq, 2005, 185p.
7) Marti, P., Leesti, P. and Khalifa, W.U., "Torsion tests on reinforced concrete slab elements", Journal of Structural Engineering, ASCE, Vol.113, No.5, May 1987, pp. 994-1010.
8) Polak, M.A., and Vecchio, F.J., "Nonlinear analysis of reinforced-concrete shells", Journal of Structural Engineering, ASCE, Vol.119, No.12, December 1993, pp. 3439-3461.

أسلوب تصميم حديد ألتسليح للمنشأت ألخرسانية ألقشرية

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ألخلاصة:

أن غياب الحلولِ المقبولةِ عالمياً في ألمدونات الخرسانيةِ ألأنشائية لتصميم حديد ألتسليح في المنشأت ألقشرية تُسبّبُ بعض ألصعوبات لحِساب التسليح المطلوبِ في هذه المنشأت. التطوير المستمر في أداءِ الحاسوبَ وقابلية الخزن المتزامنة مع الطرق العدديةِ الكفوءة أدت ألى ظهور ألحاجة ألى أكتشاف طرق لتصميم حديد ألتسليح للمنشأت ألقشرية ألمعرضة ألى أحمال مختلفة.

في هذا ألبحث، تم تقديم طريقة حسابية متتابعة كاملة لتصميم حديد التسليح للعناصر ألقشرية ألمعرضة لأحمال غشائية و عزوم.

في معادلاتِ التصميمَ، يساهم حديد ألتسليح في تحمل قوى ألشد ألمتولدة أما ألخرسانة ألموازية لأتجاه ألتشققات فأنها تتحمل قوى ألانضغاط ، تم افتراض حديد ألتسليح بشكل طبقتين متعامدتين و ضعتا في ألسطحين ألعلوي والسفلي مع غطاء حرساني مناسب.كل طبقة من حديد ألتسليح تتكون من قضبان حديد تسليح متعامدة. تم أعتبار كل طبقة من حديد التسليح عمودية على الطبقة الأخرى.كما افترض ان الأجهاد لأنضغاط عمود ألخرسانة موزعاً بأنتظام خلال عمق منطقة أجهاد (Whitney)

تم تنفيذ طريقة التصميم بكتابة برنامج تصميمي (DRCSH) اعتمادا على طريقة حسابية متتابعة، هذا البرنامج يمكن استخدامه لا يجاد قوة ألتحمل في المناطق الحرجة للألواح وألقشريات ألخرسانية ألمسلحة. و للتأكد من صحة برنامج التصميم تمت عملية تصميم لخمسة نماذج سبق أن فحصت عملياً. أظهرت نتائج التحليل أحمال فشل أعلى بمقدار ٧-١٨% من أحمال الفشل العملية، عدا حالة واحدة. مما يؤكد يمثل صلاحية ألطريقة و برنامج التصميم ألمعد.