Adaptive Filtering Method using of Non- Gaussian Moving Average model from First order (simulation study)

Mr. Wadhah S. Ibraheem
Assistant Lecturer
College of Economy & Administration
The University of Mustansiriya
Hellw_274@yahoo.com

Abstract

The interest of the time series focuses on the relations that linking the variables phenomenon and time, and to build a model suitable for time series requires the attention of parametric phenomenon, and Adaptive filtering method is one of the most important ways to build time series, In this paper, we used the Adaptive filtering method of non-Gaussian moving average model from first order MA (1) for several distributions of intermittent and continuous for a number of different volumes of samples and using simulation techniques.

The values of estimators $\hat{\vartheta}_i$ by Iterative process and ϑ_i^* by Adaptive filtering method increases as the size of the sample increased, The values MSE and MAPE to MA(1) model decrease as the sample size is increased, and for all kinds of discrete and continuous distributions.

Keyword: Time series, Moving Average model**,** Iterative process, Adaptive filtering

1- Introduction

Data analysis is important because it is the main source of information, and provides verification for the health of the theories and models as well as improvements in their own, That any kind of time-series data, whether from experience or from the dynamic system or any aspect of the economic, social or biological, Usually include white noise, and often results in analysis of these data in the presence of this white noise to an erroneous interpretation of the data, So we need to reduce the white noise of data by using the Adaptive filtering method To make the data compatible for further analysis and can therefore be important to maintain objectivity in the time series with high efficiency.

In 1966 (Holff & Widro) published the first research the Adaptive filtering method, which reflect the theoretical basis upon which the method and applications depend on (Box, Jenkins & Reinsel 1994, Brockwell & Davis 1996)[2][3].

In paper 2003, a technique based on the estimation of the gradient of the mean square error is suggested in the realm of adaptive filtering formulation. Simulation results confirmed that the method could be used as an efficient adaptive filtering algorithm (Alhabsi, Iqbal & Al-Rizzo 2003)[1].

2- Moving Average Model from first order MA(1)

By using Back shift operator B in the following formula:

$$Z_t = \vartheta(B)a_t \qquad ....(2.1)$$

$$\because \vartheta(B)a_t = (1-\vartheta_i B)a_t$$

So, the general formula of the Moving Average Model from first order MA(1) can be written as follows[(5)(6)]:

$$z_t = a_t - \vartheta_1 a_{t-1} \qquad ...(2.2)$$

Where:

z_{t-i} $,i = 0,1$
$,t = 1,2,.....,n$ values of time series observations

ϑ_1 Moving Average Parameter

a_{t-1} $,i = 0,1$ random error

2-1 The Theoretical aspects of the MA(1):

a- Inevitibility:

To achieve inevitibility, root of the equation $\vartheta(B) = 1 - \vartheta_1 B = 0$ must be outside the Unit Circle[(2)(3)], where: $|B| > 1$

This leads to make the parameter:

$$|\vartheta_1| = \frac{1}{|B|} \qquad .....(2.1.1)$$

$$\therefore |\vartheta_1| < 1$$

To make the model inevitible it must be:

$$-1 < \vartheta_1 < 1 \qquad ....(2.1.2)$$

b- Auto Covariance[(5)]:

$$Z_t = (1 - \vartheta_i B) a_t$$

$$Z_t = a_t - \vartheta_i B\, a_t$$

$$E(Z_t Z_{t-k}) = E(a_t - \vartheta_i a_{t-1})(a_{t-k} - \vartheta_i a_{t-k-1})$$

$$\gamma(B) = \sigma_a^2 (1 - \vartheta_i B^1)(1 - \vartheta_i B^{-1})$$

$$= \sigma_a^2 (-\vartheta_1 B^{-1} + (1 + \vartheta_1^2) - \vartheta_1 B^1)$$

The general formula of the Auto Covariance of the MA(1) can be written as follows:

$$\gamma_k = \begin{cases} (1+\vartheta_1^2)\sigma_a^2 & ,k = 0 \\ -\vartheta_1 \sigma_a^2 & ,k = 1 \\ 0 & ,k \ge 2 \end{cases} \qquad ....(2.1.3)$$

c- Cross Covariance:

The general formula of the Cross Covariance between Time Series x_t and random error a_t of the MA(1) can be written as follows[(6)]:

$$\gamma_{ax}(k) = \begin{cases} \sigma_a^2 & ,k = 0 \\ \neq 0 & ,k < 0 \\ 0 & ,k > 0 \end{cases} \qquad ....(2.1.4)$$

d- Auto Correlation (ACF):
The general formula of the Auto Correlation of the MA(1) can be written as follows[6]:

$$\rho_k = \begin{cases} 1 & ,k=0 \\ -\vartheta_1/(1+\vartheta_1^2) & ,k=1 \\ 0 & ,k\geq 2 \end{cases} \quad ....(2.1.5)$$

e- partial Auto correlation (PACF):
The general formula of the Partial Auto Correlation of the MA(1) can be written as follows[5]:

$$\phi_{KK} = \frac{-\vartheta_1^K(1-\vartheta_1^2)}{1-\vartheta_1^{2(K+1)}} \quad K\geq 1 \quad ....(2.1.6)$$

2-2 Estimation of the Parameter:

The model[5]:

$$a_t = \vartheta^{-1}(B)z_t$$

We notice that the above model is non-linear in its parameter:
To estimate parameter ϑ_1 Moving Average Model first order according to the method Iterative process will be[9]:

$$\because \gamma_0 = (1+\vartheta_1^2)\sigma_a^2$$

$$\gamma_1 = -\vartheta_1\sigma_a^2$$

So we estimate σ_a^2 according the following formula[12]:

$$\hat{\sigma}_a^2 = \frac{c_0}{(1+\vartheta_1^2)} \quad ....(2.2.1)$$

After we substitute in the formula (2.2.1) the Initial value Zero for the parameter ϑ_1.
So we make :

$$\hat{\sigma}_a^2 = c_0$$

Then:
We estimate the parameter ϑ_1 as the following formula:

$$\hat{\vartheta}_1 = -\left[\frac{c_1}{\hat{\sigma}_a^2}\right] \quad ....(2.2.2)$$

So that:

$$\left.\begin{aligned} C_0 &= \hat{\gamma}_0 \quad , \quad C_1 = \hat{\gamma}_1 \\ C_0 &= \frac{1}{n}\sum_{t=1}^{n} z_t^2 \\ C_1 &= \frac{1}{n}\sum_{t=1}^{n-1} z_t z_{t-1} \end{aligned}\right\} \quad ....(2.2.3)$$

We continue the replication process till we reach the stationary stage for the value of the parameter ϑ_1.

3- Adaptive Filtering Approach:

This method is suggested by Wheel Wright & Makridakis and Can estimate parameter model under stady by using the Steepest Descent method, That means to reach a point on the curved surface of the derived so the value of MSE at that point must be less than what can be done through moving towards the bottom of the downward curve to depend on the values of the new features, According to the base created by Widrow[(10)(11)(13)]:

$$\vartheta_i^* = \vartheta_{i-1} - K\overline{\nabla} a_t^2 \quad \text{............}(3.1)$$

Where:

ϑ_{i-1} Moving Average Parameter and prior for the parameter (i) in (2.1)

ϑ_i^* Now Parameter.

K Fixed amount of information depends on the values of views in the case of seasonal time series we impose ($1/S$).

$\overline{\nabla} a_t^2$ The gradient which can be approximated derivative.

And equation (1) we get:

$$a_t = Z_t + \vartheta_i a_{t-1}$$

$$a_t^2 = (Z_t + \vartheta_i a_{t-1})^2$$

$$\overline{\nabla} a_t^2 = \frac{\partial a_t^2}{\partial \vartheta_i} = 2(Z_t + \vartheta_i a_{t-1})(a_{t-1}) = 2a_t a_{t-1} \quad \text{....}(3.2)$$

$$\because \ \vartheta_i^* = \vartheta_{i-1} - K\overline{\nabla} a_t^2$$

$$\therefore \ \vartheta_i^* = \vartheta_{i-1} - 2Ka_t a_{t-1} \quad \text{........}(3.3)$$

many papers has published that included the views and different directions to the selection of the value of K related to the estimated values as soon as possible, and to determine the value of the constant K:

$$a_t = Z_t + \vartheta_i a_{t-1}$$

$$a_t^* = Z_t + \vartheta_i^* a_{t-1}$$

$$|\Delta a_t| = |a_t^* - a_t| = (\vartheta_i^* - \vartheta_i) a_{t-1}$$

$$|\Delta a_t| = 2Ka_t a_{t-1}^2 \quad \text{.......}(3.4)$$

$$0 < \frac{|\Delta a_t|}{a_t} < 1$$

$$0 < \frac{2Ka_t a_{t-1}^2}{a_t} < 1$$

$$0 < 2Ka_{t-1}^2 < 1$$

$$0 < K < \frac{1}{2(a_{t-1}^2)_{\max}} \qquad ....(3.5)$$

4- Simulation study

To investigate whether the expression for time varying coefficient leads to improvement in estimation accuracy, simulation study is preformed, where four experiments were carried out to investigate the properties of Iterative process estimators for various versions of first order Moving Average Schemes. The experiments are carried out for N=5000 samples and sample size (25, 75, 100, 150, 200, 300). And white noise to a specimen divided MA(1) spotty distribution (binomial, Poisson) or continuous distribution (exponential, Laplace), and then find the estimate parameters by the Adaptive filtering method to MA(1) Note that the value of (K) was between (zero , $\frac{1}{2(a_{t-1}^2)_{\max}}$) and to repeat 5000 times and sample sizes (25, 75, 100, 150, 200, 300). and discrete and continuous distributions[(4)(7)].

least squares (MSE)) calculated are in accordance with the following formula:

$$MSE(\hat{\vartheta}_i) = \frac{\sum_{i=1}^{k}(\hat{\vartheta}_i - \overline{\hat{\vartheta}}_k)^2}{k} \qquad ....(4.1)$$

it was also calculating the Mean Absolute Percentage Error (MAPE) as this measure is best used in the case of unequal samples, according to the following formula:

$$MAPE(\hat{\vartheta}_t) = \frac{1}{k}\sum_{i=1}^{k}\left|\frac{\hat{\vartheta}_i - \overline{\hat{\vartheta}}_k}{\hat{\vartheta}_i}\right| * 100\% \qquad ....(4.2)$$

Our findings are reported in table (4.1) the simulations performed have been quite extensive.

Table (1.1)

Distribution	Formula
Binomial (n,p)	$a_t = \begin{cases} 1 & if \quad 0 < u \le p \\ 0 & if \quad p < u \le 1 \end{cases}$
Poisson (λ)	$a_t = \begin{cases} 1,2,...... & if \quad -\ln\prod_{i=1}^{n} u_i < \lambda \le -\ln\prod_{i=1}^{n+1} u_i \\ 0 & if \quad -\ln u > \lambda \end{cases}$
Exponential (α, β)	$a_t = -\beta \ \ LOG(1-u)$
Laplace (α, β)	$a_t = \alpha - \beta \ln[2(1-u)]$

The simulation results as in the following tables:

Table (2.1)

n	Binomial p=0.5		Poisson $\lambda = o.5$		Exponential $\alpha = 1, \beta = 2$		Laplace $\alpha = 1, \beta = 2$	
	$\hat{\vartheta}_i$	ϑ_i^*	$\hat{\vartheta}_i$	ϑ_i^*	$\hat{\vartheta}_i$	ϑ_i^*	$\hat{\vartheta}_i$	ϑ_i^*
25	-.7849417	-.8333399	-.6828834	-.8342913	-.8047646	-.9596192	-.7425172	-.9163824
50	-.8085155	-.9618654	-.7241478	-.8929185	-.8177929	-.9804666	-.7598951	-.9638194
100	-.8186873	-.9641085	-.7503236	-.9348978	-.8237074	-.9815447	-.7670362	-.9745979
150	-.8223535	-.9656436	-.7574279	-.9517281	-.8255934	-.9818314	-.7687125	-.975592
200	-.8240168	-.9671552	-.7617986	-.9598299	-.8266244	-.9827514	-.7702869	-.9764361
300	-.8260347	-.9698722	-.7653856	-.9699967	-.8275818	-.9826467	-.7714176	-.9774883

Table (3.1)

n	Binomial p=0.5		Poisson $\lambda = o.5$		Exponential $\alpha = 1, \beta = 2$		Laplace $\alpha = 1, \beta = 2$	
	MSE		MSE		MSE		MSE	
	$\hat{\vartheta}_i$	ϑ_i^*	$\hat{\vartheta}_i$	ϑ_i^*	$\hat{\vartheta}_i$	ϑ_i^*	$\hat{\vartheta}_i$	ϑ_i^*
25	.00558887	.005065328	.01643347	.0173513	.003778168	.002651705	.00690092	.006636016
50	.00237326	.000582993	.00776961	.00716413	.00184672	.000597554	.00311827	.00200415
100	.00102761	.000450204	.00308742	.00215086	.000915331	.000563582	.00154298	.001017786
150	.00064119	.000421198	.00193225	.00107780	.000614119	.000567667	.00102994	.000982803
200	.00046853	.000380189	.00147512	.00070504	.000465385	.000498462	.00078452	.00090621
300	.00031355	.000320852	.00096727	.00036069	.000291524	.00048919	.00048042	.000833491

Table (4.1)

n	Binomial p=0.5		Poisson $\lambda = o.5$		Exponential $\alpha = 1, \beta = 2$		Laplace $\alpha = 1, \beta = 2$	
	MAPE		MAPE		MAPE		MAPE	
	$\hat{\vartheta}_i$	ϑ_i^*	$\hat{\vartheta}_i$	ϑ_i^*	$\hat{\vartheta}_i$	ϑ_i^*	$\hat{\vartheta}_i$	ϑ_i^*
25	.07848245	.06761326	.213032	.1681963	.06150698	.03999512	.09324228	.07436571
50	.04849187	.02050119	.1149945	.0853316	.04183441	.01769673	.05941926	.03382392
100	.03131711	.01909548	.05998142	.0391982	.02948489	.01677355	.04122425	.02316518
150	.02457167	.01811348	.04685086	.0278999	.02390833	.01679955	.03368908	.0228691
200	.02099386	.01694436	.04030912	.0223421	.02084315	.01575669	.0291567	.02170745
300	.01712085	.01499695	.032604	.0161553	.01645027	.01597022	.02280326	.02082366

5- Conclusion:

1- All the parameter values estimated $\hat{\vartheta}_i$ by Iterative process was negative for all sizes of samples, as well as for all kinds of discrete and continuous distributions.

2- All the parameter values estimated ϑ_i^* by a modified purification was negative and all the volumes of samples and also for all kinds of discrete and continuous distributions.

3- The values of estimators $\hat{\vartheta}_i$ and ϑ_i^* increases as the size of the sample increased , and for all kinds of discrete and continuous distributions.

4- The values of MSE to model MA(1) and the values of estimators $\hat{\vartheta}_i$ and ϑ_i^* are decrease when the size of the sample in increased, and for all kinds of discrete and continuous distributions.

5- The values of MSE to model MA(1) and parameter ϑ_i^* Estimated by the Adaptive Filtering are smaller than The values of MSE to model MA(1) and parameter ϑ_i^* Estimated by the Iterative process.

6- The values of MAPE to model MA(1) and the values of estimators $\hat{\vartheta}_i$ and ϑ_i^* decrease when the size of the sample in increased, and for all kinds of discrete and continuous distributions.

7- The values of MAPE to model MA(1) and parameter ϑ_i^* Estimated by Adaptive Filtering are smaller than The values of MAPE to model MA(1) and parameter ϑ_i^* Estimated by the Iterative process.

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استخدام أسلوب التنقية المعدلة لنموذج الأوساط المتحركة غير الطبيعي من الدرجة الأولى (دراسة محاكاة)

المدرس المساعد
وضاح صبري إبراهيم
كلية الإدارة والاقتصاد\الجامعة المستنصرية
Hellw_274@yahoo.com

المستخلص : Abstract

يتركز اهتمام السلاسل الزمنية بالعلاقة الدالية التي تربط بين متغيرين الظاهرة والزمن، ولبناء إنموذج ملائم للسلسلة الزمنية يتطلب اهتمام بمعلمة الظاهرة، وان طريقة التنقية المعدلة من اهم الطرائق لبناء السلاسل الزمنية وفي هذا البحث استخدمنا أسلوب التنقية المعدلة لنموذج الأوساط المتحركة غير الطبيعي من الدرجة الأولىMA(1) ولعدة توزيعات متقطعه ومستمرة ولعدد مختلف من حجوم العينات وباستخدام اسلوب المحاكاة.

ان قيم المقدرات $\hat{\vartheta}_i$بطريقة Iterative process و ϑ_i^* بطريقة التنقية المعدلة تزداد كلما ازدادت حجم العينة و ان قيم MSE و MAPE لأنموذج MA(1) تصغر كلما ازدادت حجم العينة و لجميع انواع التوزيعات المتقطعة والمستمرة.