حساب المقاطع العرضية لتفاعل $^{22}\text{Na}(n,\alpha)^{19}\text{F}$ لتفاعل $^{19}(\alpha,n)^{22}\text{Na}$ باستخدام نظرية التفاعلات في المستوي الأرضي.

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الخلاصة

في هذه الدراسة، اُجريت حسابات المقاطع العرضية لتفاعلات النووي (التاكونتريوم) و (بوترون، الماء) للنواة الحبيبة $^{22}\text{Na}$، $^{19}\text{F}$ (حيثما نوافّر الديانات والمذاب الطائفي من طاقة المرشدة إلى $10\text{MeV}$) لجسيمات الدما والبيوترنات. وُجدت المقاطع العرضية الأكثر حداثة لتفاعلات (التاكونتريوم) و (بوترون، الماء) قد استُدعت $^{19}\text{F}(\alpha,n)^{22}\text{Na}$ بخطوات طاقية ($86.4\text{KeV}$) لتفاعل $^{22}\text{Na}$ في السطح. وكذلك حسبت المقاطع العرضية (التاكونتريوم) من القطاع العرضية (التاكونتريوم) المذاب في الديانات والمذاب الطائفي واعتقدت السطح بالخطوات الطائفة نفسها باستخدام مبدأ التفاعلات المعاكس.

$^{19}\text{F}(\alpha,n)^{22}\text{Na}$ تتضمن هذه الحسابات فقط المستوى الأرضي للنواة $^{22}\text{Na}$، $^{19}\text{F}$ في التفاعلات $^{22}\text{Na}(n,\alpha)^{19}\text{F}$. 

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Determining of Cross Sections for $^{22}$Na (n,α) $^{19}$F reaction from Cross Sections of $^{19}$F(α,n) $^{22}$Na reaction using the reciprocity theory for the ground state

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Abstract

In this study, light elements $^{19}$F, $^{22}$Na for (α,n) and (n,α) reactions as well as α-particle energy from a threshold energy to 10 MeV are used according to the available data of reaction cross sections. The more recent cross sections data of (α,n) and (n,α) reactions are reproduced in fine steps 86.4 KeV for $^{22}$Na (n,α) $^{19}$F in the specified energy range, as well as cross section (α,n) values were derived from the published data of (n,α) as a function of α-energy in the same fine energy steps by using the principle inverse reactions. This calculation involves only the ground state of $^{19}$F, $^{22}$Na in the reactions $^{19}$F (α,n) $^{22}$Na, $^{22}$Na (n,α) $^{19}$F.

Introduction

The interaction of particles with matter is described in terms of quantities known as cross sections which is defined in the following way (1). Consider a thin target of area (a) and thickness (X) containing (N) atoms per unit volume, placed in a uniform mono-directional beam of incident particles (neutrons for example) of intensity $I_0$, which strikes the entire target normal to its surface as shown in fig.(1). It is found that the rate at which interactions occur within the target is proportional to the beam intensity and to the atom density, area and thickness of the target.

Summarizing this experimental result by an equation, we define the interaction rate in the entire target as $\sigma = \sigma / I N \alpha X$ ----[1]

Where the proportionality constant $\sigma$ is known as the cross section,

Thus $\sigma = \text{interaction rate} / I N \alpha X$ ----[2]

As $N \alpha X$ is equal to the total number of atoms in the target, it follows that $\sigma$ is the interaction rate per atom in the target per unit intensity of the incident beam (2).

Reciprocity theory

If the cross-sections of the reaction $A(\alpha,n)B$ are measured as functions of $T\alpha$ (Kinetic energy of α-particle) the cross-sections of the inverse reaction $B(n,\alpha)A$ can be calculated as a function of $Tn$ (Kinetic energy of neutron) using the reciprocity theorem (3) which states that:

$$\frac{\sigma_{\alpha,n}}{g_{\alpha,n} \lambda_{\alpha}^2} = \frac{\sigma_{n,\alpha}}{g_{n,\alpha} \lambda_{n}^2}$$

Where $\sigma_{\alpha,n}$ and $\sigma_{n,\alpha}$ represent cross-sections of (α,n) and (n,α) reactions respectively, $g$ is a statistical factor and $\lambda$ is the de–Broglie wave length divided by 2π and is given by

$$\lambda = \frac{h}{Mv}$$

Where $h$ is adirac constant ($h / 2\pi$), $h$ is aplanck constant, $M$ and $v$ are mass and velocity of α or n particle.

From eq.[4], we have
\[ \chi^2 = \frac{h^2}{2 \, MT} \quad \text{[5]} \]

The statistical g-factors are given by (3)

\[ g_{\alpha,n} = \frac{2J_c + 1}{(2I_{\alpha} + 1)(2I_a + 1)} \quad \text{[6]} \]

And

\[ g_{n,\alpha} = \frac{2J_c + 1}{(2I_B + 1)(2I_n + 1)} \quad \text{[7]} \]

The conservation law of the momentum implies that:

\[ I_A + I_\alpha = J_c = I_B + I_n \quad \text{[8]} \]

And

\[ \pi_A \cdot \pi_\alpha (-1)^{I_\alpha} = \pi_c = \pi_B \cdot \pi_n (-1)^{I_n} \quad \text{[9]} \]

\( J_c \) and \( \pi_c \) are total angular momentum and parity of the compound nucleus.

\( I_A \) and \( \pi_A \) are total angular momentum and parity of nucleus A.

\( I_B \) and \( \pi_B \) are total angular momentum and parity of nucleus B.

\( I_n \) and \( \pi_n \) are total angular momentum and parity of \( \alpha \)-particle.

\( I_{\alpha} \) and \( \pi_{\alpha} \) are total angular momentum and parity of neutron.

\[ \pi_\alpha = \pi_n = +1 \quad \text{[10]} \]

\[ I_\alpha = s_\alpha + \ell_\alpha \quad \text{[11]} \]

Where \( I_\alpha \) is the total angular momentum of alpha particle

\[ s_\alpha \] is spin of \( \alpha \)-particle = 0

\[ \ell_\alpha \] is the orbital angular momentum of \( \alpha \)-particle

And

\[ I_n = s_n + \ell_n \quad \text{[12]} \]

Where \( I_n \) is the total angular momentum of the neutron

\[ s_n \] is spin of neutron = 1/2

\[ \ell_n \] is the orbital angular momentum of neutron

From eq.[1-8], we have:

\[ | J_c - I_A | \leq I_\alpha \leq | J_c + I_A | \quad \text{[13]} \]

And

\[ | J_c - I_B | \leq I_n \leq | J_c + I_B | \quad \text{[14]} \]

The reactions \( A(\alpha, n)B \) and \( B(n, \alpha) \) can be represented with the compound nucleus \( C \) as in the following schematic diagram. It is clear that there are some important and useful relations between the kinetic energies of the neutron and alpha particle. One can calculate the separation energies of \( \alpha \)-particle \( (S_\alpha) \) and neutron \( (S_n) \) using the following relations:

\[ S_\alpha \] and \( S_n \) are separation energies of \( \alpha \) and \( n \) from \( C \). Then

\[ E = S_\alpha + \frac{M_A}{M_A + M_\alpha} T_\alpha \quad \text{[15a]} \]

\[ E = S_n + \frac{M_B}{M_B + M_n} T_n \quad \text{[15b]} \]

With

\[ S_\alpha = 931.5 \left[ M_A + M_\alpha - M_c \right] \quad \text{[16]} \]
Combining [15a], [15b], [16] and [17]

\[ S_n = 931.5 \left[ M_B + M_n - M_c \right] \quad \text{[17]} \]

Schematic diagram of the reactions

and as the Q-value of the reaction \( A(\alpha, n)B \) is given by:
\[ Q = 931.5 \left[ M_A + M_\alpha - M_B - M_n \right] \quad \text{[18]} \]

Then
\[ Q = \frac{M_B}{M_B + M_n} T_n - \frac{M_A}{M_A + M_\alpha} T_\alpha \quad \text{[19]} \]

Or:
\[ T_n = \frac{M_B + M_n}{M_B} \left[ \frac{M_A}{M_A + M_\alpha} T_\alpha + Q \right] \quad \text{[20]} \]

The threshold energy \( E_{th} \) is given by:

\[ E_{th} = - \frac{M_A + M_\alpha}{M_B} Q \quad \text{[21a]} \]

Or
\[ Q = - \frac{M_A}{M_A + M_\alpha} E_{th} \quad \text{[21b]} \]

Then
\[ T_n = \frac{M_B + M_n}{M_B} \frac{M_A}{M_A + M_\alpha} (T_\alpha - E_{th}) \quad \text{[22]} \]

Thus eq. [1-3] can be written as follows:
\[ \sigma_{(n,\alpha)} = \frac{g_{n,\alpha} M_\alpha T_\alpha}{g_{\alpha,n} M_n T_n} \quad \sigma_{(\alpha,n)} \quad \text{[23]} \]
It is clear from this equation that the cross sections of reverse reaction are related by a variable parameters which can be calculated if the nuclear characteristics of the reactions are known.

**Previous studies**

The only reported direct measurement (4) of the $^{22}\text{Na}(n,\alpha)^{19}\text{F}$ cross section is at thermal neutron energies. In (1973) measurements of $^{19}\text{F}(\alpha,n)^{22}\text{Na}$ cross section have been made by Balakrishnan et al. (5) and Van der Zwan and Geiger (6). Balakrishnan used paraffin-moderated $4\pi$ detector to measure the cross section between 2.6 MeV and 5.1 MeV while van der Zwan and Geiger used a stilbene crystal to measure the $0^\circ$ cross section from threshold to (4.7 MeV). Earlier efforts including those by Ehehlt et al.(7) who measured the $^{19}\text{F}(\alpha,n)^{22}\text{Na}$ cross section near the neutron threshold, Freeman and Mani (8) and Williamson et al.(9) measured the $^{19}\text{F}(\alpha,n)^{22}\text{Na}$ excitation function from 3.05 to 4.9 MeV. In California Institute of Technology P. R. Wrean and R. W. Kavanagh reported total cross section for $^{19}\text{F}(\alpha,n)^{22}\text{Na}$ reactions from threshold to 3.1 MeV, respectively. The absolute efficiency of the $4\pi$ neutron detector was determined by Monte Carlo calculations and validated by using a standard source for a nuclear reaction. Cross section for the inverse reactions (between ground states) was calculated by using the principle of detailed balance, and reaction rates for the reactions and these inverses are determined for temperatures between 0.01 and 10GK (10).

**Results and discussion**

The cross section of $(\alpha,n)$ reactions for the elements $^{19}\text{F}$ and $^{22}\text{Na}$ available in the literature, was taken and re-plotted for a defined energy level as shown in Fig.(2). These plots were analyzed by using the Matlab computer program to obtain the cross sections for the selected energies. The cross sections of $^{22}\text{Na}(\alpha,\alpha)^{19}\text{F}$ reaction are measured and declared by JEFF-3.0(11) and ADL-3.0(12) were taken and re-plotted as shown in Fig.(3) and Fig.(4). Fig. (5) gives the cross sections $(\alpha,n)$ of the p.work from ADL-3.0 with JENDL -2005 (13) for $^{22}\text{Na}(\alpha,\alpha)^{19}\text{F}$ reaction as a function of neutron energy with thresholds of 2.3638 MeV.

**References**

12. The ADL-3T library of 20 000 activation cross sections from Russia (2003).
Fig. (1) A schematic diagram illustrating the definition of total cross section in terms of the reduction of intensity(1).

Fig. (2) The cross sections of the reaction $^{19}$F($\alpha$,n)$^{22}$Na as given by JENDL library-2005 (13)

Fig. (3) The cross sections of the reaction $^{22}$Na(n,$\alpha$)$^{19}$F as given by JEFF-3.0 library (11)

Fig. (4) The cross sections of the reaction $^{22}$Na(n,$\alpha$)$^{19}$F as given by ADL-3.0 library (12)

Fig. (5) The cross sections of the reaction $^{19}$F($\alpha$,n)$^{22}$Na
Data 1 : P. Work
Data 2 : from JENDL-2005