

## High Resolution Direction-of-Arrival Estimation Using Genetic Algorithm

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### Abstract

This paper presents an application of a performance analysis of a genetic algorithm GA developed for extraction of the directions of arrival DOA of several signals impinging on uniform linear arrays. The first part of this paper describes the maximum likelihood ML technique of direction of arrival estimation with genetic algorithm. The second part presents some illustrative simulation cases of ML-DOA estimation by using GA. Results are statistically analyzed in order to conclude from it the algorithm's accuracy and reliability.

**Keywords:** Direction of Arrival (DOA); Genetic Algorithm (GA).

### استخدام الخوارزمية الوراثية في التخمين الدقيق لاتجاه الإشارات الواصلة

#### الخلاصة

يتناول هذا البحث تطبيق الخوارزمية الوراثية المطورة لاستخراج اتجاهات الوصول لإشارات متعددة ساقطة على مصفوفة الهوائيات المنتظمة. يصف الجزء الأول من هذا البحث تقنية الإمكانية القصوى من تحديد اتجاه الوصول مع الخوارزمية الوراثية. يعرض الجزء الثاني بعض حالات المحاكاة الموضحة من تقنية الإمكانية القصوى من تحديد اتجاه الوصول باستخدام الخوارزمية الوراثية. تم تحليل النتائج بشكل إحصائي لكي تستنتج منها الدقة والوثوقية.

### Introduction

In recent years, source localization using array signal processing has captured the attention of the research community because of its important applications in radar, sonar, wireless systems, seismology, and so on. A basic source localization problem is that of Direction of Arrival (DOA) estimation for narrowband signals. One of many techniques has already been proposed for the DOA estimation of narrowband sources using an array of

sensors [1,2]. This technique is called Maximum Likelihood (ML).

ML is a nearly optimal technique. In theory, it gives a superior performance compared to other methods, providing asymptotically unbiased and efficient estimates, especially in the threshold region. However, the likelihood function is complex, multimodal, multivariate and highly nonlinear, thus it is very difficult to be solved efficiently. The difficulty in optimizing the likelihood function prevents the ML technique from popular use [3].

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In this paper, an approach is proposed. A Genetic Algorithm (GA) is used for estimating the DOA of single and two narrowband signals impinging on an antenna array. GAs have become very popular for design purposes, due to their robustness and efficiency in problems where an optimal solution should be searched in a vast search space [4].

**Direction Of Arrival**

In signal modeling, we assume that a plane wave signal  $s(t)$  at time  $t$  is propagating in a medium at speed  $c$  in the direction  $\theta$ . An array of  $L$  sensors (antennas) is linearly present in the medium as shown in Fig. (1). Each sensor is assumed to record the acoustic field at its spatial position with perfect fidelity. The waveform measured at the  $m^{th}$  sensor is denoted by  $x_m(t)$  and is given by [5]

$$x_m(t) = s(t + kd \sin \theta) + n_m(t) \tag{1}$$

where

$$k = \frac{2\pi}{\lambda}, \lambda \text{ being the wavelength}$$

$d = \frac{\lambda}{2}$ , the distance between any two sensors

$n_m(t)$ , additive noise due to medium

One of the oldest ideas in array processing for determining the angle of the source  $\theta$  is the "beam-forming technique". Here the outputs are summed with weights  $W_m$  and delays  $\tau_m$  to form a beam  $y(t)$  [5]

$$y(t) = \sum_{m=1}^L W_m x_m(t - \tau_m) \tag{2}$$

Further insight into beam-forming is gained by considering these expressions in frequency domain. Defining  $X_m(f)$  to be the continuous-

time Fourier Transform of  $x_m(t)$ , the Fourier Transform of  $y(t)$  is [5]

$$Y(f, \theta) = \sum_{m=1}^L W_m X_m(f) e^{-j2\pi f \tau_m} \tag{3}$$

$$Y(f, \theta) = \sum_{m=1}^L W_m X_m(f) e^{-jkdsin\theta}$$

The energy in the beam as a function of angle  $\theta$  is computed by

$$E(\theta) = \int_{-\infty}^{\infty} |Y(f)|^2 df \tag{4}$$

Assume that all the signal energy is concentrated at a particular frequency  $f_o$ . So the power spectral density (or equivalently the beam energy) is given by

$$E(\theta)_{f=f_o} = |Y(f_o, \theta)|^2 \tag{5}$$

where  $\lambda_o = c/f_o$ . Defining the steering vector

$A = [A_1, A_2, \dots, A_L]^T$  where  $A_m$  (for  $m = 1, 2, \dots, L$ ) is

$$A_m = W_m^* e^{+jkdsin\theta} \tag{6}$$

Thus, Eq (3) can be written in matrix form as

$$Y(f_o, \theta) = \sum_{m=1}^L A_m X_m(f) \tag{7}$$

$$= [A_1^*, A_2^*, \dots, A_L^*] \begin{bmatrix} X_1(f_o) \\ X_2(f_o) \\ X_3(f_o) \\ \vdots \\ X_L(f_o) \end{bmatrix} \tag{8}$$

That is, in matrix formulation,

$$Y(\theta) = A^H X \tag{9}$$

Note the following about notation:

$A_m^*$  denotes the conjugate of  $A_m$

$A^T$  denotes the transpose of  $A$

$A^H$  denotes the Hermitian transpose of A

Thus,

$$A = [A1, A2, \dots, AL]^T$$

$$X = [X1, X2, \dots, XL]^T \text{ at frequency } f_c.$$

Matrix  $E$  which represents the ideal plane wave propagating in the direction of  $\theta$  is defined as

$$E = [E_1, E_2, \dots, E_L] \dots(10)$$

where  $E_m$  for  $m = 1, 2, \dots, L$  is given by

$$E_m = e^{-jk_m d \sin \theta} \dots(11)$$

As assumed previously,  $k1 = k2 = \dots = k_m = \dots k_L = k = 2\pi/\lambda$ . With this, we can write Eq (1) as

$$X = ES + N \dots(12)$$

where  $N = [N1, N2, \dots, NL]^T$  represents the noise vector. The noise is assumed to be statistically independent of the signal. The energy in the beam is given by [5]

$$\begin{aligned} P(\theta) &= \varepsilon[Y^2(\theta)] \\ &= \varepsilon[(A^H X)^2] \dots(13) \\ &= \varepsilon[A^H X X^H A] \\ &= A^H R A \end{aligned}$$

where  $\varepsilon$  is the expectation operator and

$$R = \varepsilon[X X^H] \dots(14)$$

$R$  is called the “spatial correlation matrix” of the sensor outputs.

In this method the estimate is derived by finding the steering vector  $A$  which minimizes the beam energy  $A^H R A$  (see Eq 13) subject to the constraint  $A^H E = 1$ . The purpose of the constraint is to fix the processing

gain for each direction of look to unity. Minimizing the resulting beam energy reduces the contributions to this energy from noise not propagating in the direction of look. We thus minimize [5]

$$F = A^H R A + \alpha(A^H E - 1) \dots(15)$$

When the gradients of A and  $A^H$  are evaluated, they are found to be complex conjugates of each other. Setting one of them to zero results in the solution

$$A = -\alpha R^{-1} E / 2 \dots(16)$$

The quantity  $\alpha$  is determined from the constraint  $A^H E = 1$ . Hence,

$$A = R^{-1} E (E^H R^{-1} E)^{-1} \dots(17)$$

Thus, the power spectrum in the beam is, from Eq (13) and Eq (17). It is given by

$$\begin{aligned} P(\theta) &= A^H R A \\ &= (E^H R^{-1} E)^{-1} \dots(18) \end{aligned}$$

As expected, the peaks of  $P(\theta)$  correspond to the direction of arrival of the given signal.

The following is a summary of the ML technique for the case single and two emitter sources: [5]

1. Collects the data samples X.
2. Estimates the correlation matrix R from Eq (14).
3. Estimates the number of signals (given).
4. Evaluates  $P(\theta)$  from Eq (18).

### Genetic Algorithm

A genetic algorithm (GA) is a directed random search technique, which can find the global optimal solution in multidimensional search space. It is one of a relatively new

class of stochastic search algorithms. Stochastic algorithms are those that use probability to help guide their search [6].

In this section, the GA used to estimate the signal parameters is described. In the following flowchart, we are going to define the terms used throughout the paper. A population is an array of individuals. Each individual is a complete solution to the problem consisting of a number of chromosomes, representing the set of signal parameters. Each chromosome is an array of genes, which are indices into the allele array (the set of alleles, the possible values that each parameter can take) [7].

A simplified flow chart of the GA applied for signal parameter estimation is shown in Fig.(2). Data is obtained measuring the complex responses of the antenna array elements.

The first step of the GA is to generate an initial population of solutions (individuals) containing signal parameter sets (DOA, normalized amplitude and relative phase). In absence of any hint, the parameters are chosen randomly following uniform distribution. A large initial population contains a more representative sample of the search space, which enhances the possibility of some of the initial generation individuals being near the global solution. However, the larger the number of individuals, the greater becomes the computational load and the slower the algorithm convergence.

The next step is to evaluate each individual's fitness and rank the population in declining fitness order.

This procedure enables the GA to resolve the total generation fitness (or cost function) and pass each solution's fitness information on to the central part of the algorithm.

The core of the Genetic Algorithm consists of two genetic operators: crossover and mutation. Crossover is the method of exchanging genetic information between two individuals (parents) in order to form another pair of individuals (offsprings) and aims at recombining the qualities of the parents and producing a higher quality pair for the next generation. This operator randomly chooses a locus (a position on each chromosome) and exchanges the subsequences before and after it.

Each chromosome is separated on a different locus in order to maximize the recombination probabilities. However, the separation takes place in a single locus (single-point crossover). In several GA implementations crossover can be applied on two or more loci [8]. This is useful in problems where gene order is crucial (i.e. a Travelling Salesman Problem), as there is a need to preserve robust sets of genes amid a chromosome. In the signal parameter estimation problem, the order of genes is not important (signals are not ordered), so for reasons of simplicity, single point crossover is chosen with crossover rate ( $p_c = 0.45$ ).

Since crossover is the main mechanism for improving the overall population fitness, it is critical to adopt an efficient way to choose the individuals that will give birth to the new generation. This is a problem of

defining the individual's fertility, the probability of producing offsprings. In most cases fertility is determined using the fitness of each individual as a proportion of the cumulative generation fitness. This strategy, called roulette-wheel selection [8], favors the highest fit individuals, guiding the GA to focus and converge faster to a solution. Since it discourages the research of the entire search space, it is not advisable to apply on the first generations, where a less discriminatory approach should be used. In the present implementation of the GA, all individuals have the same probability of producing offsprings until the entire population becomes highly correlated and the selection strategy changes to roulette-wheel selection in order to ensure faster convergence, the better value of selection rate about 50% ( $X_{rate} = 0.5$ ).

Another aspect of generation updating is the population refreshes percentage, namely the number of offsprings that will be produced as a fraction of the entire population. For static cases, where the population size doesn't change, the less fit individuals are replaced by the new breed while a number of the old generation fittest individuals are preserved (generation overlapping). This ensures that the state of the old generation can only be improved.

As long as only the crossover operator alters the generations, the population tends to become uniform, converging to a fitness maximum that is not certain to be the global maximum. In order to diversify the population and force the algorithm to

search in previously unexplored areas of the search space, there is a need to introduce mutation, an operator that randomly alters the population state. Each individual sustains mutation in one or multiple loci with a specific probability, referenced as mutation probability. Since the alleles are constant values, mutation performs an alteration by multiplying the old allele by the randomly generated mutation factor ( $P_m = 0.05$ ). The mutation factor follows uniform distribution bounded by two values symmetric to unity. A wide value range enables the algorithm to escape from a local fitness maximum, but may also disorientate the individuals being near the global maximum stalling or even preventing convergence. Preserving the fittest individuals can protect each generation's maximum fitness. The group of individuals preserved, the elite group, can be defined as a portion of the whole population, the elite percentage. If the elite group is small, it is likely to leave unprotected medium-fit individuals that are nearer the global maximum than the fittest ones, but when the elite group is large, the mutation factor is applied on a small number of less fit individuals and affects the efficiency of the mutation operation. Therefore, a trade-off between the aforesaid considerations should be made in order to choose the problem-specific elite percentage.

The algorithm execution comes to an end when the number of generations exceeds a predefined maximum ( $Gen_{max} = 100$ ), or the cost function falls below the anticipated threshold. These quantities should be

carefully chosen to avoid premature termination or unproductive continuation after reaching a satisfactory solution [7].

### Results

In this paragraph, the performance of the GA in DOA estimation is examined. There are several cases simulated using software developed to function under Matlab @ 7.4 environment. For each case, the GA is executed 100 times ( $Gen_{max}$ ) in order to conclude on the accuracy and reliability of the results. The receiver antennas are supposed to be uniform linear arrays. Different cases examine the effect a number of sensor and distance (d) between them on performance of the DOA.

For single emitter source, a simulated narrow-band emitter is used with number of sensors ( $L=10$ ), number of snapshots ( $N=100$ ) and suppose the emitter signal with 5dB additive noise is impinging on sensors array from azimuth of  $50^\circ$  degree.

The ML searches for the peak is illustrated in Fig.(3). From the graph, it is easily shown that only one emitter signal is available due to the only peak in the spectrums. Search for the peak of emitter signal has shown that the estimated DOA is  $50^\circ$  degree in azimuth, when the ML is implemented by using GA; it gave the result for one emitter in direction  $50^\circ$  degree is  $50.6892^\circ$  degree.

For two emitter sources with coherent and non-coherent scenes (ML technique works successfully both coherent and non-coherent sources), with the assumptions that two emitter sources with 5dB additive noise is impinging on a sensor array

from azimuth of  $50^\circ$  and  $100^\circ$  degrees. The ML searches for the peak is illustrated in Fig.(4), from the graph, it is easily shown that two emitter signal is available due to the two peaks in the spectrums. When the ML is implemented by using GA, it gave the result for two emitter sources in direction  $50^\circ$  and  $100^\circ$  degrees is ( $50.7633^\circ$ ) and ( $100.6281^\circ$ ) degrees.

For two closely spaced emitter sources, with the assumptions that two emitter sources with 5dB additive noise is impinging on a sensor array from azimuth of  $60^\circ$  and  $65^\circ$  degrees. The ML searches for the peak is illustrated in Fig. (5), from the graph, it can easily be noticed that there is only one peak in ML spectrum. The failure in the ML technique is due to the few sensors (L) used in this simulation. When the ML is applied with GA, it gives the result for two emitter sources in direction  $60^\circ$  and  $65^\circ$  degrees is ( $60.7713^\circ$ ) and ( $64.3979^\circ$ ) degrees.

As the number of sensors (L) is increased to 25 and 50, the ML technique will succeeds in limiting two peaks as shown in Fig.(6) and Fig.(7).

### Conclusions

Many conclusions can be derived in this paper; the most important results can be summarized as follows:

1. The GA succeeds in giving reliable results of DOA-ML without the need of any spectral search for DOA angles.

2. The ML-GA technique can handle both coherent (the signals are coherent when the signals copies arrive at the receiving sensors at the same time with the original signal and

these signals have the same wavelength) and non-coherent sources (the signals are non-coherent when the incident signals on sensors array arrive at different times and these signals have different wavelengths).

3. The efficiency of the ML-GA technique increases with the increasing number of sensors.

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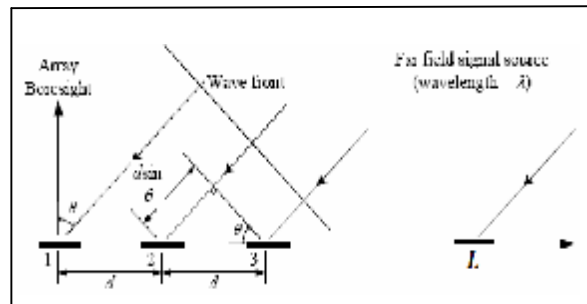


Fig. (1) Geometry of uniform linear array

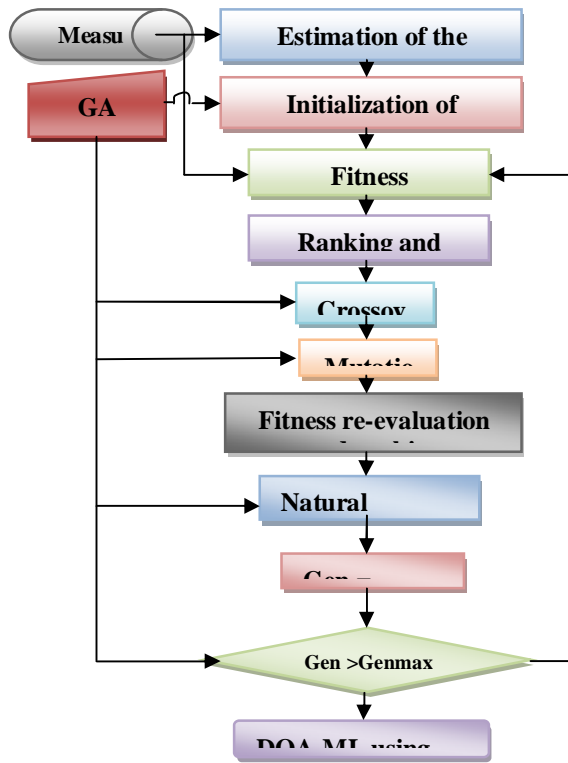


Fig. (2) Flow chart of DOA-ML using GA.

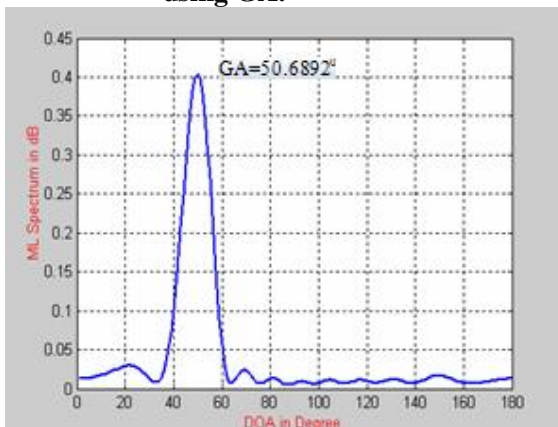


Fig.(3) DOA-ML for single emitter source

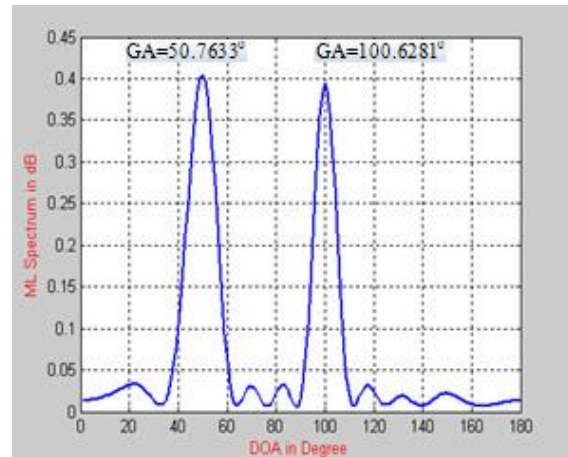


Fig.(4) DOA-ML for two emitter sources.

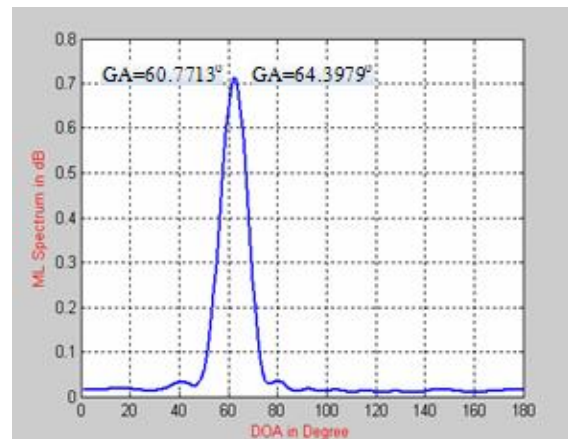
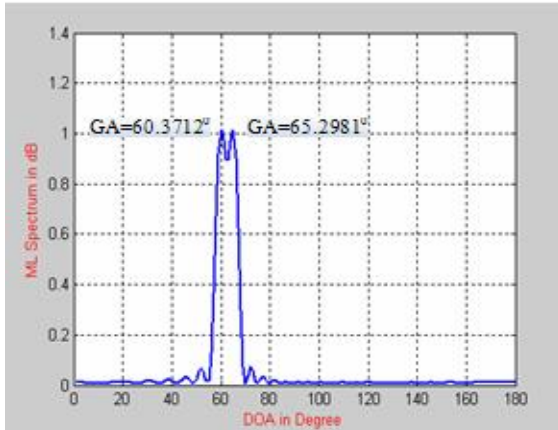
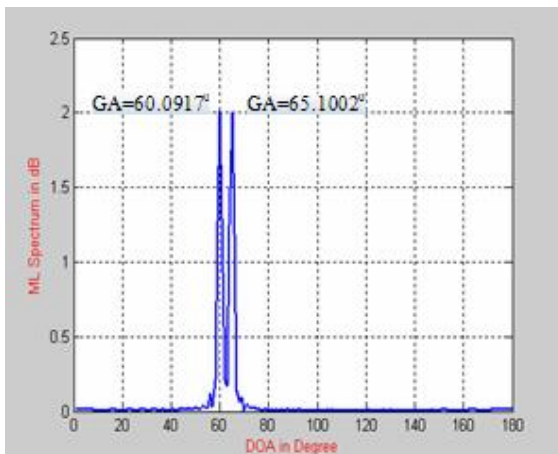


Fig.(5) DOA-ML for two closely spaced emitter sources





**Fig.(6) DOA-ML for two closely spaced emitter sources with L=25.**



**Fig.(7) DOA-ML for two closely spaced emitter sources with L=50.**